QUANTUM

SEPTEMBER/OCTOBER 1996

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GALLERY Q

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Right and Left (1909) by Winslow Homer

YOUR IMMEDIATE RESPONSE TO THIS PAINTING IS probably to wonder at the bizarre postures of the two ducks. Only after careful inspection do we notice the red flash in the background, the boat, the hunter. Then the drama becomes clear, and the scene takes on a chilling poignancy.

Why "Right and Left"? It's more customary in English to name the directions in the reverse order (perhaps because that's the direction of our written language—from left to right). Winslow Homer seems to be emphasizing the fact that we're observing the scene from the "other" direction. Just as we are often compelled to say, "The person on my left," or "The chair on your right," Homer says: "Here's how it looks from *their* point of view."

Physicists often deal with shifting frames of reference. Sometimes the transition is easy and intuitive, sometimes not. Several articles in this issue will test your ability to keep your bearings in a world where "everything is relative." You might want to begin with the Kaleidoscope, which is devoted entirely to the subject of relativity.

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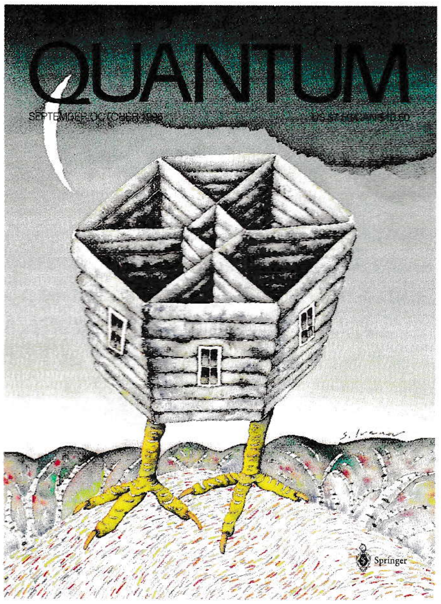

Cover art by Sergey Ivanov

As if a hut on chicken's legs weren't magical enough—something uncanny has happened to Baba Yaga's home. It used to be an ordinary square structure. Then the walls took on a life of their own, and to what end? They thought the most famous witch in all of Russian fairyland deserved something better than a run-of-the-mill, three-dimensional home. They wanted to give their mistress a *multidimensional* hut!

Several articles in this issue are devoted to the multidimensional cube and its applications. Begin your tour on page 4, where you will catch a glimpse of what inspired the fanciful treatment on our cover.

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DEPARTMENTS

PUBLISHER'S PAGE

Solar calculator

The easy precision of microelectronics can be too much of a good thing!

As ELECTRONIC CALCULATORS have become commonplace and so inexpensive that they are sometimes given away as an advertising promotion, their use has become extensive. Where I once prided myself on the speed with which I could add a column of numbers without use of a calculator, I now find myself resorting to a calculator to be reassured that my checkbook balance is correct. But there is a problem with the use of such calculators, one more serious than just my own loss of a skill.

Calculators are available that express numbers to ten, twelve, or even sixteen places. If ever there were an important distinction between mathematics and the engineering and sciences, it is with the use of these decimal places.

Mathematics doesn't care about precision or accuracy. It is concerned only with correct logic. Precision and accuracy is the domain of measurement. The theory of measurement is mathematical, but its application is the real world of science and engineering. You probably recall that *accuracy* is how well a given measurement instrument compares with a standard. If it's a meter stick, how closely do the marks on the meter stick match the marks on a platinum-iridium secondary standard meter at standard temperature? *Precision* is concerned with the fineness of the subdivisions on the instrument, or how narrow the distribution of a set of measurements is when made by the same instrument. A measurement can be very precise, but terribly inaccurate, as would be the case for a meter stick actually constructed to be 120 cm long, instead of 100 cm long.

Now, what has this to do with decimal places? When you make a measurement, each decimal place that you show in the result represents *ten times* more precision! Those extra digits are very difficult to come by, and often they reveal much more science as we try to improve the precision.

This was made apparent to me most recently when I tried to compute the apparent diameter of the Sun for every day of the year. As you probably know, the Earth–Sun distance changes only very slightly over the year. The Sun is closest to Earth on about January 1. (Many people erroneously believe that the Sun is closest in the summer and wrongly attribute the seasons to this change in distance. But actually the seasons are a consequence of the orientation of the Earth relative to the Sun, due to Earth's $23\frac{1}{2}$ degree tilt in its rotation axis.)

As a first approximation, the Earth's orbit around the Sun can be treated as a circle. So, as long as you want only one or two significant figures, you can make this assumption. You can then use the following values: $2 \cdot 10^{30}$ kg for the Sun's mass; $1.4 \cdot 10^{9}$ m for the Sun's diameter; $1.5 \cdot 10^{11}$m for its distance from Earth; and $6.7 \cdot 10^{-11}$kg-m^3/kg-s^2 for the value of G from the Law of Universal Gravitation. These quantities can be used to determine the length of the year; or, alternatively, given the length of the year, two of these quantities can be used to find the mass of the Sun. That is how planetary masses are determined ($M = 4\pi^2R^3/T^2$, where R is the orbital radius and T is the period of an orbiting satellite, like the moons of a planet). Under this assumption, it's easy to figure out the apparent size of the Sun. The image size is just $d = A(D/R)$, where A is the distance from aperture to image, D is the diameter of the Sun, and R is the distance from Earth to the Sun. Alternatively, using the image size and known distance to the Sun, you can calculate the Sun's diameter. In *this* approximation, the Sun's apparent diameter stays the same all year. But we know that, to two significant figures, the apparent diameter does *not* stay the same.

When you examine images of the Sun carefully, you find that, in fact, its apparent diameter does change over the course of a year. At apogee it's about 97% of its apparent diameter at perigee.

Earth does not travel around the Sun in a circle. It travels in an ellipse, whose general equation is given by $1/R = C[1 + e \cos(\theta - \theta_0)]$, where C and θ_0 are constants associated with the conic section involved and its characteristics, and e is the eccentricity, which for an ellipse has a value of less than one. When I made my calculations, I wanted to be as precise as possible, so I sought precise values for the various quantities needed. I used the best value for the eccentricity—namely, $e = 0.0167044$ (Julian date

8280.5). Then I started by using the apogee position as my starting point for the motion (setting the angle equal to zero and getting the constant C in terms of a and cos (θ_0), where a is the semimajor axis of Earth's orbit (which is also equal to 1 AU). Then I could look at perigee, where the angle was equal to π, to evaluate θ_0 as equal to $-\pi$. I could also evaluate C as being $1/a(1-e^2)$. The resulting equation

$$R = a\frac{1-e^2}{1+\cos(\theta-\pi)}$$

was then used in a computer program to determine R and then the apparent size of the Sun for each day of the year. The best values for these quantities that I could locate are as follows: $M = 1.9891 \cdot 10^{30}$ kg, $D = 1.393 \cdot 10^9$ m, $a = 1.4959787066 \cdot 10^{11}$m, and $G = 6.67259 \cdot 10^{-11}$kg-m^3/kg-s^2. My result shows the apparent diameter of the Sun at apogee to be 96.714% of that at perigee.

The problem with using these numbers is that with five or six significant figures, all kinds of problems crop up. For example, Earth's eccentricity isn't constant. It changes over the years. The distance from Earth to the Sun is not the distance that should be used. Earth has a moon, and the center of mass of the Earth–Moon system is really what is moving about the Sun. Thus Earth itself wobbles somewhat, and its distance changes with that wobble. Then there are the other planets. If all or most planets were on the same side of the Sun as Earth, the center of mass of the solar system would be shifted slightly, and the Sun's center would not be the center of the orbit.

The whole point here is that decimal places on calculators are meaningless unless you know the precision of the measurement for the number being entered. I use my calculator extensively, and have no intention of giving it up. But calculators need to be used properly. Since almost no common measurements you will ever make have more than three or four significant figures, you seldom need even six decimal places except as place holders.

—Bill G. Aldridge

QUANTUM

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The multidimensional cube

An introduction and a quick tour

by Vladimir Dubrovsky

THIS YEAR MARKS THE 400th anniversary of the birth of the great French mathematician and philosopher René Descartes. One of his greatest mathematical achievements, shared with Pierre Fermat, is the foundation of analytic geometry. In the course of its development, this branch of mathematics brought mathematicians to the notion of multidimensional space, which soon became perhaps the most popular mathematical abstraction among the general public, not without various mystical and spiritualistic misinterpretations. We decided to celebrate this anniversary with a series of articles about the simplest of multidimensional objects—the cube—and its applications. Although all these articles are, in principle, self-contained, you may want to start with this one, where we build the n-dimensional cube from scratch and try to explore its geometric structure.

Step by step

The easiest way to understand the multidimensional cube is to "grow" it from the simplest of all cubes, the point (which can be viewed as a zero-dimensional cube), step by step, adding one dimension at a time.

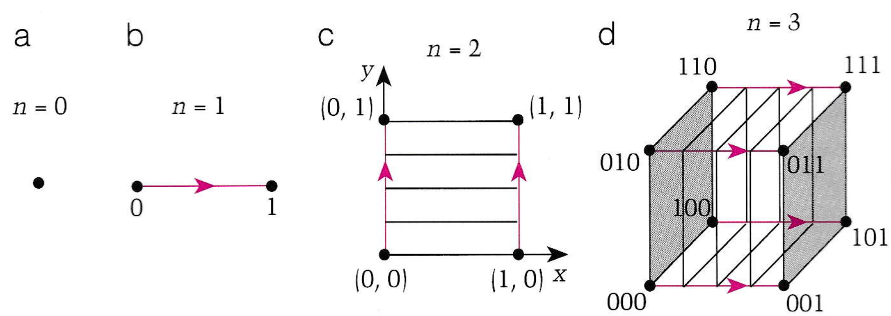

Figure 1

Let's take a point (fig. 1a) and move it a unit distance. It sweeps out a segment, or a one-dimensional cube (fig. 1b). We can think of it as the segment $0 \le x \le 1$ of the x-axis. Now let's shift the segment perpendicular to itself through a unit distance (fig. 1c). It sweeps out a square—the two-dimensional cube. We need two coordinates, x and y, to describe it: in the frame shown in figure 1c, it is given by the two pairs of inequalities $\{0 \le x \le 1, 0 \le y \le 1\}$. Shifting the square perpendicular to its plane (fig. 1d), we obtain the three-dimensional cube $\{(x, y, z): 0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1\}$.

Now let's do the next step and consider the figure traced by our three-dimensional cube when it is dragged a unit distance. It's no problem drawing this figure on the plane—see figure 2. What *is* really perplexing is to imagine that the cube is shifted "perpendicular to itself." Some people claim that they have managed to develop the ability to see this fourth dimension. Those who have not reached this degree of perfection can, as a first step, rely on the power of analogy, the step-by-step "dragging" construction, and, of course, the formal algebraic definition. Taken together, this will be quite sufficient for exploring even

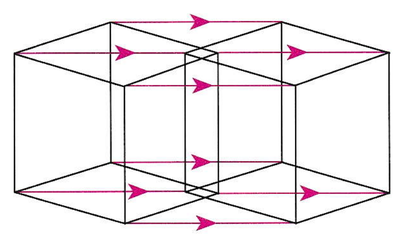

Figure 2

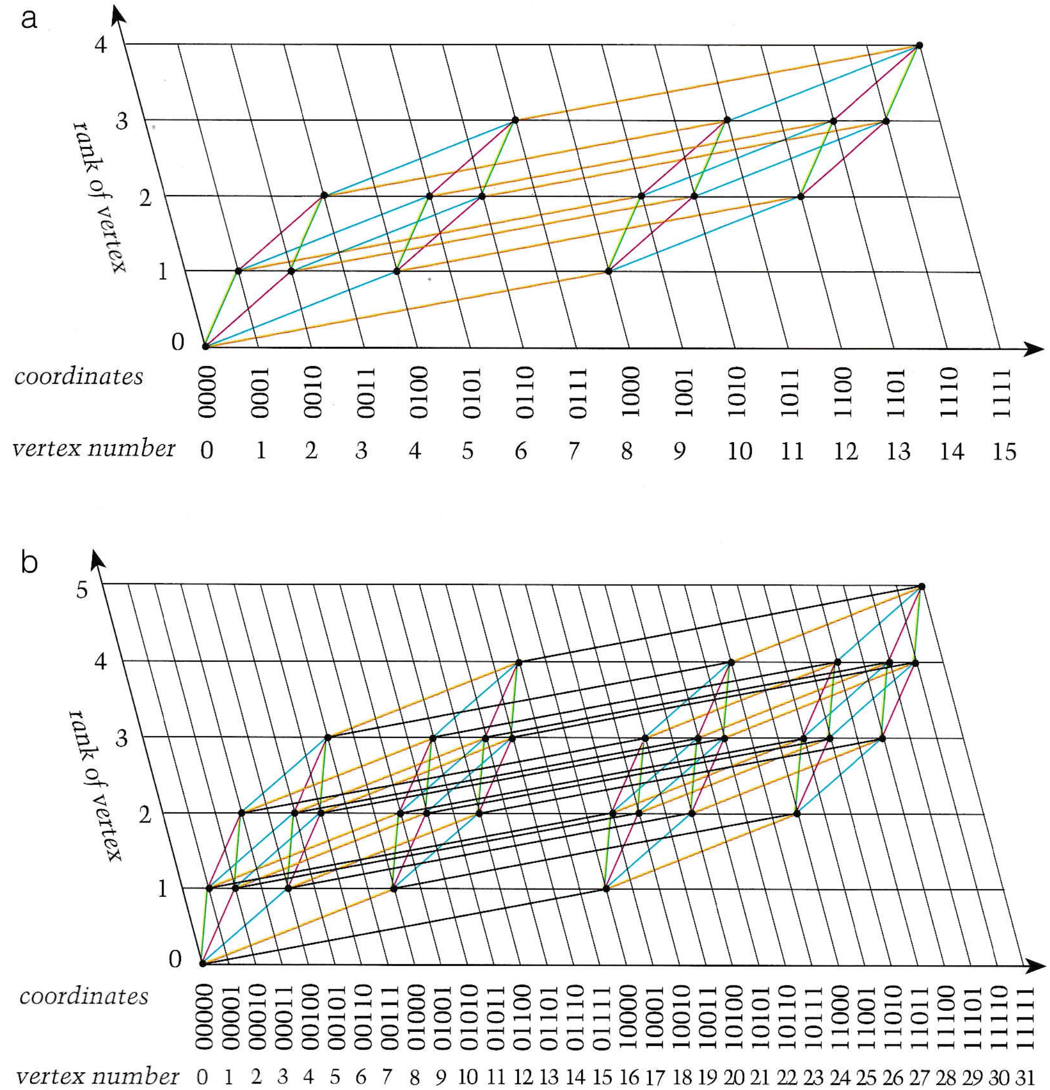

Figure 3

represents one square face of our cube in space). Then we drag the parallelogram along a third (say, blue) segment, which gives the image of an ordinary cube, and so on. This job is easy and even pleasant, because taking different directional segments we can produce diverse patterns (compare figures 2 and 3a and the symmetric portrait of the tesseract in figure 4). It's interesting that no matter how the guiding n segments are chosen (except when they lie on the same line) they produce the drawing, which is a *really possible* parallel projection of the n-cube on the plane. It correctly conveys the mutual arrangement of vertices and edges and shows which edges are parallel to one other. But the drawings in figure 3 have some additional properties and were made by a special rule.[1] Let's consider them in more detail.

such an unearthly object as the four-dimensional cube and cubes of even higher dimensions.

So, by shifting the ordinary cube we get the four-dimensional cube. (It was considered so important that it received two special names: "hypercube" and "tesseract," from the Greek *tessera*, "fours," and *aktis*, "ray of light.") The four-dimensional cube generates the five-dimensional cube, and so on.

In coordinates, the nth step of this construction amounts to appending a new, nth coordinate varying from 0 to 1 to the $n-1$ old ones. Thus the hypercube is defined as the set of number quadruples (x, y, z, u) specified by the inequalities $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$, $0 \le u \le 1$. A similar system of n double inequalities for the coordinates $x_1, x_2, \ldots, x_n$ defines the n-dimensional cube. Although strictly speaking this definition describes only one particular (unit) cube in any given coordinate frame, the generality is not lost, because for any cube we can choose a frame with respect to which it will satisfy the same inequalities. (Can you explain how?)

Performing the "dragging" process on the plane yields a portrait of the n-dimensional cube. We draw a segment from the origin (say, the green one in figure 3a or 3b), then draw another segment (say, the red one) from the same point and drag the first segment along the second to obtain a parallelogram (which

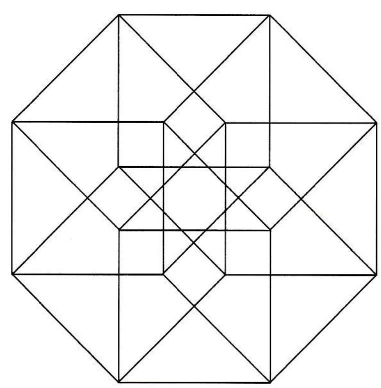

Figure 4

[1]Described by N. B. Demidovich in "How to Draw the N-dimensional Cube?" (*Kvant* 8, 1974).

Drawing and counting

From the definitions above it's clear that the coordinates $(a_1, a_2, \ldots, a_3)$ of any vertex A of the n-cube are zeros and ones. Compute two numbers: $x(A) = a_1 2^n + a_2 2^{n-1} + \ldots a_n 2^0$ (that is, the number whose binary representation is $a_1 a_2 \ldots a_n$) and $y(A) = a_1 + a_2 + \ldots + a_n$ (the number of unit coordinates, called the *rank* of vertex A). Draw an arbitrary, not necessarily rectangular, coordinate system on the plane, and for each vertex A mark the point with coordinates $(x(A), y(A))$. All these points are nodes of the integer grid (with respect to the chosen coordinates). Now join with a segment each pair of points A and B whose x-coordinates differ by a power of two ($|x(A) - x(B)| = 2^k$). I leave it to the reader to verify that the resulting diagram is indeed a drawing of the n-dimensional cube. One way to draw this cube step by step is to start with a segment connecting the origin to the point $(1, 2^0)$ and shift this segment in the direction of the segment connecting the origin to $(1, 2^1)$, then $(1, 2^2)$, then $(1, 2^3)$, and so on. The final shift is along the segment connecting the origin to $(1, 2^{n-1})$. One curious feature of this drawing is that all the vertices lie on integer lines $x = k$, $k = 0, 1, \ldots, 2^n - 1$, one on each. "Horizontally," they lie on the $n + 1$ lines $y = 0, 1, \ldots, n$, each of which contains all the vertices of the same rank. So this method of drawing comes in handy whenever the notion of rank is used, and such situations arise repeatedly in the problems about the n-dimensional cube considered in this issue. By way of example, we'll use these figures in counting the elements (vertices, edges, faces) of the n-cube.

First of all, we see that the number of vertices V_n is equal to 2^n, simply because they can be enumerated with the numbers from 0 to $2^n - 1$, as in figure 3. Or we might notice that each step of the "dragging" construction duplicates the number of vertices of our cubes: to the vertices of the initial cube we add those of its shifted copy. We can also investigate how the number of edges E_n changes at each step. To the edges of the initial cube we add their shifted copies and the edges traced by the vertices while the cube is dragged. We can express this by the formula

$$E_{n+1} = 2E_n + V_n, \qquad (1)$$

where E_n and $V_n = 2^n$ are the numbers of the edges and vertices of an n-dimensional cube.

