QUANtoons

Metaphysical Illustrations by Tomas Bunk

Physical Explanations by Arthur Eisenkraft and Larry D. Kirkpatrick

NSTApress
National Science Teachers Association
Arlington, Virginia
Introduction .................................................................................. vii
A snail that moves like light ...................................................... 2
The leaky pendulum ................................................................. 6
What goes up … ...................................................................... 10
The clamshell mirrors ............................................................. 14
Shake, rattle, and roll ............................................................. 18
Sources, sinks, and gaussian spheres ................................. 22
The tip of the iceberg ............................................................... 26
A topless roller coaster ......................................................... 30
Row, row, row your boat ......................................................... 34
How about a date? ................................................................. 38
Animal magnetism ................................................................. 42
Atwood's marvelous machines ........................................... 46
Thrills by design .................................................................... 50
Electricity in the air ............................................................... 54
Stop on red, go on green … .................................................. 58
Fun with liquid nitrogen ......................................................... 62
Laser levitation ................................................................. 66
Mirror full of water ............................................................. 70
Rising star ........................................................................... 74
Superconducting magnet .................................................... 78
Cloud formulations ............................................................ 84
Weighing an astronaut .......................................................... 88
The first photon ................................................................. 92
Pins and spin ...................................................................... 96
Split image ........................................................................... 100
Gravitational redshift .......................................................... 104
Focusing fields................................................................. 108
<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea sounds</td>
<td>112</td>
</tr>
<tr>
<td>Moving matter</td>
<td>116</td>
</tr>
<tr>
<td>Boing, boing, boing ...</td>
<td>120</td>
</tr>
<tr>
<td>The bombs bursting in air</td>
<td>124</td>
</tr>
<tr>
<td>The nature of light</td>
<td>128</td>
</tr>
<tr>
<td>Do you promise not to tell?</td>
<td>132</td>
</tr>
<tr>
<td>Mars or bust!</td>
<td>136</td>
</tr>
<tr>
<td>Color creation</td>
<td>140</td>
</tr>
<tr>
<td>A physics soufflé</td>
<td>144</td>
</tr>
<tr>
<td>Cool vibrations</td>
<td>148</td>
</tr>
<tr>
<td>Elephant ears</td>
<td>152</td>
</tr>
<tr>
<td>Local fields forever</td>
<td>156</td>
</tr>
<tr>
<td>Around and around she goes</td>
<td>160</td>
</tr>
<tr>
<td>Depth of knowledge</td>
<td>164</td>
</tr>
<tr>
<td>Doppler beats</td>
<td>168</td>
</tr>
<tr>
<td>Up, up and away</td>
<td>172</td>
</tr>
<tr>
<td>Warp speed</td>
<td>176</td>
</tr>
<tr>
<td>Sportin’ life</td>
<td>180</td>
</tr>
<tr>
<td>Elevator physics</td>
<td>184</td>
</tr>
<tr>
<td>The eyes have it</td>
<td>188</td>
</tr>
<tr>
<td>Image charge</td>
<td>192</td>
</tr>
<tr>
<td>Breaking up is hard to do</td>
<td>196</td>
</tr>
<tr>
<td>A question of complexity</td>
<td>202</td>
</tr>
<tr>
<td>Tunnel trouble</td>
<td>206</td>
</tr>
<tr>
<td>Magnetic vee</td>
<td>210</td>
</tr>
<tr>
<td>Rolling wheels</td>
<td>214</td>
</tr>
<tr>
<td>Batteries and bulbs</td>
<td>220</td>
</tr>
<tr>
<td>Curved reality</td>
<td>226</td>
</tr>
<tr>
<td>Relativistic conservation laws</td>
<td>232</td>
</tr>
<tr>
<td>A good theory</td>
<td>236</td>
</tr>
<tr>
<td>The fundamental particles</td>
<td>240</td>
</tr>
</tbody>
</table>
Q UANTOONS BRINGS PHYSICS to you through masterful illustrations, quotes, text and challenging problems. The simple classic physics problem of crossing a raging river and determining where you land on the other shore turns into a metaphor of traversing the river of life from birth to death in a cartoon illustration filled with humor and poignancy. The Heraclitus quote “You can never step into the same river twice” adds another aspect of appreciation to the physics story of relative motion, movement in two dimensions, and calculations of least time.