Exercise 1. Derive a formula for E_n as a function of n.

Alternatively, imagine that we paint all the edges parallel to one another their own color as in figure 3 (for $n = 4$ and $n = 5$). There are as many colors as edges issuing from the origin, because these edges are all colored differently. At the same time, edges of *any* given color can be thought of as traced by the vertices of a cube of one dimension less when it is dragged, because the order in which the directional edges appear in our construction is irrelevant. Now, can you tell in ten seconds how many edges there are altogether?

This coloring can also be used to calculate the number of vertices of a given rank k. Starting from the origin, we can reach any of them by following a path of k edges, and all the edges we pass on the way are a different color, because the "swelling" cube in our construction is moved in a new direction at each step. The color of the first segment can be chosen in n ways, that of the second in $n - 1$ ways (one color has been used), that of the third in $n - 2$ ways, and so on. So the total number of such paths is $n(n - 1)\ldots(n - k + 1)$. To pass along a segment of a certain color means to move a unit distance along the corresponding coordinate axis— that is, to replace the corresponding zero coordinate with one. Therefore, the coordinates of the endpoint of a path depend only on the set of colors of its segments rather than on their order. So the number of paths leading from the origin to the same vertex of rank k is equal to the number of permutations of their k colors—that is, to $k! = k \cdot (k-1) \cdot \ldots \cdot 2 \cdot 1$, and the number of rank-k vertices is equal to

$$\frac{n(n-1)\ldots(n-k+1)}{k!} = \frac{n!}{(n-k)!k!},$$

which is denoted by $\binom{n}{k}$. These numbers are the well-known *binomial coefficients*. (This will come as no surprise to readers familiar with the combinatorial use of these coefficients. In counting vertices of rank k, we are choosing k out of n coordinates to have the value 1, and the rest to have the value 0.) By the way, since the total number of vertices is 2^n, we get the relation

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n.$$

The one-dimensional skeleton of the n-cube formed by its vertices and edges gives only a rough idea of its structure. The edges must be joined by two-dimensional faces, and those by three-dimensional faces, and so forth, up to the $(n - 1)$-dimensional faces that constitute its boundary. Each face is a cube of a certain dimension. Figure 5, another portrait of the hypercube, can help us understand the arrangement of its three-dimensional "hyperfaces." In fact, this figure is a two-dimensional drawing of a three-dimensional configuration—the *central* projection of the hypercube on three-dimensional space. A similar projection of the ordinary cube on the plane is shown in figure 6. In figure 5 we can also clearly see the 24 two-dimensional faces of the tesseract. But can you imagine the four-dimensional *interior* enclosed by the eight cubes (six of them in the shape of a truncated quadrilateral pyramid) seen in this figure?

In our step-by-step construction the k-dimensional faces of the n-cube appear as those of the $(n - 1)$-

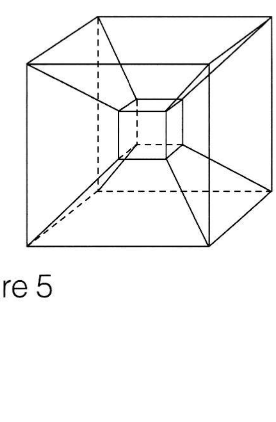

Figure 5

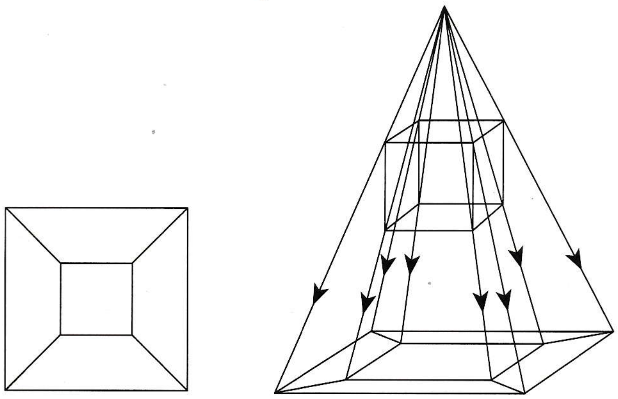

Figure 6

dimensional generating cube and its shifted copy and those swept out by its $(k-1)$-dimensional faces as it moves. This yields a recurrent formula for calculating the number $F_{n,k}$ of k-dimensional faces similar to equation (1).

Exercise 2. Show that $F_{n,k} = 2F_{n-1,k} + F_{n-1,k-1}$ for $\le k \le n-1$ with $F_{n,n} = 1$, $F_{n,0} = V_n = 2^n$. Derive a formula for $F_{n,k}$ as a function of n and k.

In coordinates, a k-dimensional face consists of points whose n-k coordinates are fixed and are each equal to 0 or 1, while the other k coordinates vary from 0 to 1. From this description, the formula for $F_{n,k}$ can be obtained directly.

Building up one's hyperintuition

It would be very difficult to imagine a multidimensional cube without diagrams like those in figure 3 or 5. But on the other hand, they are rather deceptive. For instance, looking at figure 5, can you conceive of a sphere (hypersphere!) that passes through all the vertices of the hypercube, or the spheres that touch all of its 32 edges or 24 square faces? It's even more difficult to picture the inscribed sphere that touches all the three-dimensional faces at their centers so that the faces themselves stay outside the sphere. Nevertheless, all these spheres do exist, and we can calculate their radii using the familiar definition of the (Euclidean) distance between the points $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ given by the formula $\sqrt{(x_1-y_1)^2+\cdots+(x_n-y_n)^2}$.

Let's calculate the radius of the sphere inscribed in an n-dimensional cube. For convenience, we assume that the edge length of the cube is 2, and we place the origin of our coordinates at its center. Then any coordinate of any vertex is either 1 or -1, and its distance from the center equals $\sqrt{1^2+\cdots+1^2} = \sqrt{n}$. So all the vertices lie on the sphere given by the equation $x_1^2 + x_2^2 + \ldots + x_n^2 = n$. Now take an $(n-1)$-dimensional face—say, the one given by the equation $x_1 = 1$. Its center—the point $(1, 0, \ldots, 0)$—is a unit distance from the cube's center, whereas any other of its points $(1, x_2, \ldots, x_n)$ is a greater distance away, since $\sqrt{1^2+x_2^2+\cdots+x_n^2} > 1$. This means that this face, and similarly any other face of the cube, touches the unit sphere centered at the cube's center—that is, this sphere is inscribed in the cube.

Exercise 3. For any n and k, find the diameter of the sphere touching all the k-dimensional faces of the n-dimensional unit cube.

The radius of the sphere inscribed in the unit cube is the same (1/2) in any dimension. On the other hand, the argument above, when applied to a cube of unit edge length, shows that the circumradius $\sqrt{n}/2$ grows indefinitely with the growth of the dimension n.

I want to demonstrate one paradoxical fact about the n-dimensional cube and its spheres. Take a cube of edge length 2. In each of its 2^n corners inscribe a sphere of diameter 1 (see the diagram in figure 7 for dimension 2). Any two of these spheres that are adjacent along an edge touch each other, and their centers form an n-dimensional cube with edge length 1. Now consider the sphere centered at the cube's center that touches all the corner spheres. Our intuition tell us that it must lie inside the cube. But look again: its diameter is equal to the distance between the centers of two opposite corner spheres minus their radii—that is, to $\sqrt{n} - 1$. And this is greater than 2 for $n \geq 10$. So for large enough n this sphere bulges out of the cube!

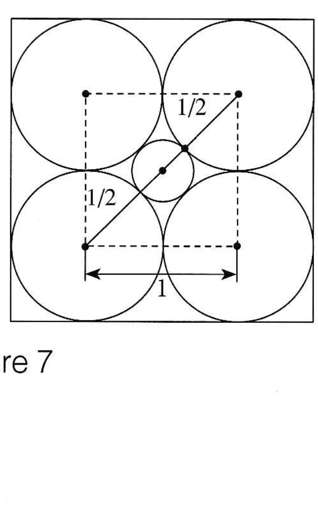

Figure 7

Our attempts to visualize the multidimensional cube are similar to what inhabitants of Flatland,[2] an imaginary two-dimensional world, would have tried to do in order to understand the structure of the ordinary cube. They could try to draw its projections on the plane, which is like the approach we used so far. But they could also try to construct and examine cross sections of the cube. To get a more detailed picture, they might have drawn a series of tomograms ("CAT scans"), so to

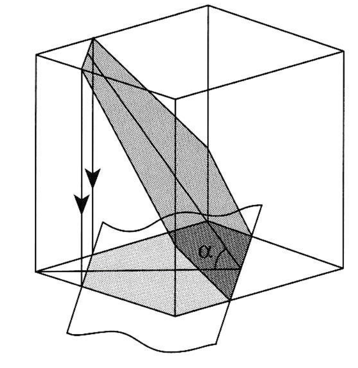

Figure 8

[2]*Flatland: A Romance of Many Dimensions* by the English scholar, theologian, and writer Edwin A. Abbott (1838–1926). The book has been reprinted by Dover Publications, Inc., and is widely available. It has also been digitized as part of Project Gutenberg. (A copy of the Gutenberg text can be downloaded from *Quantum*'s FTP server—ftp.nsta.org/pub/quantum/flat10a.txt.)—*Ed.*

speak, of the cube—that is, sections by a plane moving in a fixed direction, say, perpendicular to the cube's main (longest) diagonal. One such section is shown in figure 8. If our Flatlanders were intelligent enough, they would probably find an easy way to draw the *projections* of these sections on the cube's base: they can be obtained simply as the intersections of the base with a strip of width $w = \sqrt{2}/2$, half the diagonal of the base, perpendicular to the diagonal. We can see this immediately from the figure. The Flatlanders could derive it algebraically.

Indeed, the coordinate equation of a plane perpendicular to the cube's diagonal drawn from the origin takes the form $x + y + z = c$. The points of the section satisfy the additional inequalities $0 \le x, y, z \le 1$. The projection of the point (x, y, z) on the base—that is, on the (x, y)-plane—is simply $(x, y, 0)$. So, in (x, y)-coordinates, the projection of this section is given by the inequalities $0 \le x \le 1$, $0 \le y \le 1$, and $c - 1 \le x + y \le c$ (since $x + y = c - z$, where $0 \le z \le 1$). The first two double inequalities specify the base of the cube, while the third defines the strip.

Notice that under this projection the number of sides that a figure has does not change. Thus the Flatlanders will find out that the moving section is a growing triangle (which in this case is equilateral). At a certain moment its corners get cut off, and it gradually transforms into a regular hexagon. Then the whole process is reversed.

Like the Flatlanders, we can subject the hypercube to three-dimensional tomography, cutting it with a moving hyperplane—that is, 3-space—perpendicular to its diagonal. The sections are certain three-dimensional polyhedrons, and their projections on the hypercube's base can be found as the solids cut out from the base, which is an ordinary cube, by the layer between two parallel planes perpendicular to this cube's diagonal, 1/3 of the diagonal apart from one another (fig. 9). You can prove this using coordinates in

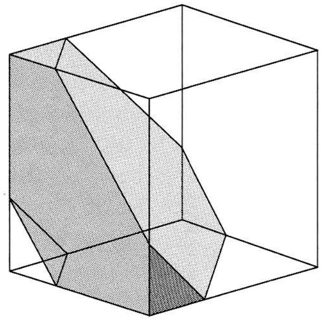

Figure 9

exactly the same way we did above. To obtain an exact copy of a section from its projection, we must stretch the latter by a factor of two perpendicular to the layer. Thus, the section starts as a point; it turns into a growing regular tetrahedron; at a certain moment its corners are cut off; and the truncated pieces keep growing until the cuts reach the midpoints of the edges of the tetrahedron. At this moment the section passes through the center of the hypercube and turns for an instant into a regular octahedron (see figure 10, which shows it shrunk by half, according to our construction). From this point on the movie is repeated in reverse.

Another way to think of our "tomograms" is to represent the (ordinary) cube as the intersection of the two trihedral angles formed by the triples of faces at its two opposite vertices. At any moment, each of these angles cuts an equilateral triangle out of the moving plane, and the section of the cube is the intersection of these triangles (fig. 11), one of them inflating, the other deflating. I leave it to the reader to look at the sections of the

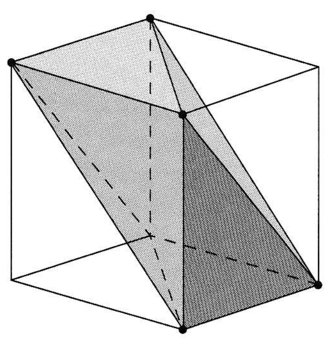

Figure 10

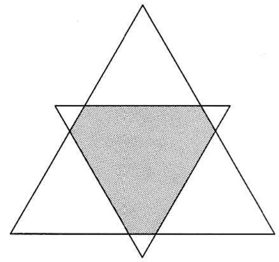

Figure 11

hypercube from this point of view. The main difference will be that the triangles will be replaced with regular tetrahedrons. Also, you may want to draw the hypercube's "tomograms" taken in other directions, or investigate what polygons can emerge when the hypercube is cut by a two-dimensional plane.

It's clear that we can apply the above considerations to cubes of an arbitrary dimension n. In particular, the cutting hyperplane can be drawn through the vertices of a given rank k (they all satisfy the linear equation $x_1 + x_2 + \ldots + x_n = k$). Thus we arrive at the following curious fact:[3] the section of the n-dimensional cube by the hyperplane drawn through its vertices of rank k "coincides" (up to contraction) with the layer of the $(n - 1)$-dimensional cube between the two sections drawn through the vertices of rank $k - 1$ and k. As a consequence of this, by counting the vertices of all these sections and comparing the results, we get the familiar formula for binomial coefficients: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

Applications

It's no wonder that the multidimensional cube appears in the recent, sensational disproof of the 60-year-old Borsuk Conjecture (which you can read about in the article by A. Skopenkov in this issue). The very wording of the problem involves n-dimensional figures. One remarkable thing about it is that the cube's vertices in the construction are identified with subsets of a finite set! This link between geometry and combinatorics proves to be helpful

[3]Observed by the Moscow mathematician D. Ryzhkov.

in some purely combinatorial problems, too.

Another reincarnation of the cube is found in information theory, where its vertices are viewed as points of the simplest binary "code space" (see, for instance, "Errorproof Coding" in the March/April 1993 issue of *Quantum*). Our calculation of the number of vertices of a certain rank is used there to estimate the greatest possible size of a "k-error correcting" code.

Some uses of the multidimensional cube are really "puzzling." In "Nesting Puzzles" (see the January/February and March/April issues) you would have encountered the remarkable sequence

121312141213121...

that solves the famous Tower of Hanoi and a number of similar puzzles. If we start at the origin and, reading this sequence digit by digit, move along the edges of the n-dimensional cube, choosing the edge parallel to the ith coordinate axis whenever the next digit in the sequence is i, we'll visit all the cube's vertices without walking the same edge twice. (Check this!) Thus we solve, for the case of the n-dimensional cube, another puzzle—one created by the outstanding Irish mathematician W. R. Hamilton as an illustration of some of his findings. It is simply this: to visit all the vertices of a given polyhedron (originally it was a dodecahedron) so as to trace any edge no more than once. Such paths are called *Hamiltonian walks*.

In conclusion, let's look at an algebraic rather than combinatorial application of the cube. We'll prove the well-known Cauchy inequality for arithmetic and geometric means. We'll do this for the case of three numbers, but it will be clear how the proof generalizes.

Let's begin with this "oldie but goodie": *cut the cube into three equal pyramids*. The solution, if you don't know it, is not so easy to find. We must take the three quadrilateral pyramids that have three of the cube's faces with a common vertex

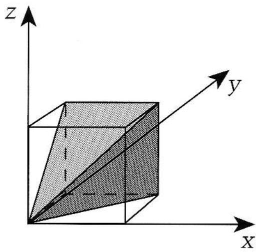

Figure 12

as their bases and the opposite vertex of the cube as their common apex. Placing the cube in the coordinate system as we have done throughout this article, we can call them x-, y-, and z-pyramids, according to which coordinate axis is perpendicular to the pyramid's base. The y-pyramid is shown in figure 12. We can see that the volume of each pyramid is 1/3 that of the cube.

Cauchy's inequality says that for any three positive numbers x, y, z, we have

$$\frac{x+y+z}{3} \geq \sqrt[3]{xyz}.$$

So let's take three positive numbers x, y, z. Without loss of generality we may assume that $x \geq y \geq z$. Then set $\sqrt[3]{x} = a$, $\sqrt[3]{y} = b$, $\sqrt[3]{z} = c$. The numbers a, b, c are also positive, and $a \geq b \geq c$. Consider a rectangular parallelepiped with edges a, b, and c, running along the x-, y-, and z-axes, respectively (fig. 13). From the origin, draw a ray along the cube's diagonal. The ray will first meet the face $z = c$ of the parallelepiped at point $C(c, c, c)$, then the extended face $y = b$ at $B(b, b, b)$, and then $x = a$ at $A(a, a, a)$. Consider the three cubes with diagonals joining the origin to the points A, B, and C. Take the

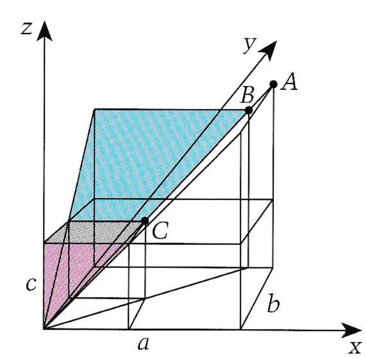

Figure 13

x-pyramid in the first of them, the y-pyramid in the second, and the z-pyramid in the third (see figure 13). These pyramids cover the parallelepiped, and their volumes are equal to $a^3/3$, $b^3/3$, and $c^3/3$. Since the volume of the parallelepiped is abc, we have $abc \leq (a^3 + b^3 + c^3)/3$. Taking $x = a^3$, $y = b^3$, $z = c^3$, we can rewrite this inequality as

$$\frac{x+y+z}{3} \geq \sqrt[3]{xyz},$$

with the arithmetic mean on the left and the geometric mean on the right, which is the inequality we were going to prove. In the case of n variables, the n-dimensional cube is cut into n equal n-dimensional pyramids whose bases are its $(n-1)$-dimensional faces coming together at a vertex, and the number 3 is everywhere replaced by n.

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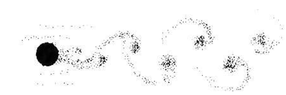

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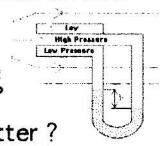

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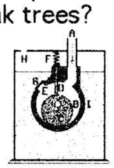

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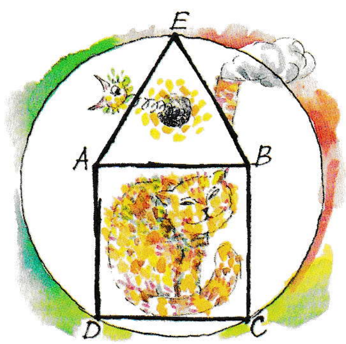
Just for the fun of it!

B181

Cowboy math. A farmer has a cow, a horse, a goat, and a stack of hay. His son calculated that this hay would suffice to feed the horse and the goat for a month, or the goat and the cow for 3/4 of a month, or the cow and the horse for 1/3 of a month. The father told his son that he must not have been too good at math in school. Did the father have grounds for his acid remark? (G. Kukin)

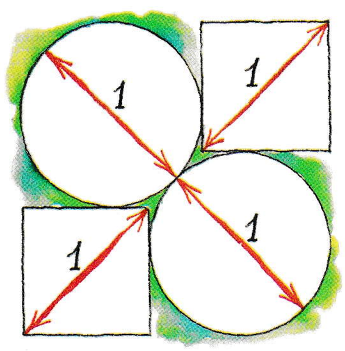


B182

Little house circumscribed. An equilateral triangle ABE is constructed on the top of a square $ABCD$ (see the figure). Find the radius of the circle drawn through C, D, and E if the side length of the square is a. (A. Savin)

B183

Cutting kerosene. You have two large, opaque vessels. One contains kerosene, the other contains kerosene and water. How can you tell the one from the other using a spring scale and a weight on a string?

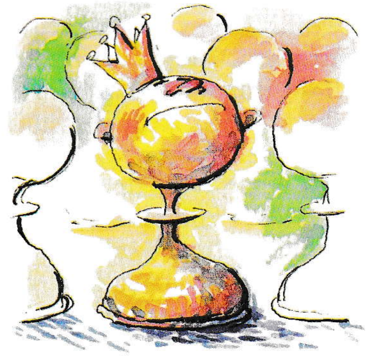


B184

With squares and circles. Mark six points on the plane such that any five of them can be covered with two squares whose diagonal length is 1, but all six can't be covered with two circles of diameter 1. (V. Proizvolov)

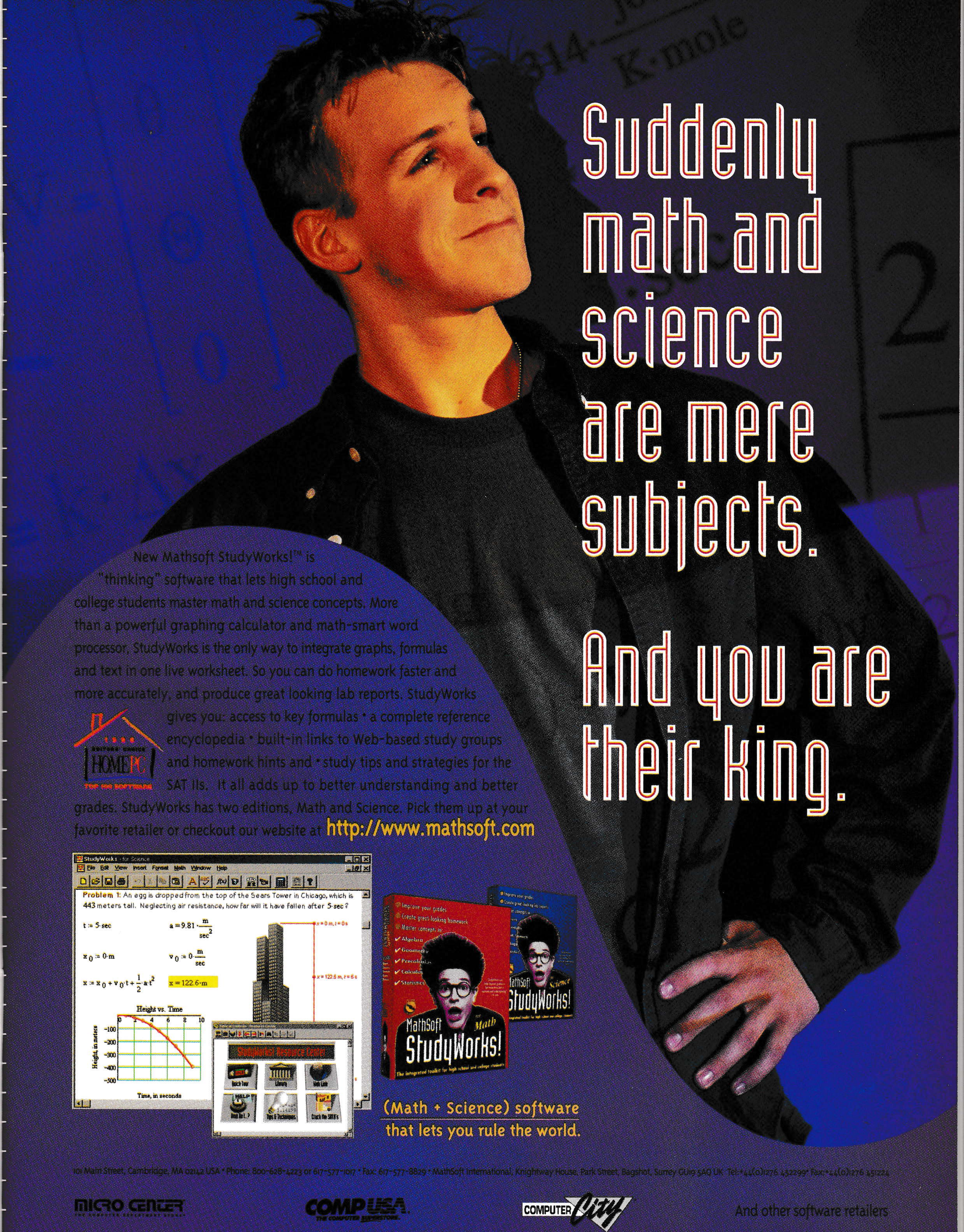
B185

Indelicate bureaucrats. A hundred officials were invited to the annual meeting at their Ministry of Affairs. They were seated in a rectangular hall with ten rows of chairs, ten chairs in each row. The opening was delayed, and the officials could find nothing better to do than compare their salaries. To consider oneself "highly paid," an official had to determine that no more than one person seated to the left, right, front, or rear or at a diagonal was paid as much or more. What is the greatest number of officials who could count themselves as "highly paid"? (A. Shapovalov)



Art by Pavel Chernusky

ANSWERS, HINTS & SOLUTIONS ON PAGE 61



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Resistance in the multidimensional cube

First you'll need to overcome your resistance ***to*** *the multidimensional cube!*

by F. Nedemeyer and Y. Smorodinsky

A POPULAR SUBJECT IN MATHematics clubs[1] in Moscow at the end of the 1940s was the problem of the electrical resistance of a wire cube. We don't know who thought it up or found it in an old book. It was very popular, and soon everyone knew about it. Later it became a common question on examinations, and the problem came to be considered almost trivial.

We can formulate it as follows: calculate the resistance R_3 between points A and B in the circuit in figure 1 if all its resistors have a resistance of 1 Ω.

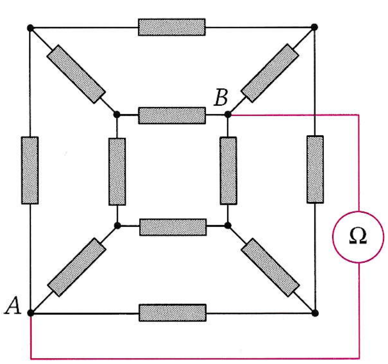

Figure 1

"What's so interesting about that?" a skeptical reader may ask. "We only have to undertake a rather long and boring calculation using Kirchhoff's laws and everything will emerge all by itself. Basically, it's just another dull physics problem."

Asked to count the resistance R_4 between nodes A and B of the circuit in figure 2, this reader will probably get really angry—what a strange idea even to *think* of such cumbersome calculation!

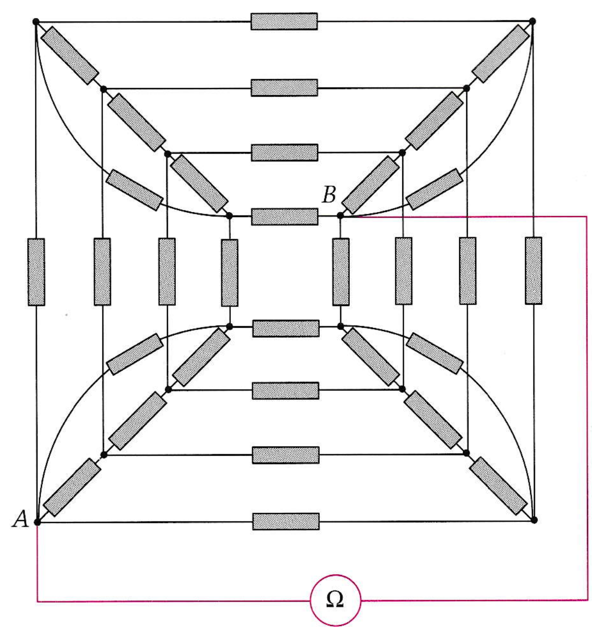

Figure 2

However, these problems conceal some beautiful geometric and algebraic relations (it's not without reason that this problem was discussed in *mathematical* circles) that will allow us to solve it without any "boring calculations" and will lead to an unexpected generalization.

In dimensions three and four

Let's start with an obvious geometric observation: the circuit in figure 1 is simply the network corresponding to the edges of an ordinary cube (compare figure 1 and figure 3a). As a model for our cube, consider the standard unit cube in coordinate space with nodes A and B represented by the cube's vertices (0, 0, 0) and (1, 1, 1), and each of the

[1]This form of extracurricular advanced mathematical education for high school students—"mathematical circles" in Russian (математические кружки)—was developed in Moscow, Leningrad (St. Petersburg), and other Russian cities, big and small, and helped launch the careers of many prominent Russian mathematicians of today.—*Ed.*

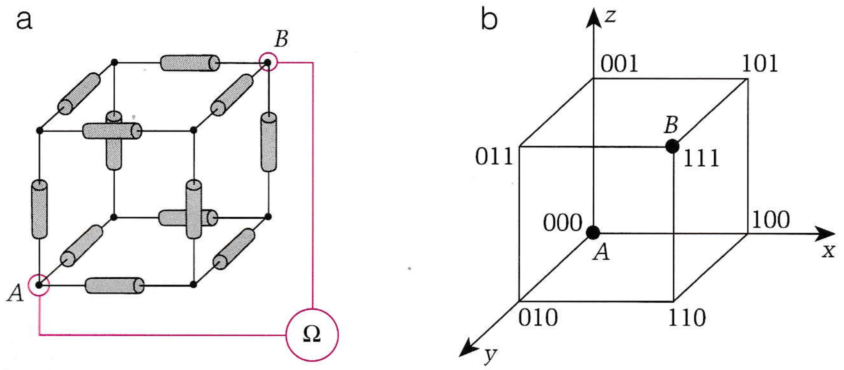

Figure 3

cube's edges thought of as a 1-Ω resistor. Notice that all the coordinates of the cube's vertices (and only these coordinates) are ones and zeros. Let's define the rank of a vertex as the sum of its coordinates. If we apply a voltage difference between points A and B, then *the vertices of the same rank will have the same potential* (this is clear from the symmetry of the configuration). Therefore, we can short-circuit such vertices without changing the overall resistance of the circuit. As a result, we get a circuit consisting of three groups of parallel resistors connected in series, as shown in the right-hand side of figure 4). And for this circuit the problem can be solved in your head: resistance R_3 equals 5/6 Ω.

To compute the second resistance (fig. 2), we notice that this circuit can be interpreted as the network of edges of the four-dimensional cube (see the introductory article "The Multidimensional Cube" in this issue). This is a less obvious geometric observation. However, you can check that it is true by comparing figure 2 to the portraits of the four-dimensional cube in that article. Now the calculation is done the same way we did it for the ordinary cube—see figure 5 on the next page. Again we use the fact that *the vertices of the same rank are all at the same voltage* and so can be short-circuited without changing the total resistance between A and B. The answer is $R_4 = 2/3$ Ω.

Exercise 1. Find the resistance R_5 for the five-dimensional cube between its diagonally opposite vertices if the resistance of each edge is 1 Ω.

Resistance of the *n*-dimensional cube

It's only natural to generalize our problem to cubes of dimension $n = 5$, 6, 7, 8, and so on. This could be done along the same lines as for $n = 3$ and $n = 4$. (By the way, what is the answer R_2 for $n = 2$?) However, we couldn't rightfully consider ourselves mathematicians if we were unable to compute the resistance R_n between two opposite vertices of an n-dimensional cube for all n at once (the definition of the n-dimensional cube can be found in the article mentioned above).

You may wonder whether such a setting of the problem is legitimate at all, because the n-dimensional cube for $n > 3$ is only a mathematical abstraction—it doesn't exist "in reality," and it isn't clear whether there's any point in calculating its resistance. But it turns out there is! This problem is an absolutely "real" *physical* question. While the n-dimensional cube itself for $n > 3$ can't be imbedded in our three-dimensional space, its "two-dimensional skeleton"—the framework of its edges—fits into our space without any problem. Figure 2 (or figure 5) shows how this can be done for the four-dimensional cube. It presents no difficulty in the general case either. In fact, it can be proved that any graph (not only that of the cube's edges) can be embedded in three-dimensional space without self-intersections. This is a rather simple mathematical theorem and we won't dwell on its proof.

It is interesting that the graph of the edges of the three-dimensional cube can be embedded without self-intersection not only in space (where it resides by definition) but in the plane as well (fig. 1). However, such an embedding is impos-

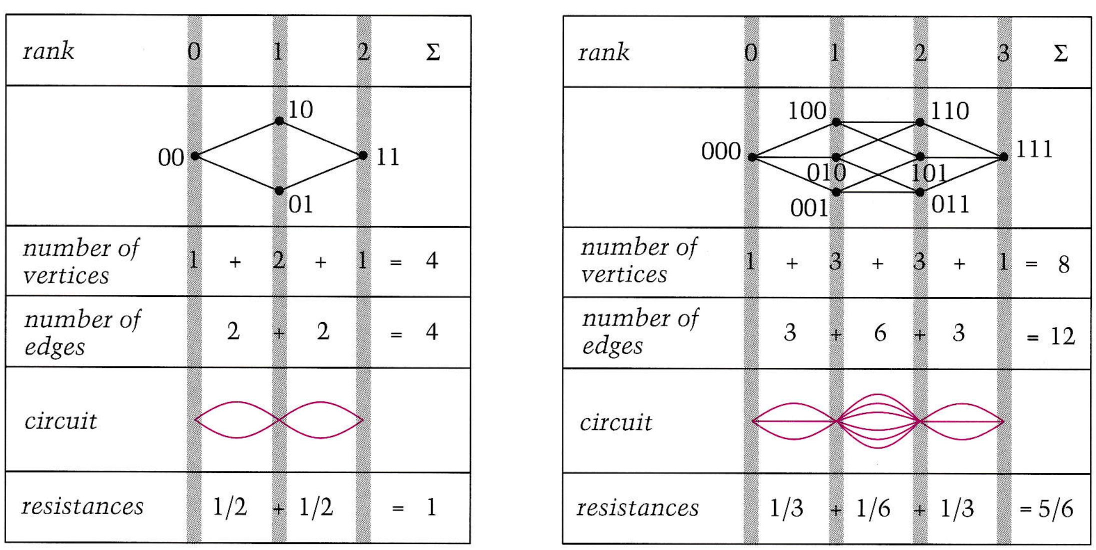

Figure 4

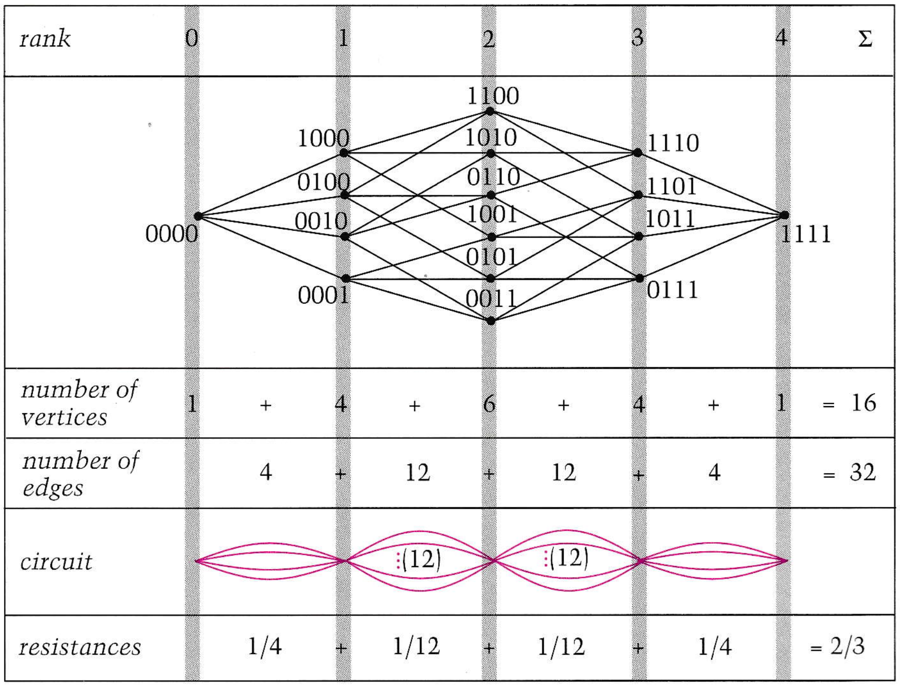

Figure 5

sible for the four-dimensional cube, to say nothing of higher dimensions. This follows from the general Kuratowski theorem on planar graphs (see, for instance, "Graphs and Grafs" in the November/December 1995 issue of *Quantum*).

It must certainly be clear to you that the computation of R_n can be done in essentially the same way as for R_3 and R_4. You can follow it by referring to figure 6. The answer is

$$R_n = \sum_{k=0}^{n-1} \frac{1}{(n-k)\binom{n}{k}}, \qquad (1)$$

where the numbers $\binom{n}{k}$ are . . . well, let's say $\binom{n}{k}$ is the number of the rank-k vertices of the n-dimensional cube. (Of course, many of our readers know that the notation $\binom{n}{k}$ is used for *binomial coefficients* given by the formula

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

And of course we used this notation deliberately, because the numbers in equation (1) are indeed binomial coefficients—for details see the introductory article, page 6. But here this connection is, in fact, irrelevant.)

To prove equation (1), we must again use the fact that the vertices of the same rank are equipotentials, so the whole problem reduces to counting all the edges joining vertices of two successive ranks. The number of the vertices of rank k is $\binom{n}{k}$, and each of them is joined to $n-k$ vertices of rank $k+1$ (namely, to the vertices whose coordinates are obtained by replacing one of the $n-k$ zero coordinates of the rank-k vertex with a one). So the number of edges in question is $(n-k)\binom{n}{k}$. They can be regarded as connected in parallel, and their total resistance equals $1/[(n-k)\binom{n}{k}]$, which leads to equation (1).

Exercise 2. Prove that equation (1) can be rewritten as

$$R_n = \frac{1}{n}\sum_{k=0}^{n-1} \frac{1}{\binom{n-1}{k}}. \qquad (2)$$

This exercise should pose no serious difficulties for many of our readers. And here are two other beautiful formulas for R_n. It won't be difficult for you to derive them from each other, but to prove that either of them gives a correct value for R_n is a real challenge.