The colors in a rainbow, the sounds of rustling leaves, and the splattering of waves crashing into rocks communicate to us through our senses and our imaginations. Physics can add to our appreciation and understanding of these natural occurrences if the insights of physics are made accessible.

Quantoons is a compilation of Contest Problems that were published in Quantum magazine during its 11-year run [1990–2001]. Quantum was a collaborative effort of the United States and Russia. Published by the National Science Teachers Association (NSTA), this semi-monthly magazine was targeted at students and teachers and anyone else interested in science and mathematics. Borrowing from original notes and articles in the Russian publication Kvant, Quantum added articles and problems from American authors. The Contest Problem was one such addition. In every issue, a physics problem was presented and interested readers were invited to submit solutions. The best of these solutions were acknowledged in a subsequent issue, and often used as the basis for the published solution. The Contest Problems were also intended to be descriptions of physics enhanced by the creative cartoons that accompanied them.

In the first issue of Quantum, Bill Aldridge quotes the great Russian scientist and poet Mikhail Lomonosov as he viewed the Northern Lights: “Nature, where are your laws? The dawn appears from the dark northern climes! Does not the sun there set up its throne? Are not the ice-bound seas emitting fire?” For 11 years, Quantum attempted to answer some of these questions while illuminating the minds of so many who will one day provide us with other glimpses into the wonder of the universe.

Bill Thurston, in that same first issue, reflected on the beginning of his illustrious career as a mathematician. “As a child, I often hated arithmetic and mathematics in school. Pages of exercises were tedious and dull … I stared out the window and let my mind wander. Sometimes I tried to puzzle something out …. Might the square root of 2 eventually be periodic if you write it out in base 12 instead of base 10? How many ways are there to fold a map into sixteenths, in quarters each way?” This spirit of inquiry pervaded the many issues of Quantum and stimulated readers to in-
vent their own questions and allow their minds to explore.

The history of Quantum is worth recounting briefly. Arthur Eisenkraft was first introduced to the Russian publication, Kvant, by his friend Sergey Krotov of the former Soviet Union during their years as Academic Directors for their respective Physics Olympiad teams. The problems, articles, and humor in Kvant seemed like something that could be imported and massaged for United States audiences. Lots of interested people stepped up to the plate. Bill Aldridge, then Executive Director of NSTA, led the charge to create a magazine of “the highest quality.” Bill came through on his commitment. He enlisted the help of Sheldon Glashow, a Nobel Laureate in physics, William Thurston, a Fields medalist in mathematics, and Yuri Ossipyan, vice-president of the Academy of Sciences of the USSR, to launch the magazine. Edward Lozansky served as an international consultant and Tim Weber took on the responsibility of managing editor. NSTA, under Bill Aldridge’s leadership, with financial support from the National Science Foundation (NSF), committed resources to insuring that Quantum met the needs of our intended audience. He also brought the American Association of Physics Teachers (AAPT) and the National Council of Teachers of Mathematics (NCTM) on board. Larry Kirkpatrick and Mark Saul became the field editors for physics and mathematics, respectively.

Arthur Eisenkraft and Larry Kirkpatrick collaborated from the beginning on the physics Contest Problems. The physics was great, as were the literary quotes accompanying each, but better illustrations were needed. Tomas Bunk, a professional cartoonist with credits including MAD magazine and Garbage Pail Kids, was approached. His first reaction was “But I don’t know physics.” Arthur responded, “Well, that might be an asset. The next article is about light. How would you draw light?” Tomas replied, “I guess like a super-hero because it travels so fast.” And so the collaboration began. Of course, as Tomas learned enough physics to illustrate that first picture, Arthur learned about art. The illustration had light at the circus moving through hoops, ricocheting off mirrors, and darting through a water-filled aquarium. Arthur loved the sketch but the light did not always obey the laws of physics. What to do? Tomas explained that to shift the path of light would require a new illustration because the balance and structure of the art would be off in this portrayal. Arthur decided it would be better to state below the illustration, “Light is misbehaving in this picture. Can you find where?” The cartoon became another dimension to the Contest Problem. As the Contest Problems evolved, so did Tomas’s illustrations. They began to take on political commentary, historical ideas, and larger issues of philosophy while always providing insights into the physics with whimsy and humor.