Problem 1. Prove that

$$R_n = \sum_{k=0}^{n-1} \frac{1}{(n-k)2^n}. \qquad (3)$$

Problem 2. Establish the recurrent relation

$$R_n = \frac{1}{n} + \frac{1}{2}R_{n-1}$$

algebraically (using equation (3)) and geometrically. Check our calculations for small values of n using this relation and the initial value $R_1 = 1$.

Extensions

The method we used to calculate R_n can be applied to other problems—for instance, to this one.

Exercise 3. Find the resistance between two *adjacent* vertices of a wire three-dimensional cube if the resistance of each edge is 1 Ω.

Rather than change the points where the ohmmeter is attached, it's more interesting to change the configuration of the circuit. Here are some more examples that can be calculated in the same manner.

Exercise 4. Assuming that the resistance of all the wires that form the circuits defined below is 1 Ω, find the resistance between two adjacent nodes of a wire (a) m-gon, (b) tetrahedron, (c) circuit with m

rank	0		1		2 ...		$k-1$		k		... $n-1$		n	
number of vertices	1	+	n	+	$\frac{n(n-1)}{2}$	+	...	+	$\binom{n}{k}$	+ ... +	n	+	1	$= 2^n$
number of edges		n	+	$n(n-1)$	+	...	+	$(n-k)\binom{n}{k}$	+	...	+	n		= ?
resistances		$\frac{1}{n}$	+	$\frac{1}{n(n-1)}$	+	...	+	$\frac{1}{n\binom{n-1}{k}}$	+	...	+	$\frac{1}{n}$		= ?

Figure 6

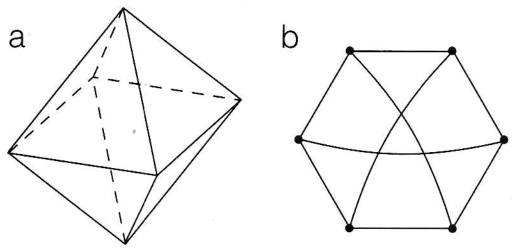

Figure 7

nodes any two of which are connected, (d) octahedron (fig. 7a), (e) hexagon with its opposite vertices joined to one another (fig. 7b).

Now we have a surprise for you: all the formulas in the last two exercises are particular cases of one general formula. Before we write it out, you may want to derive it yourself. Just one little hint: for all these circuits the resistance in question can be expressed in terms of two numbers: the number of nodes m and the number of edges s issuing from each node.

Problem 3. Suppose a circuit has m nodes and each node is connected with wires of resistance 1 Ω to s other nodes. Suppose also that all the wires are "equivalent" in the sense that for any two edges AB and CD we can establish a one-to-one correspondence between the nodes of the circuit such that nodes A and B will correspond to C and D, respectively, and any pair of nodes will be connected if and only if their corresponding nodes are connected. Then the resistance between any two adjacent nodes equals[2]

$$R = \frac{2}{s}\left(1 - \frac{1}{m}\right). \qquad (4)$$

[2]In the original Russian article, published in our sister magazine *Kvant* a while back, the authors erroneously omitted the requirement of edge equivalence. Without it the formula becomes inapplicable: even in the same circuit the resistance through different edges may be different (consider, for instance, a wire triangular prism). In all likelihood, this additional requirement suffices for the formula to be true; however, we did not verify this.—*Ed.*

Applied to the main character of our story, the n-dimensional cube ($m = 2^n$, $s = n$), this formula gives the resistance through its edge

$$R = \frac{2}{n}\left(1 - \frac{1}{2^n}\right).$$

Try to verify this on your own. It's interesting that the formula works as well for infinite circuits.

Problem 4. Prove equation (4) for the infinite grids of squares ($s = 4$, $m = \infty$), triangles ($s = 6$, $m = \infty$), and hexagons ($s = 3$, $m = \infty$) in the plane.

Even more interesting (but, unfortunately, not at all elementary!) is the problem of computing the resistance between two *diagonally* adjacent nodes of the infinite square grid of 1-Ω resistors. The answer turns out to be equal to $2/\pi$ Ω, although nothing in the problem suggests any connection with the circle! ◘

ANSWERS, HINTS & SOLUTIONS ON PAGE 63

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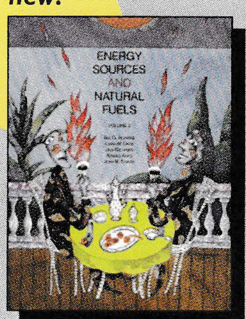

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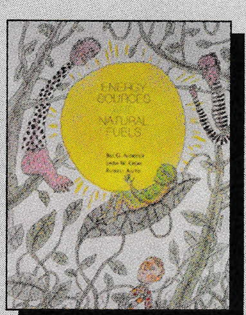

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To cherry blooms row
S. Ivanov

Borsuk's problem

"To cherry blooms I row,
But the oar froze in my hand:
Willows on the shore."
—Basho

by Arkady Skopenkov

IN 1933 THE POLISH MATHEmatician Karel Borsuk proved the following theorem:

THEOREM. *Any bounded plane figure can be divided into three pieces of smaller diameter.*

(The diameter of a figure is the maximum distance between its points.)

He also offered the following generalization of his result, which for years has been one of the most intriguing problems in combinatorial geometry:

BORSUK'S CONJECTURE. *Any bounded n-dimensional figure can be divided into $n + 1$ pieces of smaller diameter.*

This is obviously true for $n = 1$. Also, it's not difficult to find n-dimensional figures that cannot be divided into n pieces of smaller diameter. For $n = 3$ the simplest example is the regular tetrahedron: its diameter equals its edge length, and no matter how it is cut into three pieces, one of them will contain two of the four vertices—that is, it will have the same diameter as the entire solid. This example readily generalizes to any dimension: the corresponding n-dimensional polyhedron with $n + 1$ equidistant vertices is called the n-dimensional *simplex*.

Exercise 1. Write out the coordinates of the vertices of a four-dimensional regular simplex.

For dimension three, Borsuk's conjecture wasn't proved until 1955 (by the English mathematician H. G. Eggleston[1]). Later the conjecture was proved for the n-dimensional sphere and for centrally symmetric convex bodies, then for all smooth solids (those that have no "sharp points"). The complete solution seemed to be a stone's throw away. But in 1993 two Israeli mathematicians, D. Kahn and G. Kalai, following an idea of Erdős, Larman, and Boltyansky concerning the use of combinatorial considerations to construct a counterexample, found a counterexample to Borsuk's hypothesis! They showed that a certain set of vertices of the n-dimensional cube can be broken into pieces of smaller diameter only if the number of pieces increases with n at an approximate rate of $1.2^{\sqrt{n}}$. This is, of course, greater than $n + 1$ for sufficiently large n.

The hypothesis simply fell apart! Well, such disasters are not so rare in mathematics.

The construction of this counterexample is one of very few significant results in modern mathematics that don't require a half-year special university course (after a two-year regular course) to be understood, though not in all its details. The main goal of this article is to describe this remarkable application of combinatorics to geometry. But before we come to grips with it, we'll make a few digressions intended to clarify the ideas behind the construction and to capture the spirit of the problem.

Borsuk's problem on the plane

I'll start with a sketch of the problem's solution in the two-dimensional case. It has practically nothing to do with the counterexample *per se*, but it's useful in itself and demonstrates the wide range of ideas connected with the problem.

Notice that a regular hexagon can be cut into three pentagons of smaller diameter (fig. 1). Therefore, it will suffice to show that any plane figure Φ can be covered by a regular

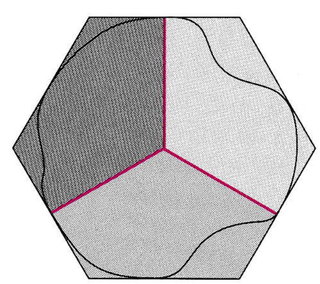

Figure 1

[1]A simplified version of the proof can be found in *Results* and *Problems in Combinatorial Geometry* by V. Boltjansky and I. Gohberg (Cambridge University Press, 1985).

Art by Sergey Ivanov

hexagon of the same diameter $d = \operatorname{diam}\Phi$—that is, with the distance d between its opposite sides. The construction of this hexagon is based on "considerations of continuity."[2]

Let's fix a directed straight line l on the plane. Circumscribe the (smallest possible) parallelogram $P_\alpha = ABCD$ about the given figure Φ with $\angle BAD = 60°$ and such that the angle from l to AB is α (fig. 2). Pull apart the opposite sides of P_α parallel to one another to a distance d so as to transform it into a rhombus R_α with the same center. Cut off the greatest possible equilateral triangles from the two 60° angles of the rhombus so that Φ is still covered by the remaining hexagon H_α and denote by $h_A(\alpha)$ and $h_C(\alpha)$ the altitudes of the two equilateral triangles (as shown in figure 2). When the direction of the side AB makes a half-turn, we obtain the same parallelogram and rhombus as we had initially ($P_{\alpha+190°} = P_\alpha$ and $R_{\alpha+180°} = R_\alpha$) except that the labels A and C (and B, D) exchange places. This means that $h_A(\alpha + 180°) - h_C(\alpha + 180°) = h_C(\alpha) - h_A(\alpha)$. Since the difference $f(X) = h_A(\alpha) - h_C(\alpha)$ is a continuous function of α, we can apply the Intermediate Value Theorem and conclude that $h_A(\alpha) = h_C(\alpha)$ for a certain angle α_0, $\alpha \le \alpha_0 \le \alpha + 180°$ (because either $f(\alpha) = f(\alpha + 180°) = 0$, or $f(\alpha)$ and $f(\alpha + 180°)$ have different signs). The distance between the cuts for $\alpha + \alpha_0$ is no greater than d, so we can pull the cuts apart, if needed, to make this distance exactly equal to d and thus obtain the required hexagon.

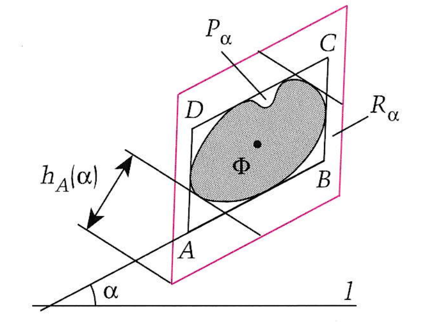

Figure 2

[2]See the article with this title in the May 1990 issue of *Quantum*, where you can find many other interesting applications of this powerful method—*Ed.*

Geometry of the set of subsets

The Kahn–Kalai construction is based on an estimate of the number of certain subsets of a finite set. So we'll start by establishing the connection between these subsets and the vertices of a multidimensional cube.

Let's represent the subsets of the finite set $X = \{1, 2, \dots, n\}$ as points on the plane. We'll arrange these points in $n + 1$ "floors"—that is, horizontal lines numbered 0, 1, ..., n from the bottom to the top. The empty set $\varnothing$ will be placed on the "ground" (zeroth) floor; the one-element sets $\{1\}$, $\{2\}$, ..., $\{n\}$ will be placed on the first floor; and so on. The last (nth) floor will be occupied by a single "tenant": the entire set X. The number of points on the kth floor is

$$\binom{n}{k} = \frac{n(n-1)\dots(n-k+1)}{k(k-1)\cdot\ \cdots\ \cdot 2\cdot 1}.$$

(See "The Multidimensional Cube" in this issue.)

Join with a line each pair of points in our diagram that correspond to sets differing by a single element—these points always reside on neighboring floors. Figure 3 shows the graphs thus obtained for $n = 0, 1, 2, 3$.

Exercise 2. Draw the graph for $n = 4$.

So what do we get? Yes, these are n-dimensional cubes—more exactly, the graphs of their vertices and edges. This is clear for small values of n. As for $n \ge 4$, refer to the article mentioned above, where you can find a detailed presentation of these sophisticated objects. However, for our purposes the "trimmed" version of the cube that involves only the vertices will be quite sufficient. In fact, we'll deal only with the vertices of the "standard" unit cube—that is, n-digit sets of zeros and ones.

Geometrically these sets are viewed as coordinates of points (the cube's vertices) in n-dimensional space. The distance between any two of them, $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ and $\beta = (\beta_1, \beta_2, \dots, \beta_n)$, is defined by the familiar formula

$$\sqrt{(\alpha_1-\beta_1)^2+(\alpha_2-\beta_2)^2+\dots+(\alpha_n-\beta_n)^2}.$$

Our goal is to choose a subset of these points that cannot be partitioned into $n + 1$ pieces whose diameter is smaller than that of the entire subset. It will be more convenient to talk about the squares of distances rather than the distances themselves (this doesn't change the problem). But for the points in question (with α_i and β_i equal to 0 or 1), $(\alpha_i - \beta_i)^2 = |\alpha_i - \beta_i|$, so we can measure distances by the formula $|\alpha_1 - \beta_1| + |\alpha_2 - \beta_2| + \dots + |\alpha_n - \beta_n|$, which defines what is known as the *Hamming distance*. This distance can be described as the number of "differences" between α and β—that is, the number of digits in one of these strings that differ from their counterparts in the other.

Combinatorially, any of our points $(\alpha_1, \dots, \alpha_n)$ can be associated with the subset A of X that consists of all the numbers i such that $\alpha_i = 1$. Thus we get a one-to-one correspondence between the cube's vertices and the subsets of X. Notice that in terms of subsets the Hamming distance equals the number of points that belong to exactly one of the two subsets. The set of all these points is called the *symmetric difference* of the subsets.

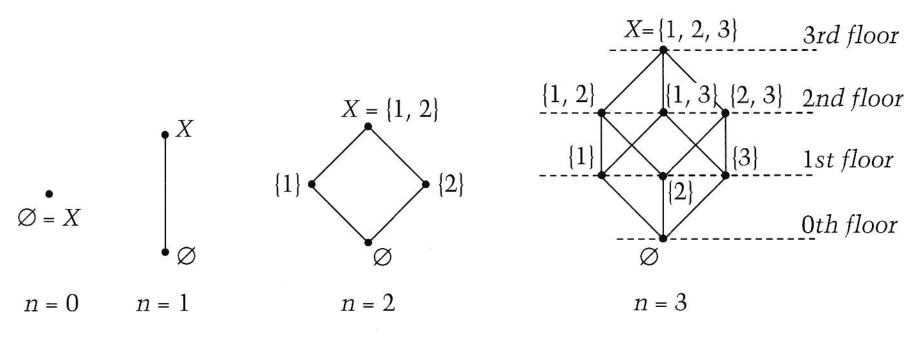

Figure 3

See how it works

To get used to this correspondence, let's solve the following problem.

Best in their own ways. Each participant in a mathematical olympiad received a personal special prize, because it was impossible to compare their results: none of them solved all the problems solved by anybody else. What was the greatest possible number of competitors, if the total number of problems was n?

Each competitor is characterized by the set of problems he or she solved. None of these sets is contained in another. If we call *incomparable* any collection of sets with this property, our problem can be reworded as "find the largest size of an incomparable family of subsets in an n-element set."

Notice that all k-element subsets of the set $X = \{1, 2, ..., n\}$ (that is, "points on the kth floor") form an incomparable family. The size of this family is $\binom{n}{k}$.

Exercise 3. Prove that $\binom{n}{k}$ is maximal for $k = [n/2]$, where $[a]$ denotes the integer part of a.

Thus there exists an incomparable family of $\binom{n}{[n/2]}$ subsets. We'll prove that this number can't be increased. Let's use our representation of subsets as vertices of the n-dimensional cube. A subset A contains a subset B if A can be obtained from B by adding a number of elements one by one. This gradual transformation of B into A corresponds to a path on our graph consisting of a continuous series of edges always going upward and joining B to A. Any such path from the lowest vertex $\varnothing$ to the highest vertex X (fig. 4) will be called a chain. So $B \subset A$ if and only if the corresponding points (we identify them with the subsets) lie on the same chain, B under A. It follows that any chain passes through at most one point of an incomparable family, which in its turn leads to the following relation:

$$\text{the size of any incomparable family} \le \frac{\text{the total number of chains}}{\text{the smallest number of chains passing through a point}}.$$

Let's count the numerator and denominator on the right-hand side. Any chain $\varnothing \subset \{i_1\} \subset \{i_1, i_2\} \subset ... \subset \{i_1, ..., i_n\} = X$ is uniquely determined by the order in which the numbers $1, 2, ..., n$ are included as we move from $\varnothing$ to X—that is, by the permutation $(i_1, ..., i_n)$ of the set X. It's well known that this number equals $n! = 1 \cdot 2 \cdot ... \cdot n$; and this is the numerator. Now consider the chains passing through a fixed point A on the kth floor. Each of these chains is divided by the point A into a lower and upper part (fig. 5). Since A has k elements, its subsets form a k-dimensional cube, which contains the lower part of any chain through A. So the number of lower parts is $k!$. Similarly, any subset that contains A is obtained from A by adding a subset of the set difference $X \backslash A$. Since the subsets of the $(n-k)$-element set $X \backslash A$ form an $(n-k)$-dimensional cube, the number of the upper parts of the chains through A equals $(n-k)!$, and the total number of these chains is $k!(n-k)!$. This number is the smallest for $k = [n/2]$ (compare with exercise 3). Therefore, the size of any incomparable family is no greater than

$$\frac{n!}{[n/2]!(n-[n/2])!} = \binom{n}{[n/2]},$$

which completes the solution.

Exercise 4. Prove that if the subsets $A_1, ..., A_m$ of an n-element set form an incomparable family and consist of $a_1, ..., a_m$ elements, respectively, then

$$\binom{n}{a_1}^{-1} + \cdots + \binom{n}{a_m}^{-1} \le 1.$$

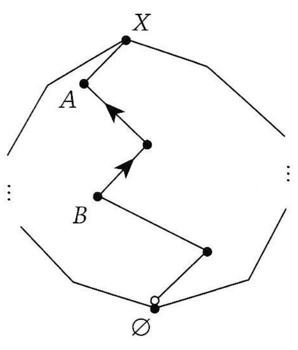

Figure 4

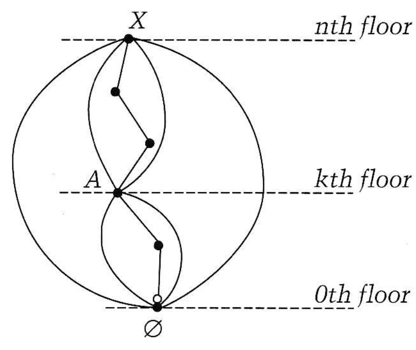

Figure 5

Counting common points

The Kahn–Kalai counterexample is a very intricate "multistoried" structure involving sets, sets of sets, and even sets of sets of sets. So I tried to invent problems in which some of its basic ideas appear in a more tangible, if not mundane, shape. It's only natural that the formulations turned out to be rather unnatural (these are artificial problems), but I hope they'll help you understand the main points of the subsequent construction. The first problem, by the way, was used in a training session of the Russian team before the International Mathematical Olympiad.

Baker's dozen. A hostess can bake k different kinds of cakes. Once she invited 66 persons to a big reception, and each group of 36 persons ate a cake. It turned out that no two groups that ate cakes of the same sort had exactly 18 persons in common. Prove that the hostess's baking skill will suffice to invite 12 guests and treat each group of 6 guests to a cake in such a way that no two groups that will eat cakes of the same sort will have exactly 3 persons in common.

Solution. Divide the 66 guests at the reception into eleven groups G_1, ..., G_{11} of six persons each. Number the guests at the party 0, 1, 2, ..., 11. Give to each group $\{i_1, i_2, ..., i_6\}$ $(0 \le i_1 < i_2 < ... < i_6 \le 11)$ a cake of the sort that was eaten at the first party by the group $G_{i_1} \cup G_{i_2} \cup \cdots \cup G_{i_6}$ if $i_1 \neq 0$, or by the group $\overline{G_{i_2} \cup \cdots \cup G_{i_6}}$ (that is, the guests not included[3] in any of the groups $G_{i_2} \cup \cdots \cup G_{i_6}$) if $i_1 = 0$. It is directly verifiable that with this distribution of cakes the condition for the party will be satisfied. (For instance, if we suppose that two groups, $\{i_1, i_2, \ldots, i_6\}$ and $\{j_1, j_2, \ldots, j_6\}$ with $i_1 \neq 0$, $j_1 \neq 0$, had three persons—say, a, b, and c—in common, then the corresponding groups at the reception, $G_{i_1} \cup \cdots \cup G_{i_6}$ and $G_{j_1} \cup \cdots \cup G_{j_6}$, would have 18 persons—the union $G_a \cup G_b \cup G_c$—in common, contrary to the assumption of the problem.)

Exercise 5. Prove that the statement of the problem remains true if we replace the numbers 66, 36, 18 and 12, 6, 3 by $(4n-1)k$, $2nk$, nk and $4n$, $2n$, n, respectively.

The solution given above was found by the olympiad team members. However, this problem in fact emerged as a consequence of the following problem. It will shed some light on the origin of the rather unusual numerical values in both of them.

Protruding edges. Each of two points of a 12-point set are joined with an edge. Let's say that an edge *protrudes* from a given subset of these points if exactly one of its endpoints belongs to this subset. Prove that any two 6-point subsets have at least 18 common protruding edges.

Solution. Consider two 6-point subsets A and B. Let y be the number of their common points. Any edge protrudes from both A and B if and only if either one of its endpoints belongs to both subsets (that is, to $A \cap B$) and the other to neither of them (it lies in $\overline{A} \cap \overline{B}$), or one of the endpoints belongs to A but not to B (it lies in $A \cap \overline{B}$) and the other to B but not to A (it lies in $\overline{A} \cap B$). The number of edges of the first kind is y^2 (see figure 6a), and the number of edges of the second kind is $(6-y)^2$ (fig. 6b). So the total number of

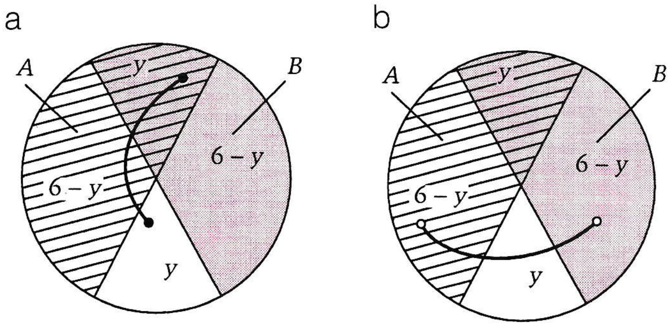

Figure 6

"double-protruding" edges is

$$y^2 + (6-y)^2 \geq \frac{(y+(6-y))^2}{2} = 18$$

(because, as you may want to demonstrate to yourself, $a^2 + b^2 \geq (a+b)^2/2$).

Notice that the value 18 is attained only for $y = 3$, so if the number of edges that protruded from two 6-point sets simultaneously is not 18, then the number of common points of these two sets is not equal to 3. Notice also that the number of all edges joining 12 points is $\binom{12}{2} = 66$ and the number of edges protruding from any 6-point subset is $6 \cdot 6 = 36$. Now compare these numbers to those in the "baker's dozen" problem!

Exercise 6. Solve the "baker's dozen" problem using the "protruding edges" problem.

The meaning of all these combinatorial exercises with regard to Borsuk's conjecture becomes clear if we introduce the Hamming distance between sets of edges. If we confine ourselves to the 36-element sets of edges that protruded from 6-point subsets of a 12-point set, as we did above, then the Hamming diameter of this family of sets is 36 (any two such sets have at least 18 common elements, so their symmetric difference consists of at most $(36-18) + (36-18) = 36$ elements—see figure 7). Not only that, we have a description of the sets that are most distant from one another: they must be generated by 6-point sets with exactly three common points. It is this construction (for the case of dimension $n = 66$) that refutes Borsuk's conjecture. Now we'll describe it more formally and generally.

The construction

For any even m, consider an m-element set S (called "points" above) and the set of all its subsets viewed as an m-dimensional cube q. Then consider the set P of all pairs of the elements of S (P consists of $m(m-1)/2$ elements—called "edges" above) and the set of all subsets of P viewed as an $m(m-1)/2$-dimensional cube Q. Take the largest "incomparable family of subsets" in S—that is, the points on the $(m/2)$th floor in q. The required set X is the image of this family under a certain map f of q into Q. More exactly, for each subset A of S (a point of q) we define $f(A)$ as the subset of P (a point of Q) that consists of all the pairs each of which has exactly one element in A. (So $f(A)$ is the set of pairs that "protruded" from A in the terminology of the previous section.) This map is illustrated in figure 8.

Exercise 7. Prove that the set X lies on the $(m^2/4)$th floor of cube Q.

Let's find the diameter of X with respect to the Hamming distance. For the sake of diversity, we'll do it a bit differently from the way we did it earlier. As we saw, the (Hamming) distance between the subsets $f(A)$ and $f(B)$ of the set of pairs P equals the number of pairs in the symmetric difference of $f(A)$ and $f(B)$, which

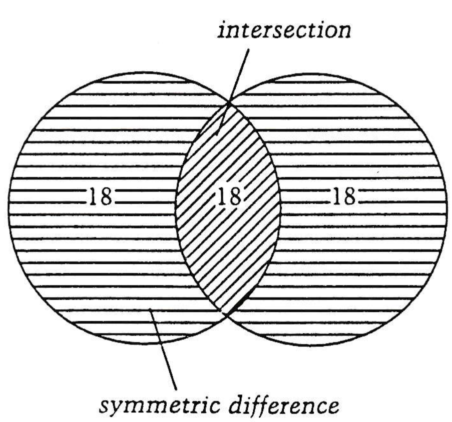

Figure 7

[3]The notation $\overline{A}$ means the complement of the set A.

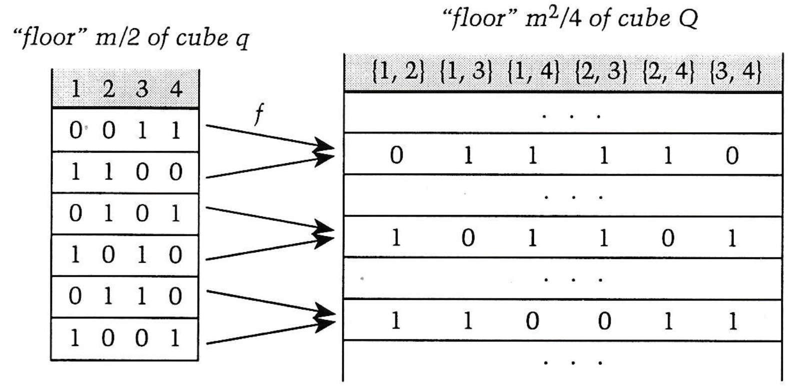

Figure 8

The 0–1 sequences on the left represent all the (m/2)-element subsets of the m-element set {1, 2, ..., m} (for m = 4): for instance, 0011 denotes {3, 4}. On the other hand, these sequences can be thought of as coordinates of the vertices of the m-dimensional cube that lie on its (m/2)th floor. The meaning of the digits on the right is similar, except that the set here consists of m(m – 1)/2 (rather than m) elements—the pairs {1, 2}, {1, 3}, ..., {m – 1, m}. The arrows join each subset on the left to the set of pairs that "protrude from" this set.

includes (1) the pairs in $f(A)$ but not in $f(B)$ and (2) those in $f(B)$ but not in $f(A)$. A pair belongs to $f(A)$ if exactly one of its elements belongs to A; it doesn't belong to $f(B)$ if either both or none of its elements belong to B. In other words, either one element of this pair must belong both to A and B and the other to B but not to A (the number of such elements is equal to $y(m/2 - y)$, where y is the size of the intersection $A \cap B$), or one element belongs to A but not to B and the other neither to A nor to B (the number of such elements is $(m/2 - y)y$). So the number of pairs of type (1) is $2y(m/2 - y)$. Clearly there are equally many type (2) pairs, so the distance between $f(A)$ and $f(B)$ is $4y(m/2 - y)$. This value is maximal for $y = m/4$; the maximum equals $m^2/4$. So if X is split into parts of smaller diameter, then *for any two points f(A) and f(B) in the same part, the number of common points in A and B is not equal to m/4.*

Deus ex machina

All our constructions were intended to fit into the conditions of the following theorem, which was proved by Frankl and Wilson long before and then found an unexpected application in the solution of Borsuk's problem.

THEOREM. *Let F be a family of distinct (m/2)-element subsets of an m-element set such that no two of them have exactly m/4 common elements. If $m = 4p^\alpha$, where p is a prime greater then 2 and α is a nonnegative integer, then the number of subsets in F is no greater than* $2\binom{m-1}{m/4-1}$.

Exercise 8. Check this theorem for $m = 4$.

This theorem tells us that if our set X is divided into pieces of smaller diameter, then the pre-image $f^{-1}(A)$ of each piece A consists of at most $2\binom{m-1}{m/4-1}$ elements. So if there are N pieces, then the number of elements in the pre-image of the entire set X—that is, in the $(m/2)$th "floor" of cube q—is no greater than $2N\binom{m-1}{m/4-1}$. But this number is equal to $\binom{m}{m/2}$, so

$$N \geq \frac{\binom{m}{m/2}}{2\binom{m-1}{m/4-1}}.$$

It remains to estimate the value on the right. This can be done by using the well-known asymptotic Stirling formula for $n!$, which says that $n!$ is approximately equal to $\sqrt{2\pi n}e^{-n}n^n$ for large enough n.

Exercise 9. Show that

$$N > \frac{m(m-1)}{2} + 1$$

for large enough m.

Thus our construction supplies a counterexample to Borsuk's hypothesis if m is a large number of the form $4p^\alpha$.

I'd like to thank Nikolay Dolbilin, from whom I learned about the solution of Borsuk's problem; the students of the Kolmogorov school and school 57 in Moscow, who learned about it from me; and Vladimir Dubrovsky, for valuable discussions and suggestions about this article.

ANSWERS, HINTS & SOLUTIONS ON PAGE 63

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Challenges in physics and math

Math

M181

Triple quadratic. Is it possible to find three quadratic polynomials $f(x)$, $g(x)$, $h(x)$ such that the equation $f(g(h(x))) = 0$ has the eight roots 1, 2, 3, 4, 5, 6, 7, and 8? (S. Tokarev)

M182

Third circle. Let A be one of the intersection points of two circles in the plane. In each of the circles a diameter is drawn parallel to the tangent to the other circle at A. Prove that the endpoints of the diameters lie on a circle. (S. Berlov)

M183

Sum of polynomial shifts. (a) Prove that for any nonzero polynomial $f(x)$ of even degree there exists a positive integer k such that the polynomial

$$F_k(x) = f(x) + f(x+1) + f(x+2) + \ldots + f(x+k)$$

has no real roots.

(b) Prove that if the degree of a polynomial $f(x)$ is odd, then for a certain k the polynomial $F_k(x)$ defined above has exactly one real root. (S. Berlov, K. Kohas)

M184

Sisyphus's pay. Sisyphus was given a new job: he must carry stones, one at a time, from one of three piles to another. For each stone Zeus gives him a number of coins equal to the difference between the number of stones in the pile to which this stone is added and the number of stones in the pile from which the stone was taken. (The stone being moved is not counted in this calculation at all, so that if Sisyphus moves a stone from a pile of a stones to a pile of b stones, he receives $b - a - 1$ coins.) If the difference is negative, Sisyphus returns the corresponding sum to Zeus. (If he's short of money, magnanimous Zeus allows him to drag the stone on credit.) At a certain moment all the stones are in the same piles they were in initially. What is the greatest amount of money that Sisyphus could have made up to that point? (I. Izmestyev)

M185

Divisibility of partial sums. Does there exist a sequence of positive integers containing each positive integer exactly once such that the sum of the first k terms of this sequence is divisible by k for any $k = 1, 2, 3, \ldots$? (A. Shapovalov)

Physics

P181

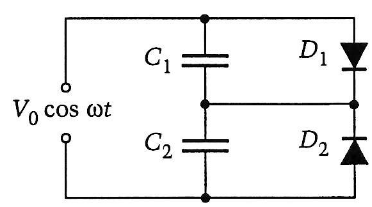
Gymnast on a trampoline. A gymnast falls from the height $H = 12$ m onto a horizontally stretched elastic trampoline, which bends a distance $h = 1$ m. Estimate how much greater the maximum force acting on the gymnast from the trampoline is than the gymnast's weight if the trampoline is much larger than h and its mass is much less than a person's. (A. Izergin, S. Manida, V. Saulit)

P182

Capillary tube. An Π-shaped capillary tube with two sides of length $l = 10$ cm and diameters $d_1 = 0.1$ mm and $d_2 = 0.2$ mm is lowered into water with its open ends down. The tube is submerged so that the level of the water in the narrow side is the same as that in the vessel. Find the height of the water in the thick side. Neglect the volume of the horizontal part of the tube. The atmospheric pressure is standard. The coefficient of surface tension of water is $\sigma = 0.070$ N/m. (B. Bukhovtsev)

P183

Circuit with diodes. A circuit composed of two capacitors with capacitance $C_2 > C_1$ and two ideal diodes D_1 and D_2 (see the figure below) is fed by an alternating current $v = V_0 \cos \omega t$.



How does the voltage across each capacitor vary with time in the steady-state regime? Draw the corresponding functions. The resistance of an ideal diode is zero when the electric field is applied in the conducting direction, and infinity in the opposite case. (V. Skorovarov)

The name game of the elements

Chemistry and politics don't mix

by Henry D. Schreiber

QUESTION: "WHAT DO THE elements with atomic numbers 104 and 106 have in common?" Answer: "Both are named rutherfordium!" But how can that be? Doesn't each element have a unique name? Even more astounding is that the same question can be asked of the elements with atomic numbers 105 and 108, in that both are named hahnium. Table 1, which gives the names for elements 102 through 109, provides the answer to this apparent paradox. The actual names for elements 104 through 108 depend on who's doing the naming! But how did the nomenclature of these most recently discovered elements fall into such disarray?

"Rules" of the name game

Figure 1 summarizes the elements, as represented by their symbols, that have been discovered up to 1995. As chemical elements are the fundamental building blocks of substances, this arrangement of elements is a central organizing concept in science. What makes an element unique is that it consists of a multitude of characteristic, and identical, atoms; accordingly, elements cannot be broken into any simpler chemical components. For example, lead is made up of indivisible lead atoms, while carbon is made up of only carbon atoms. Atoms of one element are distinguished from the atoms of another by the number of protons possessed by that atom. Thus, the element

Table 1

Atomic number	Discoverer's		IUPAC's			Compromise slate
	proposed name	nationality	interim provisional name	provisional name (1994)	provisional symbol (1994)	
102	nobelium joliotium	Swedish Russian	–	nobelium	No	flerovium
103	lawrencium	American	–	lawrencium	Lr	–
104	rutherfordium kurchatovium	American Russian	unnilquadium	dubnium	Db	dubnium
105	hahnium nielsbohrium	American Russian	unnilpentium	joliotium	Jl	joliotium
106	seaborgium	American	unnilhexium	rutherfordium	Rf	seaborgium
107	nielsbohrium	German	unnilseptium	bohrium	Bh	nielsbohrium
108	hassium	German	unniloctium	hahnium	Hn	hahnium
109	meitnerium	German	unnilennium	meitnerium	Mt	meitnerium

Names for some of the transfermium elements. IUPAC stands for the International Union of Pure and Applied Chemistry. The compromise slate was proposed by selected IUPAC representatives from the United States, Germany, and Russia in 1995.

1 ← IUPAC group number
IA ← Traditional American group number

Period	1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8 ←	9 VIIIB	10 →	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 0
1	1 H 1.0079																	2 He 4.00260
2	3 Li 6.941	4 Be 9.01218											5 B 10.81	6 C 12.011	7 N 14.0067	8 O 15.9994	9 F 18.9984	10 Ne 20.179
3	11 Na 22.9898	12 Mg 24.305											13 Al 26.9815	14 Si 28.0855	15 P 30.9738	16 S 32.06	17 Cl 35.453	18 Ar 39.948
4	19 K 39.0983	20 Ca 40.08	21 Sc 44.9559	22 Ti 47.88	23 V 50.9415	24 Cr 51.996	25 Mn 54.9380	26 Fe 55.847	27 Co 58.9332	28 Ni 58.69	29 Cu 63.546	30 Zn 65.39	31 Ga 69.72	32 Ge 72.59	33 As 74.9216	34 Se 78.96	35 Br 79.904	36 Kr 83.80
5	37 Rb 85.4678	38 Sr 87.62	39 Y 88.9059	40 Zr 91.224	41 Nb 92.9064	42 Mo 95.94	43 Tc (98)	44 Ru 101.07	45 Rh 102.906	46 Pd 106.42	47 Ag 107.868	48 Cd 112.41	49 In 114.82	50 Sn 118.71	51 Sb 121.75	52 Te 127.60	53 I 126.905	54 Xe 131.29
6	55 Cs 132.905	56 Ba 137.33	71 Lu 174.967	72 Hf 178.49	73 Ta 180.948	74 W 183.85	75 Re 186.207	76 Os 190.2	77 Ir 192.22	78 Pt 195.08	79 Au 196.967	80 Hg 200.59	81 Tl 204.383	82 Pb 207.2	83 Bi 208.980	84 Po (209)	85 At (210)	86 Rn (222)
7	87 Fr (223)	88 Ra 226.025	103 Lr (262)	104 Unq (261)	105 Unp (262)	106 Unh (266)	107 Uns (262)	108 Uno (265)	109 Une (266)									

57 La 138.906	58 Ce 140.12	59 Pr 140.908	60 Nd 144.24	61 Pm (145)	62 Sm 150.36	63 Eu 151.96	64 Gd 157.25	65 Tb 158.925	66 Dy 162.50	67 Ho 164.930	68 Er 167.26	69 Tm 168.934	70 Yb 173.04
89 Ac 227.028	90 Th 232.038	91 Pa 231.036	92 U 238.029	93 Np (237)	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)

1 ← Atomic number
H ← Element symbol
1.0079 ← Relative atomic mass—values in parentheses are mass numbers for the most stable known isotope

Figure 1

Periodic table of the elements—1995. (From a figure in Physical Chemistry *by John S. Winn, New York: Harper Collins, 1995)*

with atomic number 104 is made up entirely of atoms each with the characteristic 104 protons. The elements are systematically organized by increasing atomic number (increments of one with each additional proton) in a periodic table, as shown in figure 1.