Quantoons adds a feature that was not present in the original Quantum series. Each illustration now has a brief commentary by Tomas Bunk. This peek at the creative mind of a visual artist not only provides insight into how Tomas views the world, but also how people who are not trained in physics can appreciate the world of science and make it their own.

Some readers will move right to the cartoons while others will begin with the quotes. These readers may then be curious enough to check out the physics text that introduces the topic. Still others will be intrigued by the complexity of some of the physics problems, be tempted to invent a solution, and then may reward themselves with an investigation of the illustration. Our hope is that you will find your own, personal way to enjoy Quantoons and better appreciate the world we share.

—Arthur Eisenkraft and Larry D. Kirkpatrick
QUANTOONS
The behavior of light, which is now known as Snell’s law:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2, \]

where \( n_1 \) and \( n_2 \) are the indices of refraction. We can see that if the light enters water (\( n = 1.33 \)) from air (\( n = 1.00 \)) at an angle of 30°, the angle in water would be 22°:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2, \]
\[ 1.00 \sin 30° = 1.33 \sin \theta_2, \]
\[ \theta_2 = 22°. \]

Measuring the angle of refraction is one way to tell whether that’s a diamond or a piece of glass in that ring you bought.

What fascinates many people about the study of physics is the alternative ways of explaining phenomena. The great mathematician Pierre de Fermat recognized (in 1657) that the path of light is the path that requires the least time.¹ If you try all possible paths from the light source \( A \) to the object \( B \) after they hit the mirror, you’ll find that the shortest path, and so the quickest, is the path through point \( D \) (fig. 1 on the next page), where the angle of incidence equals the angle of reflection.

¹The “extremum path.”
Light is bending the rules a bit here. (Can you see where?)
You can demonstrate this for yourself by drawing lots of paths and measuring them. You can also prove it with some simple geometry or by using some calculus.

Fermat’s theorem is also valid for refraction: the path light takes when it passes from air to water must be the path requiring the least time. In this case least time is not identical to least distance, since light travels more slowly in water that in air. The speed of light in a substance is equal to the speed of light in a vacuum divided by the substance’s index of refraction \( n \).

Proving that the path of the light is the quickest one takes some ingenuity. You can draw lots of paths of light traveling from point \( A \) in air to point \( B \) in water (fig. 2). You can then measure the lengths of the lines in air and water. But Fermat’s theorem states that the path should take the least \( \text{time} \), not the least \( \text{distance} \). We can multiply the lengths in water by 1.33, since the light takes longer to travel in water by a factor of 1.33. Then add this distance to the distance in air. The path that minimizes this sum is the path the light takes. And—guess what? It’s the same path described by Snell’s law! Those of you who have some calculus background can prove it mathematically.

Leaving light behind, we enter the world of slow-moving mollusks to find our contest problem. A snail must get from one corner of a room (dimensions 5 m × 10 m × 15 m) to the diagonally opposite corner in the least time. The snail can walk on any of the four walls but may not walk on the floor or ceiling. What is the path that the snail should take? In part B of the contest problem, for our more advanced readers, the snail finds that the 15 meter wall that must be traveled is sticky—that is, the snail can only travel at a fraction of its normal speed. If the snail on the sticky wall travels at 1/3 of its normal speed, what is the path that requires the least time for the snail? Finally, in part C, for our most advanced readers, what happens if the snail finds that the stickiness of the first wall is not constant but increases linearly along one dimension of the wall? Specifically, the speed at one end of the wall is the normal speed and the speed at the far end of the wall is 1/3 the normal speed. What will be the path of least time? You may need to use graphical or computer techniques to solve parts B and C. Our best readers are encouraged to see if they can find general proofs for any room (dimensions \( l \times w \times h \)) and a stickiness factor of \( s \). We are not sure ourselves if such general proofs exist.

**Solution**

You were asked to help a snail find the quickest path from one corner of a room to a diagonally opposite corner.

In the first case, in which all walls were identical and the dimensions of the room were 5 × 10 × 15, there are at least three ways to solve the problem. The first is to choose different crossover points at the edge between the two walls and calculate the total distance that the snail travels. This numerical method may appear to be tedious, but it will actually converge on the correct solution quickly. A second method is to call the height of the crossover point \( x \), write the total distance traveled in terms of \( x \), and differentiate. By setting the derivative equal to zero, the minimum distance will be revealed as the solution to the equation. The third method is the elegant solution. In this case, the wall is opened up. The room is now a large rectangle of dimensions 25 × 5. The shortest distance will be the diagonal connecting the two corners of the rectangle. If the snail starts at the lower corner of the 15-meter wall, the crossover point can be found by using similar triangles. The crossover point is...
\[
\frac{x}{15} = \frac{5-x}{10}, \\
x = 3.
\]

In the second case, one of the walls was declared “sticky,” meaning that the snail could travel at only 1/3 of its speed on this wall. Unlike the first case, the shortest distance is no longer the shortest time! Since the snail travels at different speeds on the two walls, the quickest path will be the one where the snail travels a greater distance on the faster wall. Once again, the straightforward but tedious solution would be to assign the variable \(x\) to the crossover point, write an equation that describes all paths in terms of \(x\), and the minimum time will be revealed.

The more elegant solution in this case is to realize that light always takes the least time to travel, and that this snail traveling on a sticky wall is like light traveling in a slower medium. We then recognize that the solution will be Snell’s law (or, if you’ll forgive us, “Snail’s law”). Even with this knowledge, we are faced with a fourth-order equation, which we choose to solve by numerical techniques:

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2.
\]

Since the stickiness factor is 3, then \(n_1 = 3\) and \(n_2 = 1\), and it follows that

\[
3 \cdot \frac{x}{\sqrt{15^2 + x^2}} = \frac{5-x}{\sqrt{(5-x)^2 + 10^2}}.
\]

We’ll try different values of \(x\) and see if the value of the left side of the equation is equal to the value of the right side.

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<th>(x)</th>
<th>(left\ side)</th>
<th>(right\ side)</th>
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<td>2</td>
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<tr>
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<td>1.6</td>
<td>0.3182</td>
<td>0.3219</td>
</tr>
<tr>
<td>1.63</td>
<td>0.3241</td>
<td>0.3194</td>
</tr>
<tr>
<td>1.62</td>
<td>0.3221</td>
<td>0.3202</td>
</tr>
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This method can give us any accuracy we desire. It would certainly be easier to plug the equations into a spreadsheet program and have all values given “instantly.”

The third part of the problem, to solve for a wall whose stickiness varies along one dimension, was solved by Jason Jacobs of Harvard University. We will leave this problem as a tease.
ONE PERSON DESCRIBED how the bedroom wall moved across the room. Another watched as a huge wave of concrete traveled along the highway. We all saw the massive destruction when one bridge roadway collapsed on top of another. The earthquake in the San Francisco area that coincided with the 1989 World Series gave us a glimpse of the power and energy in our planet.

In the fury of the destruction, the Earth is whispering secrets about its composition. The Earth is not solid rock. The Earth is not of uniform density. Longitudinal and transverse waves, called $P$ and $S$ waves, travel through the Earth as a result of the quake. The differences in $P$ and $S$ wave behaviors can give us clues about the structure of the Earth while also allowing us to locate the epicenter of the quake.

Although the speeds of the $P$ and $S$ waves vary within the Earth, the $P$ waves always travel faster than the $S$ waves. This fact gives us the ability to locate the epicenter of the quake. By knowing the relative speeds of the $P$ and $S$ waves and measuring the delay in the arrival of the $S$ waves, we can determine the distance from the epicenter. Here's an analogy. If you run at 3 m/s and a friend walks at 1 m/s, you will always arrive at a given location before your friend. If you arrive 10 seconds earlier, the distance traveled was 15 meters. If you arrive 20 seconds earlier, the distance traveled was 30 meters.