A student of science would assume that the naming of these fundamental units, the elements, is an established process. In fact, the discoverer of a new element has, by custom, the honor of suggesting a name for it. The only real guideline governing the naming process is that new metallic elements must end in *-ium*. IUPAC—the International Union of Pure and Applied Chemistry—then reserves the right to select an official or definitive name for that element for use in the international community of scientists. However, IUPAC historically has selected names that do not deviate significantly from those suggested by the discoverers. The "trick" was usually to determine who was the first to discover, if more than one legitimate claim for discovery was made for that element.

The transfermium name game

The transfermium elements—those with atomic numbers higher than that of fermium (element 100)—have all been discovered within the past 30 years. But the discovery of each of these new elements was often based on the isolation of only a few atoms of a short-lived radioactive isotope. Furthermore, only a few laboratories in the world can manufacture these synthetic elements and, as such, replicate the experiments of others. To prepare a transfermium element, specific target atoms are bombarded with a beam of other atoms until they fuse together. For example, just one atom of element 112 was recently produced by bombarding a lead target with high-energy zinc atoms for two weeks.

More than two thirds of the known isotopes of elements 101 through 109 have half-lives less than a minute, and many are in the millisecond range. So not only is the discovery of these elements plagued with very low production rates, the atoms decay to those of another element in a matter of seconds or minutes at the longest! How long does a group of atoms have to "exist" before they constitute a new element? Further, the evidence that atoms of a new element were present is often indirect and based on postulated nuclear decay schemes, not on classical chemical separations. To isolate and identify a new transfermium element is a difficult, not to mention an often controversial and contested, process. What happens if two or more groups of researchers make (or claim to make) the discovery of a new element more or less simultaneously, and each group suggests a different name for the element? More than one name is operationally applied to the same element, as shown in table 1. For example, element 104 is known as rutherfordium or as kurchatovium depending on whether the suggested American or Russian name is used.

Surely there must be a process by which an arbiter decides on the priority of discovery and thus on the official name of the element. The Transfermium Working Group—a joint committee of IUPAC and IUPAP (IUPAC's sister organization for physics)—acts as the judge to assign priority for disputed discoveries of these elements. Sometimes the judge admits that assigning credit for the discovery could very well be a toss-up. CNIC (the IUPAC Commission on Nomenclature of Inorganic Chemistry) then acts much like a jury to recommend a suitable name to the IUPAC governing board, which finally defines the official name. However, CNIC selects the recommended name primarily on the basis of prevailing usage and practicability, and makes no judgment regarding priority of discovery. It even reserves the right to come up with a name different than those suggested by the discoverer. Confusion runs rampant if yet a third name is proposed by IUPAC for the element. Again returning to element 104, IUPAC ignored the two names proposed by the discoverers and came up with an entirely new name: dubnium. Rutherfordium, kurchatovium, and dubnium subsequently all join with unnilquadium (the IUPAC interim name—*un* = 1, *nil* = 0, *quad* = 4, and *ium* = metal) in the element 104 "name game" sweepstakes.

Name games from the past

Is this confusion over element nomenclature, as illustrated in table 1 for the transfermium elements, unusual? Surprisingly enough, it's actually common! Similar games in naming an element have erupted many times in the past, with disputes lasting decades and in at least one case over a century. The disputes are usually traced back to determining who

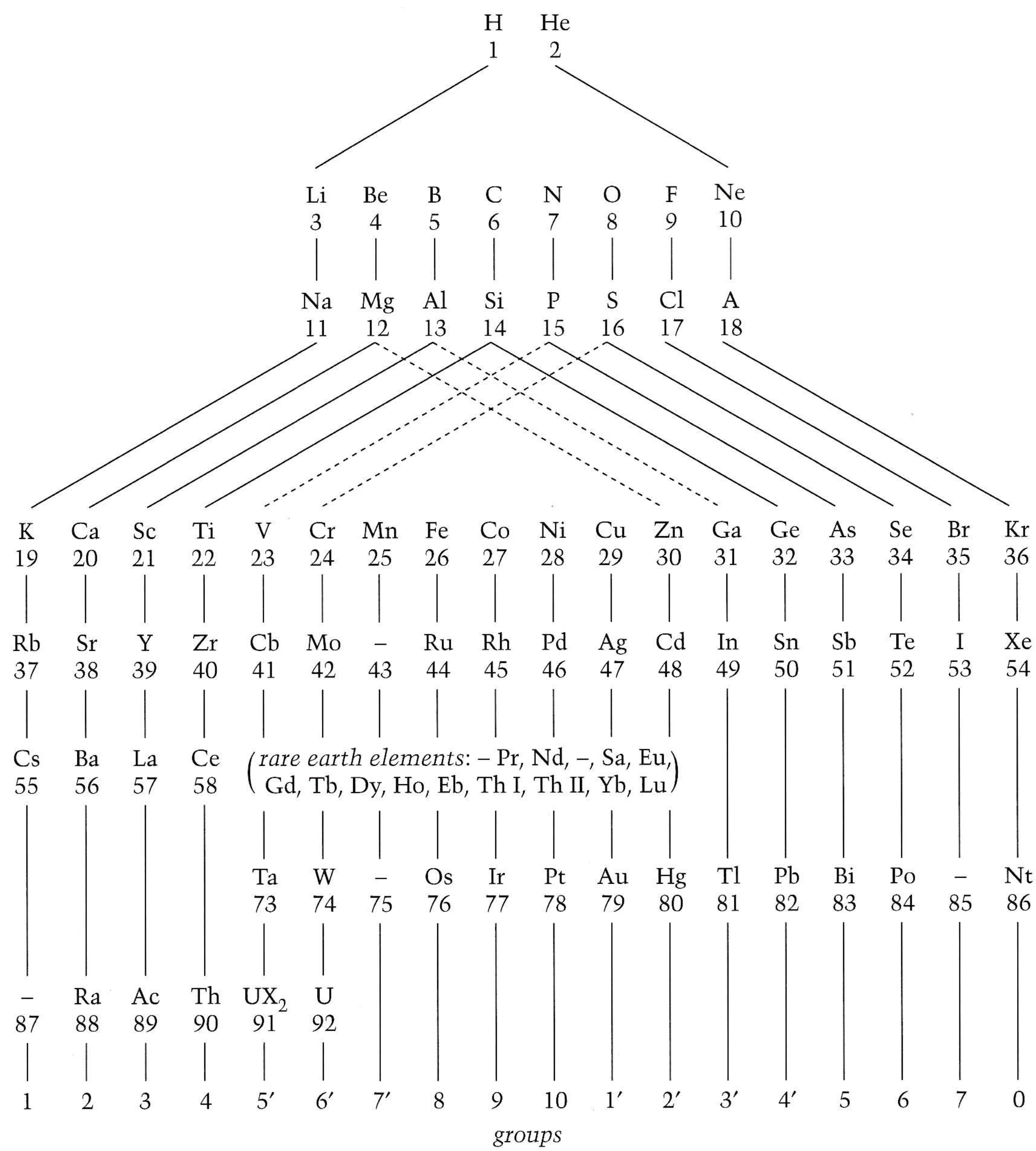


Figure 2

Periodic table of the elements—1920. (From a figure in Principles of Chemistry *by Joel H. Hildebrand, New York: MacMillan, 1920)* rightfully discovered the element and, thus, whose suggestion for the name should have priority. However, in the past it was often the chemical isolation and identification of the element that was uncertain. With many decades of chemical understanding at our disposal, it is now very easy for us to assign priority of discovery in retrospect. But it's very different when one is involved in the heat of a dispute.

Many people believe that the chemical elements have "all" been known for some time and that only recently more elements—artificially produced elements with high atomic numbers—have been added to the end of the periodic table. However, "all" the elements have not even really been known for such a long time. For example, compare two periodic tables of the elements, one from 1995 (fig. 1) and the other from 1920 (fig. 2). These seventy-five years represent a span of only one generation.

The 1920 periodic table has many discrepancies, apparent now in retrospect. First, there are elements not yet discovered and named. Uranium (element 92) is the heaviest element, so that all elements of greater atomic number are not yet known. In addition, elements 43 (technetium), 61 (promethium), 72 (hafnium), 75 (rhenium), 85 (astatine), and 87 (francium) had not been isolated and identified. Second, there are some inconsistencies in chemical symbols with A (instead of Ar) for argon, Sa (instead of Sm) for samarium, and UX_2 (instead of Pa) for protactinium. Finally, some participants in the name game being played in 1920 are apparent: element 41 was then Cb for columbium (instead of Nb for niobium), and element 86 was then Nt for niton (instead of Rn for radon). It's also evident that the modern periodic table was just developing in the 1920s—for example, there are too many rare earth elements and no place for hafnium.

A compilation of the elements in 1933 still listed columbium as element 41, even though the modern name for element 86 (radon) was established by this time. But this list also identified alabamine as element 85 and virginium as element 87. Recently discovered elements announced at this time were illinium and masurium. Most of you have probably never heard of these ill-fated elements!

Many of these ongoing battles in the name games of the elements in the main portion of the periodic table were not resolved until 1949. Table 2 lists the decisions of IUPAC at this time—it was always an either/or decision for IUPAC, which would choose one name that was already in common use for that element. In fact, since the claimed discovery of "columbium" in the early 1800s, the battle between niobium versus columbium had raged for over 100 years. This name game was very nationalistic—columbium was the favored name for many years in the United States (Columbia was an early name for America) and niobium elsewhere. But it was not always the United States against the rest of the world. Glucinium was the favored name for element 4 in France, and cassiopeium for element 71 in Germany. Each nation and its neighbors tend to use the name suggested by its favorite son, the purported discoverer of that element from that country. It's not unexpected, then, that for the past twenty years element 104 is rutherfordium in American and English texts but kurchatovium in Russian and Scandinavian textbooks.

As another example of a name game from the past, the search for the missing alkali element—element 87, or eka-cesium—was in full gear in the 1920s and 1930s. The various names suggested by those who believed that they had isolated the element were russium, alcalinium, virginium, and moldavium, before final credit in 1949 was given to the one who provided the name francium. The not very subtle nationalism in most of these names identifies the location of the claimed discoverers. Is this really that much different from the four current names for element 104?

Figure 3 (on the next page) shows the steady progression in the discovery of elements over time. All the elements through atomic number 112 have now been discovered. The only new elements can be those appended to the end of the periodic table. The linear extrapolation of figure 3 would indicate that scientists will continue to manufacture and discover new elements. Is there any limit to incrementally adding protons to the atoms to synthesize more and more transfermium elements? Some scientists actually predict a sea of more stable atoms as analogs of thallium, lead, and bismuth follow on the periodic table. So scientists will continue naming the elements, and probably continue

Table 2

Atomic number	Official name	Symbol	Competitive usage
4	beryllium	Be	glucinium (France)
41	niobium	Nb	columbium (US)
43	technetium	Tc	masurium
61	promethium	Pm	ilinium, florentium
71	lutetium	Lu	lutecium (US), cassiopeium (Germany)
72	hafnium	Hf	celtium
74	wolfram	W	tungsten (English and Romance languages)
85	astatine	At	alabamine
87	francium	Fr	virginium
91	protactinium	Pa	protoactinium

Acceptance of new official names for the elements, circa 1950.

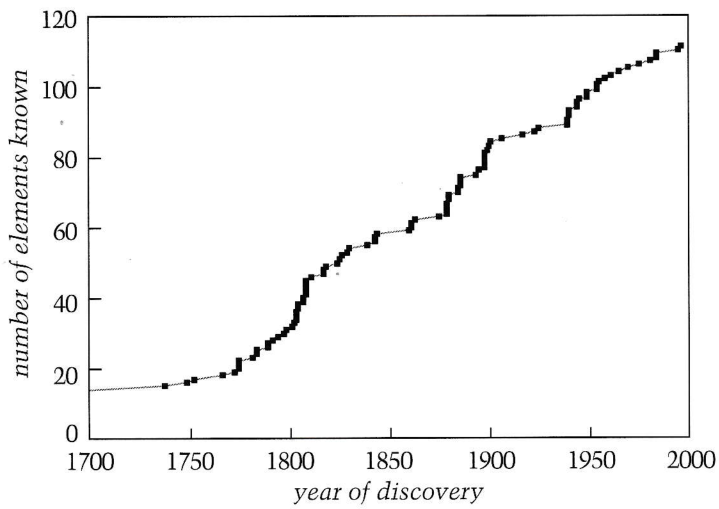


Figure 3
Discovery of elements versus time.

disagreeing about what the names should be!

Fashions in naming

Much of the wisdom in naming new elements has relied on the discretion of the discoverer. But over the course of the last three hundred years, there have been subtle but dramatic shifts in the nomenclature of the elements.

One way to illustrate these shifts is to group the element names into several classifications. The first group contains those elements whose names were constructed from words portraying a special chemical or physical property of the element, or from words indicating a unique mode of discovery. Typically, the root words for such names have a Greek or Latin basis. Examples include chlorine from the Greek *khloros*, meaning yellow green, the color of the gas; lanthanum from the Greek *lanthano*, meaning to hide or to escape notice, as it had been "hiding" in a mineral that had been previously used to isolate cerium over 30 years prior; and radium and radon from the Latin *radius*, meaning ray, both being radioactive. The second group consists of those elements named after celestial bodies or mythological figures. For example, elements 93 and 94 are named neptunium and plutonium respectively, after the planets Neptune and Pluto. Just as these two elements are beyond uranium (element 92, named after the planet Uranus) on the periodic table, their respective planets are beyond Uranus in the solar system. From mythology, vanadium is named after Vanadis, the Norse goddess of beauty, to signify the beautiful colors imparted by the compounds of vanadium. A third group of elements has names representing places or locations—whether the place of discovery, the homeland of the discoverer, or the location of the mineral from which the element was isolated. Americium, californium, and berkelium were all synthesized in laboratories in Berkeley, California. Erbium, terbium, ytterbium, and yttrium are all named for Ytterby, a quarry near Stockholm containing the rare-earth minerals from which the elements were extracted. The final group contains elements named after individuals, such as curium after Marie and Pierre Curie, and einsteinium after Albert Einstein.

Several elements have been known since antiquity. Names such as carbon, iron, lead, and tin among others simply represent those materials. Of the next twenty elements discovered, from 1699 to 1802, three quarters of them had constructed names, primarily originating from Latin or Greek root words describing some unique property of the element. Most of the remaining elements in this time frame were named after heavenly or mythological figures. Likewise, the next twenty elements isolated (1803 through 1829) had names that could be placed in the same groups with about the same percentages. Of the next twenty elements discovered (1830 through 1886), about half still had constructed names, but now about 40% had names signifying places or locations. The same distribution was followed for the names of the next twenty elements in chronological order. Through these first two centuries of modern chemistry there appears to be a subtle shift from constructed names with Latin/Greek roots or with planetary origins to names representing places and locations.

Even more surprisingly, of the first 93 elements discovered, only two were named for individuals—albeit indirectly as quirks of fate. The elements were actually named for the minerals from which they were isolated. However, the minerals happened to have been named after an individual. Thus, the Finnish chemist Gadolin became the origin for gadolinium; and Col. Samarsky, a Russian mine official, was elevated to a status probably never envisioned by him with the naming of samarium.

When the last 16 elements were discovered, from 1944 to the present, about three quarters of their names honor a certain individual. Most of the rest identify a certain place or location. But such are the trends or fashions in the elemental name game—several centuries ago, the fashion was to construct a name for the discovered element based on some property described with Latin or Greek root words. More recently in the late 1800s and early 1900s, there was a trend toward using names that represent locations, often very nationalistic names. Recently, there has been a very dramatic shift toward using the proposed name of an element to honor a distinguished individual.

What, oh what, to name a transfermium element?

When it comes to naming artificial elements past uranium, it's hard to argue against names such as curium, einsteinium, or mendelevium—in honor of great scientists, regardless of nationalities. However, of the names of the transfermium elements listed in table 1, many of the individuals being honored through such names are not as immediately recognizable to the average scientist. For example, an American scientist would probably not have direct knowledge of the individuals after which kurchatovium and flerovium are named; likewise, a Russian scientist would not easily recognize the individuals behind lawrencium or seaborgium. Because the discovery of new elements currently relies on nuclear chemistry and physics for preparation, isolation, and identification, the proposed names for transfermium elements honor primarily those who worked in these fields. The scientists Davy and Ramsey, who discovered 11 elements between them in the 1800s, never had their names immortalized in an element name. Such was not the fashion in naming in those years.

Element names constructed from Latin or Greek root words expressing some characteristic property of the element are not so nationalistic, and thus would not be so offensive and controversial among groups claiming priority in an element's discovery. But then how do you name an element whose most distinguishing feature is that it does not exist very long? Brevium, formed from a root word indicating that it was only observed for a brief period of time, was initially suggested for the name of element 91 (protactinium) by its discoverers. Unfortunately, there are only so many appropriate Greek and Latin words indicating radioactive and short-lived, especially for the most recently discovered transfermium elements.

Perhaps the name game of the elements has shifted to honoring individuals by default. For the transfermium elements it's difficult to prepare meaningful, constructed names. The planets out to Pluto have already been used; the significance of mythological comparisons is lost when not much is known about the element; and most of the relevant locations have pretty much been taken already. The three major locations for work on synthesizing transfermium elements involve laboratories directed by American, German, and Russian scientists. Americium, californium, and berkelium already signify the American work; germanium was taken many years ago; and ruthenium (from the Latin for Russia) was also previously employed as an element name.

What else can the American, German, and Russian teams employ for names based on locations? Even the name hassium, suggested by the German discoverers to honor *Hassia* (the Latin name for the state of Hesse in Germany) for element 108 was ruled unacceptable by IUPAC in its recent decisions. It rationalized that such a name would not be readily recognizable or be associated with Germany. But then how many scientists know hafnium and lutetium were named for Copenhagen and Paris, respectively, after their Latin names? How many American scientists would realize that dubnium, the IUPAC alternative name for element 104, honors the location of Dubna in Russia?

Using the provisional IUPAC name of unnilquadrium for element 104 is also unacceptable to most scientists. The name lacks the character and the flavor of other element names. Besides which, it is customary for the discoverers to suggest a name! So the discoverers may only be left with names of individuals after which to name elements.

The arguments over naming the elements have grown more passionate because the discoverers chose to honor individuals whom they admire. It is much easier to fight or to take up a cause for a person or a country than for a chemical property! Naming an element after someone is the highest honor that can be paid to that person. The elements contribute to the foundation of modern science—the elements will be here forever. This name will be said in the same breath as elements such as carbon, iron, and oxygen. Nobel prize winners can be forgotten after a few years or a generation. Whereas a certain level of confusion in the name game of the element has always been present over the years, the passion and fervor associated with the game has been elevated to new heights with the names of the transfermium elements.

Politics as usual

Intense national rivalry exists in the discovery of a new element. It's obviously much more impressive to be the discoverer than to be the second and simply confirm that indeed someone else had made the element previously. When others are unable to repeat the claimed discovery, problems result with defining who should receive credit for the discovery. Often years pass before conclusive and unbiased judgments are forthcoming. IUPAC commendably provided compromise choices for the element names that tried to alleviate the nationalism, pride, envy, and politics.