Let's assume that the epicenter is near the Earth's surface and that the $P$ and $S$ waves have constant but unequal velocities. If at one location on the Earth the waves arrive with a time difference of 2 minutes, we know that the epicenter of the quake must be situated a specified distance from this location. But in which direction? We don't know. We therefore trace the circumference of a circle on the surface with

**Shake, rattle, and roll**

“She stood in silence, listening to the voices of the ground . . .”
—William Blake, “The Book of Thel”
a radius specified by this time delay. The epicenter can be located on any part of this circumference. If we have a second location with a (different) time delay, this will provide us with a second circle. A third location and a third circle will uniquely determine the actual point location of the epicenter.

The \( P \) waves are able to travel through solids and liquids, while the \( S \) waves travel only through solids. The \( P \) waves arrive at locations on the opposite side of the Earth; the \( S \) waves do not. This information leads us to conclude that a portion of the interior of the Earth is liquid. By carefully observing where the \( P \) waves travel and where the \( S \) waves do not, we can infer more about the size of this liquid core of the Earth.

More curious is the observation that there are positions on the Earth where neither the \( P \) nor the \( S \) waves arrive. These shadow zones are somehow protected from disturbances at some locations. What could cause such a shadow region? One explanation is that the \( P \) waves travel at a different speed within the liquid core. A \( P \) wave traveling from a solid mantle into a liquid core will change speeds and change direction (that is, they will refract). The result of this refraction is the creation of the shadow region.

Professor Cyril Isenberg, academic leader of the British Physics Olympiad Team, challenged students worldwide in the 1986 International Physics Olympiad with a problem about the \( P \) and \( S \) waves of an earthquake. We present parts of that problem as a challenge to our readers.

Let\’s assume that the Earth is composed of a central liquid spherical core of radius \( R \) that is surrounded by a solid, homogeneous mantle of radius \( R \). The velocities of the \( S \) and \( P \) waves through the mantle are \( v_s \) and \( v_p \) respectively. An earthquake occurs at point \( E \) on the surface of the Earth and produces \( P \) and \( S \) seismic waves. A seismologist observes the waves at location \( X \). The angular separation between \( E \) and \( X \) measured from the center of the Earth \( O \) is \( \theta \), as shown in figure 1.

A. Our beginning physics students should try to show that the seismic waves that travel through the mantle in a straight line arrive at \( X \) at a time \( t \) (the travel time after the earthquake) given by

\[
t = 2R \sin \theta / v,
\]

where \( v = v_s \) for the \( P \) waves and \( v = v_p \) for the \( S \) waves.

B. After an earthquake an observer measures the time delay between the arrival of the \( S \) wave and the \( P \) wave as 2 minutes, 11 seconds. Deduce the angular separation of the earthquake from the observer using these data:

- \( R = 6,370 \) km
- \( R_s = 3,470 \) km
- \( v_p = 10.85 \) km/s
- \( v_s = 6.31 \) km/s

C. The observer in part B notices that at some time after the arrival of the \( P \) and \( S \) waves, there are two further recordings on the seismometer separated by a time interval of 6 minutes, 37 seconds. Explain this result and verify that it is indeed associated with the angular separation determined in part B.

D. For those of you who wish to plunge deeper, draw the path of a seismic \( P \) wave that arrives at an observer, where \( \theta \leq \arccos (R_s / R) \), after two refractions at the mantle–core interface. Obtain a relation for \( P \) waves between \( \theta \) and \( i \), the angle of incidence of the seismic \( P \) wave at the mantle–core interface.

E. For our advanced problem solvers, using the data above and the additional fact that the speed of the \( P \) waves in the liquid core is 9.02 km/s, draw a graph of \( \theta \) versus \( i \). Comment on the physical consequences of the form of this graph for observers stationed at different points on the Earth\’s surface.

F. Sketch the variation of the travel time taken by the \( P \) and \( S \) waves as a function of \( \theta \) for \( 0 \leq \theta \leq 90 \) degrees.

**Solution**

A. In part A, you were asked to calculate the time it would take for \( P \) or \( S \) waves emanating from the earthquake location \( E \) to reach an observation point \( X \). From figure 2 we can see that

\[
EX = 2R \sin \theta.
\]

Therefore,

\[
t = \frac{2R \sin \theta}{v},
\]

where \( v = v_s \) for \( P \) waves and \( v = v_p \) for \( S \) waves. This is valid provided that \( X \) is at an angular separation less than or equal to \( X' \), defined by the tangential ray to the liquid core. From figure 2, \( X' \) has an angular separation given by

\[
2\phi = 2 \cos^{-1} \left( \frac{R_s}{R} \right).
\]

B. Given the delay time between \( P \) and \( S \) waves, you were next asked to deduce the angular separation of \( E \) and \( X \). Using the result from part A,

\[
t = \frac{2R \sin \theta}{v},
\]

**Figure 2**
we can express the time delay as
\[ \Delta t = 2R \sin \theta \left( \frac{1}{v_s} - \frac{1}{v_p} \right). \]  
(1)

Substituting the data given, we get
\[ 131 = 2 \left( 6370 \left( \frac{1}{6.31} - \frac{1}{10.85} \right) \right) \sin \theta. \]

Therefore, the angular separation of \( E \) and \( X \) is
\[ \theta = 17.84^\circ. \]

This result is less than
\[ 10 \cos \theta - \theta = 90 - \theta = 90 - 17.84 = 72.16^\circ, \]
and consequently the seismic wave is not refracted through the core.

C. If a second set of \( P \) and \( S \) waves had a longer delay, readers first had to hypothesize an explanation for the second set of delayed waves and then see if the result is consistent with the time delay given in part B.

The observations are most likely due to reflections from the mantle-core interface. Using the symbols in figure 3, we can express the time delay
\[ \Delta \tau' \] as
\[ \Delta \tau' = (ED + DX) \left( \frac{1}{v_s} - \frac{1}{v_p} \right). \]

In triangle \( EYD \),
\[ (ED)^2 = (R \sin \theta)^2 + (R \cos \theta - R_p)^2 = R^2 + R_p^2 - 2RR_u \cos \theta, \]

and since \( \sin^2 \theta + \cos^2 \theta = 1 \). Therefore,
\[ \Delta \tau' = 2R^2 + R_p^2 - 2RR_u \cos \theta \left( \frac{1}{v_s} - \frac{1}{v_p} \right). \]

Using equation (1), we get
\[ \Delta \tau' = \frac{\Delta \tau \sqrt{R^2 + R_p^2 - 2RR_u \cos \theta}}{R \sin \theta} \]
\[ = 396.7 \text{ s} = 6 \text{ min } 37 \text{ s}. \]

Thus, the subsequent interval produced by reflection of seismic waves at the mantle-core interface is consistent with an angular separation of 17.84°.

D. Since a \( P \) wave is able to travel through the core, you were asked to draw the path of the refracted \( P \) waves and derive the relation between the angle of incidence and the angular separation of \( E \) and \( X \).

From figure 4, we get
\[ \theta = \angle AOC + \angle EOA \]
\[ = (90 - r) + (i - \alpha). \]  
(2)

The law of refraction [Snell’s law] gives
\[ \frac{\sin i}{\sin r} = \frac{v_p}{v_p}. \]  
(3)

From triangle \( EAO \) and the law of sines, we get
\[ \frac{R_\alpha}{\sin \alpha} = \frac{R}{\sin i}. \]  
(4)

Substituting equations (3) and (4) into (2) yields
\[ \theta = 90 - \sin^{-1} \left( \frac{v_p}{v_p} \right) \sin i \]
\[ + 1 - \sin^{-1} \left( \frac{R}{R} \right) \sin i. \]  
(5)

E. Our most talented readers were then asked to draw a graph of the relationship expressed in part D and to comment on the physical consequences.

Substituting \( i = 0^\circ \) into equation (5) gives \( \theta = 90^\circ \); \( i = 90^\circ \) gives \( \theta = 90.8^\circ \). Substituting numerical values for \( i = 0^\circ \) to \( i = 90^\circ \), one finds a minimum value at 55° and the corresponding minimum value of \( \theta : \theta_{\text{min}} = 75.8^\circ \) (fig. 5). As \( \theta \) has a minimum value of 75.8°, observers at positions for which \( 2\theta < 151.6^\circ \) will not observe the earthquake as seismic waves. For \( 2\theta < 114^\circ \), however, the direct, nonrefracted waves will reach the observer.