In trying to please some of the scientists some of the time with their choices, IUPAC actually only succeeded in alienating, disappointing, and angering most. As outsiders to the heavy-element community, its members didn't realize the sensitivities of the groups involved. They essentially disenfranchised the discoverer of the element from the honor of naming the element. In fact, they further aggravated the situation by playing musical chairs with the names for the transfermium elements. As recently as 1949, when IUPAC made decisions on the element names summarized in table 2, it selected either one or the other suggested name. Now, as shown in table 1, IUPAC developed new names, such as dubnium, and also scrambled names from one element to another.

To take one example, the American community of scientists feels very strongly about naming element 106 seaborgium in honor of Glenn Seaborg, a pioneer in this field and codiscoverer of 10 elements. The discovery of this element by the American group was not contested by others. However, it would also have represented the first naming of an element after a living person. Despite the absence of rules to the contrary, IUPAC decided to rationalize that it was inappropriate to do so because that person's accomplishments cannot be assessed from the perspective of history. It choose to recycle rutherfordium—the suggested name for element 104—as its choice in the name game.

IUPAC wanted to resolve the name game confusion of the transfermium elements rapidly, since it had only become more confusing in the past 30 years. In doing so, it created more confusion than was resolved. After all, it took over 100 years to clear up the niobium–columbium battle, so what's the hurry? IUPAC sought to force everyone to accept its decisions for the definitive names of elements 104 through 109 in 1994. By 1995, in an unprecedented move, it reconsidered and downgraded its slate of names back to provisional status.

Especially troublesome for IUPAC is that others may usurp its authority by ignoring or disregarding its decisions. For example, in 1949 IUPAC defined wolfram (W) as the official name for element 74 (see table 2). How many science texts, however, use wolfram instead of the commonly accepted name tungsten? Likewise the American Chemical Society in 1995 decided that it will adopt the names recommended by its committee on nomenclature (basically those names proposed by the American and German discoverers) for use in their journals and abstracts. Thus IUPAC, unless it goes along with the American Chemical Society, will find its names of the transfermium elements ignored by a major segment of scientists.

Scientists take pride in being recognized for their accomplishments, one of which is their "right" to name their discoveries. It's truly incredible that others would remove or transfer such names from one element to another without consulting the discoverers. Stay tuned for the next few years, guaranteed to bring much more confusion, attempts to compromise, anguish, and just plain bickering. The current status of the transfermium elements puts me in mind of a cartoon I saw a while back. The character Ziggy is driving along and passes a sign telling him he is leaving one town, and up ahead is another sign telling him the name of the adjoining town. The first road sign says "LEAVING CHAOS," and the second says "ENTERING UTTER CHAOS."

Much as the best way to coach a football game is on Monday morning after the game has been played, the best way to say what was right in this new period of the elemental name game is several decades from now. Perhaps thirty years in the future, a new generation of scientists will be scoffing at this confusion, or perhaps it will be just an obscure footnote in the overall history of chemistry—much like our current knowledge of the niobium–columbium controversy in the not too recent past. After all, Shakespeare once penned:

> . . . that which we call a rose
> By any other name would smell as sweet.

Element 104 has 104 protons and will always have the same nuclear and chemical properties, whether it is called rutherfordium, kurchatovium, dubnium, or just #104.

Henry D. Schreiber *is a professor of chemistry at Virginia Military Institute in Lexington, Virginia. His e-mail address is hs@vmi.edu.*

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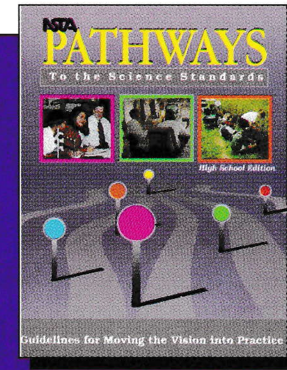

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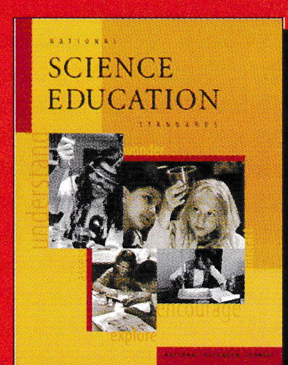

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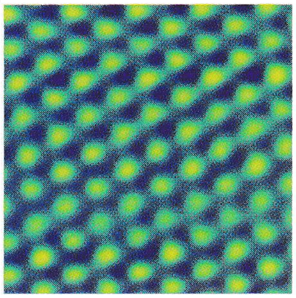

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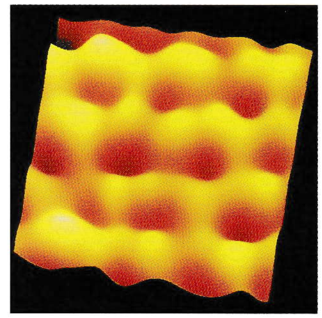

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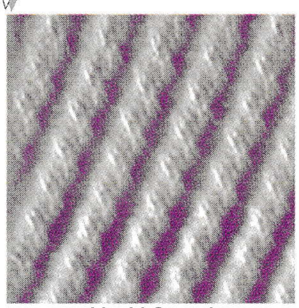

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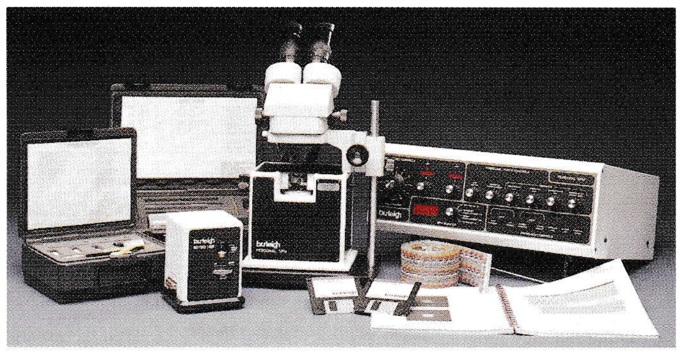

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—Chinese astronomical text, ca. 2nd century B.C.

I. The Earth is the center of the universe.
II. The Earth is stationary.
III. All the heavenly bodies move around the Earth.
—Ptolemy's postulates, 2nd century A.D.

"Off to sea from port we ventured,
The land and the towns receded."
—Virgil, 1st century B.C.

Are you relati

The great minds of the

"If we impart some movement to the Earth, this motion would manifest itself also in everything that is outside the planet, but only in the opposite direction, as if it were passing by . . ."
—Nicolaus Copernicus (1473–1543)

AS YOU SEE, THE CONCEPT of relativity has intrigued many of the outstanding minds of the past—Chinese astronomers, Roman poets, and naturalists of many countries. It was a challenging problem in ancient times, in medieval times, and in modern times. This notion is related both to very ordinary earthly phenomena and to the basic structure of the universe as a whole. Once it arose in attempts to describe the simplest forms of motion, it "incorporated" itself into the most fundamental problems of modern science, forcing a reconsideration of many concepts that had seemed unshakable cornerstones of science. One can surely say that relativity runs all through the history of physics. And you must admit there aren't many such concepts.

Now let's see how deeply the concept of relativity has become rooted in *your* mind.

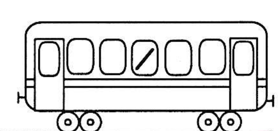
Questions and problems

1. So, which is it: does the Earth revolve around the Sun, or vice versa?

2. What shape is the Moon's trajectory?

3. What is the trajectory of a point on an airplane's propeller relative to (a) the pilot; (b) the ground?

4. The figure below shows the trajectory of a raindrop on the window of a train car. Can we determine the direction of the train's motion?



5. A stone tossed upward slows during the first half of its trip and picks up speed during the second half. Does this mean that in the first half its acceleration is negative, and in the second it's positive?

6. Smoke is coming out of smokestacks. Carried by the wind, the smoke forms long plumes. Can two of the smoke plumes intersect?

7. When is it possible for the pilot of a fighter jet to see an artillery shell flying nearby?

8. An up escalator moves with a speed of 0.75 m/s. How fast must a person walk on the up escalator so as to move downward at the same speed as persons standing on the down escalator?

9. A ball is tossed up with a velocity v_0. When it reaches the top of its trajectory, a second ball is thrown upward with the same velocity. What is the relative speed of the two balls?

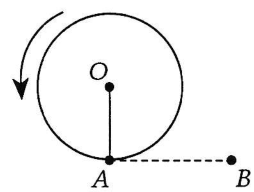
10. Can two points A and B move along parallel lines in one reference system and along intersecting lines in another?

11. A boat and a raft float side by side down a river. What would take less effort for the rower—to row 15 m in front of the raft or 15 m behind?

12. Why do airplanes almost always take off and land into the wind?

13. There was an airplane race from New York City to Washington, D.C., and back. All the while a stiff wind blew from New York to Washington. Will the flight times be better or worse because of the wind?

14. A round horizontal platform rotates about its axis as shown in the figure below. An observer A stands on the platform and another observer B stands on the ground. The distance OB is twice the distance OA. At the moment depicted in the figure, A moves toward B with a velocity of 1 m/s. What is the velocity of B relative to A?



15. A boy standing on a railway flatcar moving with a velocity of

atively sure?

f the past were, too!

"Many a time, sitting in my cabin, I asked myself whether the ship moved or not. Sometimes I believed that the ship drifted in one direction while in reality it moved in the opposite direction."
—Galileo (1564–1642)

"Two events that are simultaneous when viewed from one coordinate system are not perceived to be simultaneous when observed from a system moving relative to the given system."
—Albert Einstein (1879–1955)

"In essence, absolute space is related to nothing external and is always the same and stationary."
—Isaac Newton (1642–1727)

30 m/s shoots a pneumatic rifle. The velocity of a pellet as it leaves the rifle is also 30 m/s. Will the pellet have any kinetic energy?

16. A load suspended from a long string (a pendulum) is attracted not only to the Earth, but also to the Sun. Will it lean a little to the east in the morning and to the west in the evening?

17. When two combs whose teeth are spaced differently move relative to each other, an observer can see the shifting dark and bright bars. Can the bars move with a speed greater than the speed of light?

18. Quasars are among the most distant objects in outer space. One of them is moving away from the Earth at half the speed of light. It radiates light that can be detected on the Earth. What is the speed of this light relative to us?

Microexperiment

While traveling in a railway car, look at the train coming in the opposite direction. Why does it seems that your motion is drastically slowed just as this train goes by?

It's interesting that . . .

. . . observing the heavenly bodies, Ptolemy himself pointed out that their diurnal motion could be explained either by the Earth's rotation or by the rotation of the entire universe.

. . . the Copernican system was a revolutionary step not only with respect to the Church (the Earth and human beings were no longer the center of the universe), but also from the viewpoint of mechanics—up to that time the relativity of motion wasn't used in solving concrete problems.

. . . Galileo's classical principle of relativity was his answer to the criticism of the Peripatetics (followers of Aristotle). They considered the Earth stationary because flying birds do not lag behind it; the range of catapults aimed toward the west is no greater than their range to the east; heavy objects fall vertically; and so on.

. . . in an elevator accelerating upward, a horizontal beam of light acquires a parabolic curvature as if affected only by the gravitational field. This is just one example of the phenomena that raised doubts about the universal applicability of Euclidean geometry and led to the creation of the theory of relativity.

. . . according to the special theory of relativity, the angle between the diagonals of a square moving in the direction of one of its sides with a velocity of 270,000 km/s becomes 48ϒ due to the shortening of this side.

. . . Hendrik Lorentz, the author of equations that form the basis of the special theory of relativity, never could agree with Einstein's basic notion of the relativistic nature of simultaneity. To the end of his life he tried to prove the possibility of the existence of absolute time.

. . . a striking example of time slowing down is the disintegration of muons arriving at the Earth from outer space. (The muon is a negatively charged particle whose mass is 207 times that of the electron.) Their lifetime in the laboratory frame of reference is several times longer than in their own reference frame.

. . . Michelson and Morley used an optical interferometer in their famous experiment. This device was so sensitive that it could detect the time difference corresponding to light traveling just a few meters—a mere 10^{-16} s. Keep in mind that the experiment was conducted in 1881, when no electronic devices or computers existed!

—Compiled by A. Leonovich

ANSWERS, HINTS & SOLUTIONS ON PAGE 62

The bombs bursting in air

"Education is the art of making man ethical."
—Georg Hegel

by Arthur Eisenkraft and Larry D. Kirkpatrick

IS EDUCATION INTENDED TO expand our horizons or to make us conform? Can the knowledge we acquire be a tool to maintain the status quo and instill specific values in us?

When we learn arithmetic, we assume that the information is value free. What hidden message could be sheltered in the equation 3 + 4 = 7? In some children's books, this problem is illustrated with three apples and four apples. We can imagine another primer illustrating the problem with three machine guns and four machine guns. Does the choice of illustration promote values?

There is such a history in physics texts, as well. As we peruse the introductory physics texts on our shelves, we find the following trajectory problems:

A rescue plane is flying at a constant elevation of 1200 m with a speed of 430 km/h toward a point directly over a person struggling in the water. At what angle of sight ϕ should the pilot release a rescue capsule if it is to strike (very close to) the person in the water? (Halliday, Resnick, and Walker, *Fundamentals of Physics*, Wiley, 1993)

In the 1968 Olympics in Mexico City, Bob Beamon shattered the record for the long jump with a jump of 8.90 m. Assume that the speed on takeoff was 9.5 m/s. How close did this world-class athlete come to the maximum possible range in the absence of air resistance? The value of g in Mexico City is 9.78 m/s^2. (Halliday, Resnick, and Walker, *Fundamentals of Physics*, Wiley, 1993)

A golf ball hit with a 7-iron soars into the air at 40.0 degrees with a speed of 54.86 m/s. Overlooking the effect of the atmosphere on the ball, determine the range and where it will strike the ground. (Hecht, *Physics*, Brooks/Cole, 1994)

In contrast, the predominant problems in the older texts appear to be illustrated by these examples:

A bomber is flying at a constant horizontal velocity of 820 miles/hr at an elevation of 52,000 feet toward a point directly above its target. At what angle of sight ϕ should a bomb be released to strike the target? (Halliday and Resnick, *Physics*, Wiley, 1966)

The projectile of a trench mortar has a muzzle velocity of 300 ft/s. Find the two angles of elevation to hit a target at the same level as the mortar and 300 yd distant. (Sears and Zemansky, *College Physics*, Addison-Wesley Press, 1948)

The angle of elevation of an anti-aircraft gun is 70° and the muzzle velocity is 2700 ft/s. For what time after firing should the fuse be set, if the shell is to explode at an altitude of 5000 ft? Neglect air resistance. (Sears and Zemansky, *College Physics*, Addison-Wesley Press, 1948)

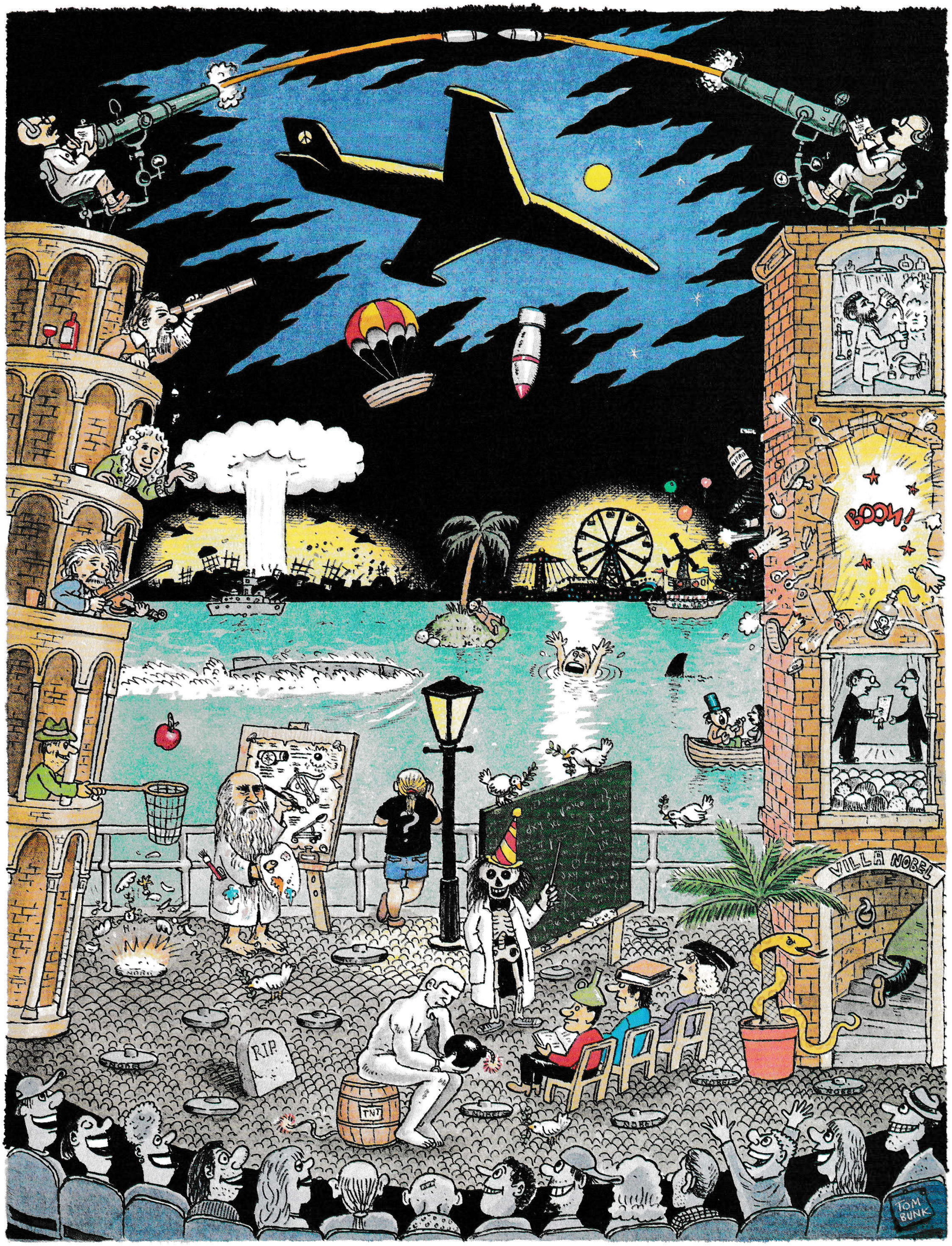
All of these problems have similar solutions. We can analyze the trajectory of any object (without air resistance) by recognizing that the horizontal and vertical motions are independent of one another. The horizontal motion has a constant velocity and the vertical motion has a constant acceleration. The standard kinematic equations for motion in one dimension—

$$s = v_{av}t,$$
$$s = \tfrac{1}{2}at^2 + v_i t,$$
$$v_f^2 = 2as + v_i^2,$$
$$v_f = at + v_i$$

—permit us to find the time in the air, the range of the projectile, or whatever else is required in the problem.

As an example, let's solve the rescue plane problem given above. The initial velocity of the capsule is the same as that of the plane. That is, the initial velocity v_0 is horizontal

Art by Tomas Bunk



BOOM!
VILLA NOBEL
R.I.P.
TNT
TOM BUNK

and has a magnitude of 430 km/h. We can find the time of flight of the capsule:

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2,$$

$$t = \sqrt{2\frac{(y_0 - y)}{g}} = \sqrt{2\frac{(1200\text{ m})}{9.8\text{ m s}^2}} = 15.6\text{ s}.$$

The horizontal distance covered by the capsule in that time is

$$\begin{aligned} x - x_0 &= v_{0x}t \\ &= (430\text{ km/h})(15.6\text{ s})(1\text{ h}/3600\text{ s}) \\ &= 1{,}860\text{ m}. \end{aligned}$$

The angle of sight can be calculated by comparing the horizontal and vertical displacements:

$$\phi = \arctan\frac{x}{h} = 57.2°.$$

The pedagogical/social question is more difficult than the physics: is the selection of examples and problems of concern? Does it make a difference if we learn to solve projectile problems using sports and rescue planes or mortar shells and bombs? What is your opinion?