F. Finally, readers were asked to sketch a comparison of the travel times for \( P \) and \( S \) waves for all angles. In this sketch (fig. 6) we can get a better sense of the “shadow” region where no earthquake waves will be observed.
WHY DO ELEPHANTS HAVE such big ears? And why do they have such thick legs? In other words, why do elephants have different shapes than horses? These questions and more can be answered using the laws of scaling that we learn in physics.

Elephant bones are made from the same basic material as human bones. Therefore, the bones must be thicker to support the extra mass of the elephant. But how much thicker? Let’s compare an elephant to a horse. A typical horse has a mass around 600 kg and a typical elephant has a mass around 4200 kg, or some 7 times larger. Because all mammals have a density near that of water, the elephant must have 7 times the volume of the horse. If we assume that the two have the same shape (they both have four legs!), the linear dimensions of the elephant must be $\sqrt[3]{7} \approx 1.9$ times the corresponding dimensions of the horse.

Each elephant leg must support 7 times as much weight as a horse leg. Because the compression strength of a beam depends on its cross-sectional area, an elephant leg bone must have 7 times the cross-sectional area of a horse leg bone. In other words, the elephant leg bone must have 2.6 times the diameter of a horse leg bone. Notice that the elephant and the horse cannot have the same shape; the legs must be proportionately larger than the other dimensions. The comparison would be even more dramatic if we compared the elephant to a mouse!

This explains why elephants have such thick legs, but what about the ears? Let’s assume for the moment that an elephant eats 7 times as much as a horse because it has 7 times as much mass. As this food is used by the body, it generates heat. Therefore, an elephant must dissipate 7 times as much heat as a horse. We know that the thermal loss is proportional to the difference in the temperature across the skin and to the area of the skin. The surface area of any solid depends on the square of its linear dimensions, so the elephant only has $1.9^2 = 3.6$ times the surface area. This means that the

Elephant ears

“Sir Isaac Newton was very much smaller than a hippopotamus, but we do not on that account value him less.”
—Bertrand Russell (1872–1970)

Tonight at the circus an elephant is being measured to find out if it makes sense to be so big or to be different from a horse or piglet, for example. A handful of specially appointed clowns are investigating with scientific unseriousness his ears, legs, memory, blood pressure, weight, height, inside mechanism that makes him tick, and the important capacity of his trunk to hold water. The results will be collected, may be analyzed, but will definitely and thoroughly be ignored. Even Einstein and Galileo stepped down from the heights of their scientific status to join the fun of circus life, performing daring stunts on the tightrope while a mouse tries to get hold of a horse on the flying trapeze. Not an easy task compared to the little piglet dancing on top of the world.

—T.B.
elephant must have a much higher body temperature or some other way of getting rid of the thermal energy. This is one of the roles of the big ears. They increase the surface area and, by moving in the air, keep the air temperature near the skin from climbing very much. The elephant also eats less per unit mass than a horse.

It is interesting to note that although elephants communicate by ultrasound, it is not necessary for them to have big ears for this purpose.

Not all scaling deals with lengths. We can use any factor as a scaling parameter. For instance, the Bohr radius for the hydrogen atom is given by

$$a_0 = \frac{h^2}{mke^2} = 0.0529 \text{ nm},$$

where $\hbar$ is Planck’s constant divided by $2\pi$, $m$ is the mass of the electron, $k$ is Coulomb’s constant, and $e$ is the electronic charge. What would be the new radius if the electron were replaced by a muon with a mass 207 times as large? (We assume that the mass of the proton is large compared to the mass of the muon.) We do not need to solve for the new radius from scratch; all we need to know is that the radius scales inversely with mass. Therefore, the radius of the muonic hydrogen atom is

$$a_\mu = a_0 \left( \frac{m_e}{m_\mu} \right) = \frac{a_0}{207} = 0.256 \text{ pm}.$$ 

This was one of a series of five problems on scaling that made up one of the three theory questions at the International Physics Olympiad held in Sudbury, Canada, in July 1997. The theory problems were developed under the direction of Chris Waltham, who is a faculty member at the University of British Columbia. Three of the other scaling problems make up the problem we offer below.

A. The mean temperature on the Earth is $T = 287$ K. What would the new mean temperature $T'$ be if the mean distance between the Earth and the Sun were reduced by 1%?

B. On a given day, the air is dry and has a density $\rho = 1.2500$ kg/m$^3$. The next day the humidity has increased and the air contains 2% water vapor by mass. The pressure and temperature are the same as the day before. What is the new air density $\rho'$? Assume ideal gas behavior. The mean molecular weight of dry air is 28.8 g/mol, and the molecular weight of water is 18 g/mol.

C. A type of helicopter can hover if the mechanical power output of its engine is $P$. If another helicopter is produced that is an exact half-scale replica (in all linear dimensions) of the first, what mechanical power $P'$ is required for it to hover?

### Solution

A correct solution to the first question was submitted by Tal Carmon from Technion, Israel.

A. The first question asked what would happen to the mean temperature $T$ of Earth if the mean distance $R$ between Earth and the Sun decreased by 1%. To do this we match the input radiation to the output radiation because Earth is in thermal equilibrium. If the power output of the Sun is $P$, the radiation reaching Earth per unit area is $P/4\pi R^2$. If we denote Earth’s radius by $R_E$ and its reflectance by $r$, the input power $P_{\text{in}}$ to Earth is

$$P_{\text{in}} = (1 - r) \frac{P}{4\pi R_E^2} \pi R^2.$$ 

Stefan’s Law gives the output power

$$P_{\text{out}} = 4\pi R_E^2 \varepsilon \sigma T^4,$$

where $\varepsilon$ is the Earth’s emissivity and $\sigma$ is Stefan’s constant. Although the emissivity is a function of temperature, the change in temperature is expected to be small, and we can neglect this dependence. Therefore,

$$T \propto \sqrt{\frac{1}{R}}$$

and a reduction of 1% in $R$ gives a 0.5% rise in $T$. For a mean temperature of 287 K, we get a rise of 1.4 K.

B. The second question asked about the change in the density of dry air with an increase in humidity when the temperature and pressure remain the same. Let’s use the subscripts $d$ for dry and $m$ for moist air, respectively. Then the number of molecules $N_d$ in the dry air is

$$N_d \approx \frac{M_d}{28.8},$$

where $M_d$ is the mass of dry air in a unit volume and the mean molecular mass of dry air is 28.8 g/mol. For moist air, we must account for the proportions of dry air and water vapor. For 2% humidity, we have

$$N_m \approx 0.02 \frac{M_m}{18} + 0.98 \frac{M_m}{28.8},$$

where the mean molecular mass of water is 18 g/mol.

We know that identical volumes of ideal gases with the same temperature and pressure have the same number of molecules. Therefore,

$$M_d = 1.012 M_m.$$ 

Because the densities of equal volumes are proportional to the respective masses,

$$\frac{\rho_m}{\rho_d} = \frac{M_m}{M_d} = 0.988,$$

and using $\rho_d = 1.25$ kg/m$^3$, we get our answer:

$$\rho_m = 1.235 \text{ kg/m}^3.$$ 

C. The last question asked how the power required for a helicopter to hover depends on the size of the helicopter. The mechanical power $P$ of the helicopter is equal to the thrust $T$ times the downward velocity component $v$ of the air below the blades. The thrust is given by the change in momentum of the air per unit time

$$T = v \frac{dm}{dt},$$

with

$$\frac{dm}{dt} = \rho Av,$$
where $\rho$ is the density of the air and $A$ is the cross-sectional area covered by the blades. Thus,

$$T = \rho A v^2.$$ 

When the helicopter is hovering, the thrust must be equal to the helicopter’s weight. Therefore,

$$v^2 = \frac{T}{\rho A} = \frac{W}{\rho A}.$$ 

If the size of the helicopter is characterized by a linear dimension $L$, then $W \propto L^3$, $A \propto L^2$, and $v \propto L^{0.5}$. Thus,

$$P = TV = Wv \propto L^{3.5}.$$ 

For a half-scale helicopter, the required power is $0.5^{3.5}P = 0.0884P$. 