As the second part of our contest problem, we ask you to solve a difficult projectile problem. A fireworks aerial display is shot into the air and explodes isotropically (uniformly in all directions) into a very large number of fragments. At some time t_1, the first fragment(s) will begin to hit the ground. At some later time t_2, the final fragment(s) will hit the ground. Neglecting the effects of air resistance, at what time will the frequency of fragments hitting the ground be the greatest?

We pose the problem with a fireworks display. One can easily imagine how the same problem could have military applications in terms of bomb fragments or interference with ground radar. There is an answer to the second part of our contest problem. As students and instructors of physics, we should reflect and discuss the first part as well.

Please send your responses to *Quantum*, 1840 Wilson Boulevard, Arlington VA 22201-3000 within a month of receipt of this issue. They will serve as the basis for a continuing discussion in a future issue of *Quantum*.

Sea sounds

In the March/April issue of *Quantum*, we asked our readers to analyze the propagation of sound waves in the sea. A very good solution was jointly submitted by André Cury Maiali and Gualter José Biscuola, who are physics teachers in Jundiaí, São Paulo, Brazil. We will follow their reasoning in our solution.

A. We can show that the sound rays follow circular paths by utilizing a spreadsheet or resorting to calculus. Knowing that the general formula for a circle in the xz-plane is given by

$$(x - x_c)^2 + (z - z_c)^2 = R^2, \qquad (1)$$

where the center of the circle is located at (x_c, z_c) and R is the radius of the circle, and recognizing that Snell's law is given in terms of the angle θ, we decide to find the coordinates x and z as functions of θ.

Let's restrict ourselves to an upward-pointing ray in the region above the minimum sound speed (that is, $z > 0$ and $\theta < \pi/2$). From the symmetry of the problem, we will obtain the same results for $z < 0$. We substitute the relationship for the variation of the sound speed into Snell's law—

$$\frac{\sin\theta_0}{v_0} = \frac{\sin\theta}{v} = \frac{\sin\theta}{v_0 + bz} \qquad (2)$$

—and solve for z to obtain

$$z = \frac{v_0}{b}\left(\frac{\sin\theta}{\sin\theta_0} - 1\right).$$

Anticipating that we will eventually need the derivative, we have

$$\frac{dz}{d\theta} = \frac{v_0}{b}\left(\frac{\cos\theta}{\sin\theta_0}\right). \qquad (3)$$

Let's now look at the slope of the curve:

$$\frac{dz}{dx} = \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta = \frac{\cos\theta}{\sin\theta}. \qquad (4)$$

We can also use the chain rule to write

$$\frac{dz}{dx} = \frac{dz}{d\theta}\frac{d\theta}{dx}. \qquad (5)$$

Substituting equations (3) and (4) into equation (5), we obtain

$$\frac{\cos\theta}{\sin\theta} = \frac{v_0}{b}\frac{\cos\theta}{\sin\theta_0}\frac{d\theta}{dx}.$$

We now solve for dx—

$$dx = \frac{v_0}{b\sin\theta_0}\sin\theta d\theta$$

—and integrate:

$$x = \frac{-v_0\cos\theta}{b\sin\theta_0} + C.$$

For $x = 0$, $\theta = \theta_0$. Therefore, $C = v_0/b\tan\theta_0$, and finally

$$x = \frac{v_0}{b\tan\theta_0}\left(1 - \frac{\cos\theta}{\cos\theta_0}\right). \qquad (6)$$

In a similar fashion we obtain

$$z = \frac{v_0}{b}\left(\frac{\sin\theta}{\sin\theta_0} - 1\right). \qquad (7)$$

Moving the constant terms to the left-hand sides of equations (6) and (7), squaring them, and adding them together, we find that

$$\left(x - \frac{v_0}{b\tan\theta_0}\right)^2 + \left(z + \frac{v_0}{b}\right)^2 = \left(\frac{v_0}{b\sin\theta_0}\right)^2,$$

which has the form of equation (1). Therefore, the path is that of a circle of radius

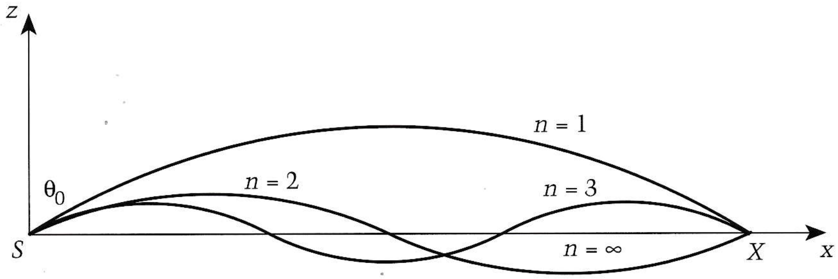
$$R = \frac{v_0}{b\sin\theta_0}$$

centered at

$$\left(\frac{v_0}{b\tan\theta_0}, \frac{-v_0}{b}\right).$$

B. The smallest value of θ_0 that can occur without the sound ray hitting the surface is obtained when the circular path is tangent to the surface of the sea. This requires that

$$R = z_s + |z_c|,$$



or

$$\sin\theta = \frac{v_0}{bz_s + v_0}.$$

C. In the figure above we have drawn four possible paths (including the direct path) the sound rays could take from S to X. For each case the length of the chord of the circle must be X/n. Therefore,

$$2R\cos\theta_0 = \frac{X}{n} = 2\frac{v_0\cos\theta_0}{b\sin\theta_0},$$

yielding a series of values for θ_0:

$$\tan\theta_0 = n\frac{2v_0}{bX}, \quad n = 1, 2, 3, \ldots.$$

D. For the given data we have

$$\theta_0 = 86.19° \text{ for } n = 1,$$
$$\theta_0 = 88.09° \text{ for } n = 2,$$
$$\theta_0 = 88.73° \text{ for } n = 3,$$
$$\theta_0 = 89.05° \text{ for } n = 4.$$

Note that the limiting value is $\theta_0 = 90°$, as expected.

E. For the axial path, we simply divide the distance by the speed to obtain the time taken:

$$\Delta t_\infty = \frac{X}{v_0} = 6.667 \text{ s}.$$

For the circular arc we need to add up the times for a large number of small pieces of the path. This can be done using a spreadsheet or by integrating. Let's do the latter:

$$dt = \frac{ds}{v} = \frac{Rd\theta}{v}.$$

Rather than simply plugging in the expressions for R and v, we note that

$$R = \frac{v_0}{b\sin\theta_0} = \frac{v}{b\sin\theta},$$

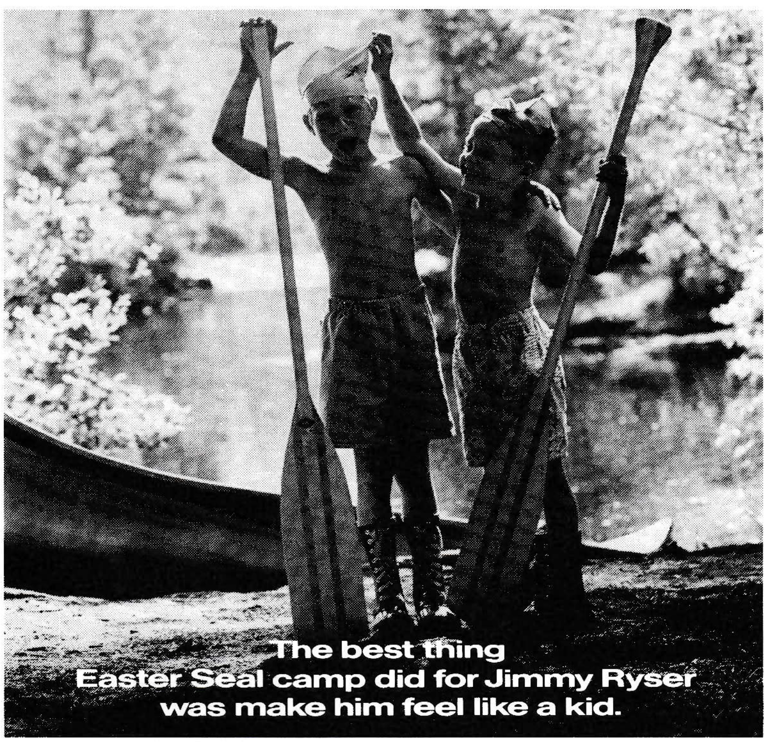
according to Snell's law (equation (2)). Therefore,

$$dt = \frac{d\theta}{b\sin\theta}.$$

To simplify the integration we take advantage of the symmetry and only integrate to the top of the path and then multiply by 2:

$$\Delta t_1 = 2\int_{\theta_0}^{\pi/2}\frac{d\theta}{b\sin\theta} = \frac{2}{b}\left[\ln\tan\frac{\theta}{2}\right]_{\theta_0}^{\pi/2}$$
$$= -\frac{2}{b}\ln\tan\frac{\theta_0}{2}.$$

This gives a time Δt_1 = 6.6546 s, which is a *shorter* time than for the direct ray. ◘



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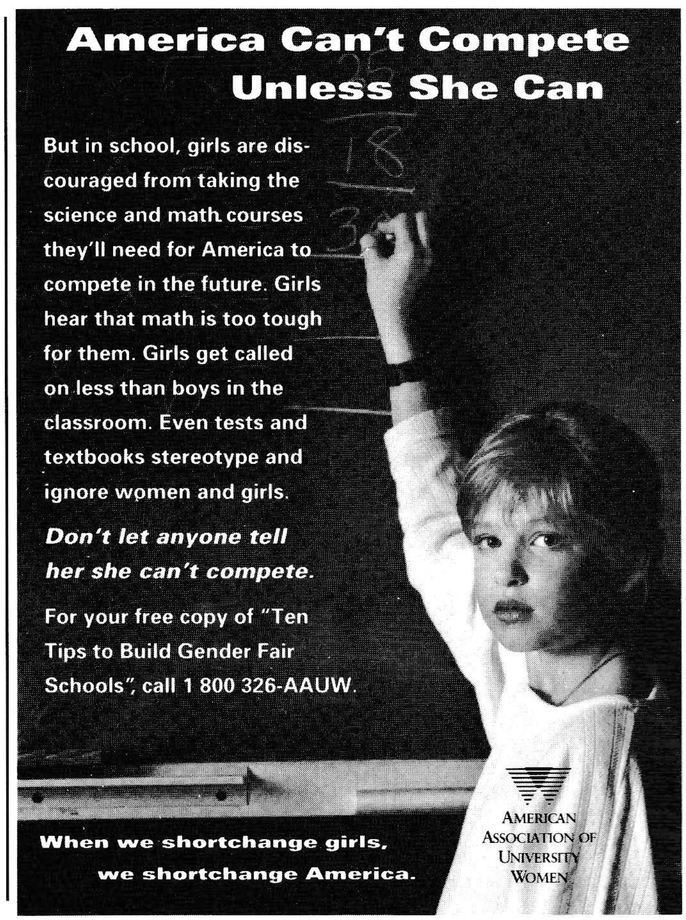
Easter Seals



Embedding triangles in lattices

With thanks to the Math.Note enthusiasts of the '80s at DEC

by George Berzsenyi

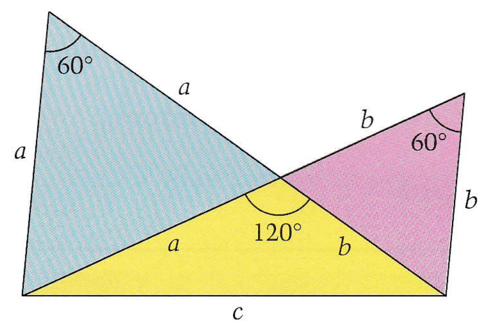
TWELVE YEARS AGO I SENT the following problem to my friend Stanley Rabinowitz, who was working for DEC (Digital Equipment Corporation). At that time, Stan and his colleagues maintained a file named Math.Note at DEC, through which they shared with one another interesting problems and their deliberations on them. Later Stan sent me a hard copy of this file, which is the source of much of the information below. Here is the problem I sent: **Are there three points in the three-dimensional lattice (of points with integer coordinates) that form a triangle with integer sides and a 120° angle?**

The answer to this question turned out to be negative, and I hereby challenge my readers to rediscover the elegant proof devised by Stan's colleagues. Before doing so, they may wish to treat the same problem in the two-dimensional setting, which should be somewhat easier.

When Stan posed the problem to the participants of Math.Note, he remarked that if $A = (0, 0, 0)$, $B = (1, 1, 10)$, $C = (4, 1, 15)$, then ΔABC has a 120° angle (at C), though none of its sides are of integer length. Hence one may also ask: **Are there triangles with a 120° angle in the three-dimensional lattice such that one or two of their sides are of integer length?**

Interestingly, the answer in the four-dimensional lattice is positive, as exemplified by the triangle with vertices $A = (0, 0, 0, 0)$, $B = (10, 0, 0, 0)$, $C = (-3, 5, 1, 1)$, which has a 120° angle at the origin. It should be noted that the angle enclosed by two vectors $\mathbf{u}$ and $\mathbf{v}$ is given by $\cos^{-1}(\mathbf{u} \bullet \mathbf{v}/|\mathbf{u}||\mathbf{v}|)$, where $\mathbf{u} \bullet \mathbf{v}$ is the dot-product of $\mathbf{u}$ and $\mathbf{v}$. The question remains: **Are there infinitely many such triangles in the four-dimensional lattice?** Moreover: **Is it possible to realize all of the post-Pythagorean triangles in the four-dimensional lattice?** If the answer is no, can one succeed in higher dimensional space? More specifically: **Is there a minimum value of *d* such that the answer is yes in the *d*-dimensional lattice?** For more information on such triangles, the reader is referred to the author's column in the March/April 1992 issue of *Quantum*, where triangles with integer sides and an angle of 120° were called post-Pythagorean. To generate all post-Pythagorean triangles with sides a, b, c, one can use the formula $(a,b,c)=(m^2-n^2, 2mn-m^2, m^2+n^2-mn)$, where m and n are positive integers with $n < m < 2n$.

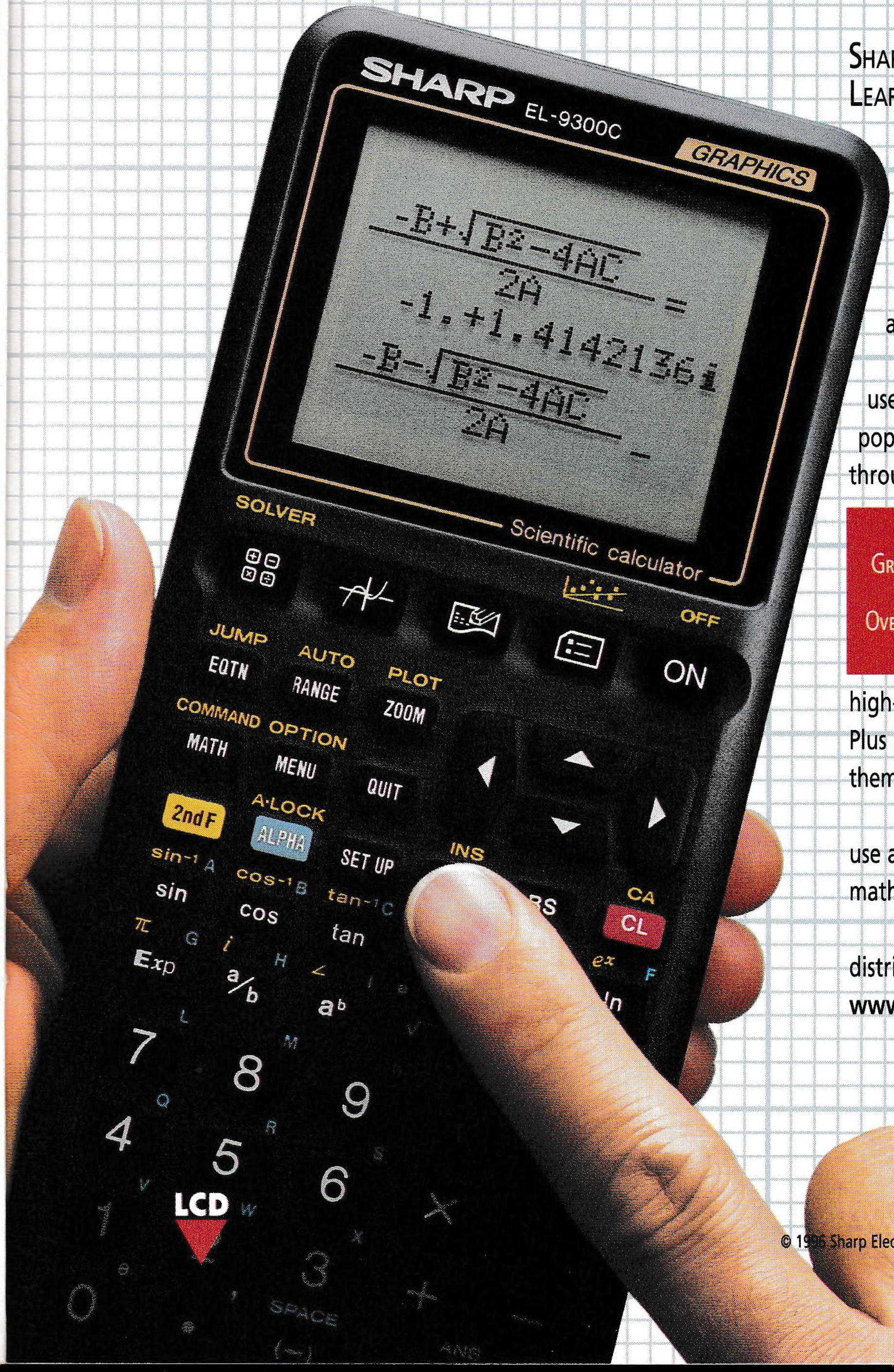
Similar questions need to be asked about pre-Pythagorean triangles, which were defined in the column referenced above as triangles with integer sides and an angle of 60°. One can generate the pre-Pythagorean triangles by adding equilateral triangles to the post-Pythagorean ones; this can be done in two different ways, as shown in the figure above. Stan's colleagues at DEC didn't consider pre-Pythagorean triangles, so all of the above questions are still open.



In conclusion it should be mentioned that in addition to the Pythagorean triangles (which have integer sides and an angle measuring 90°, only the post- and pre-Pythagorean triangles have integer sides and an angle measuring an integral number of degrees—hence the special interest in them.

Please send your findings to me c/o *Quantum*, 1840 Wilson Boulevard, Arlington VA 22201-3000. Perhaps they will generate further discussion in this space.

NOTHING MAKES PYTHAGORAS LESS GREEK TO YOUR STUDENTS.



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The ashen light of the Moon

The how, when, and why of a faint lunar glow

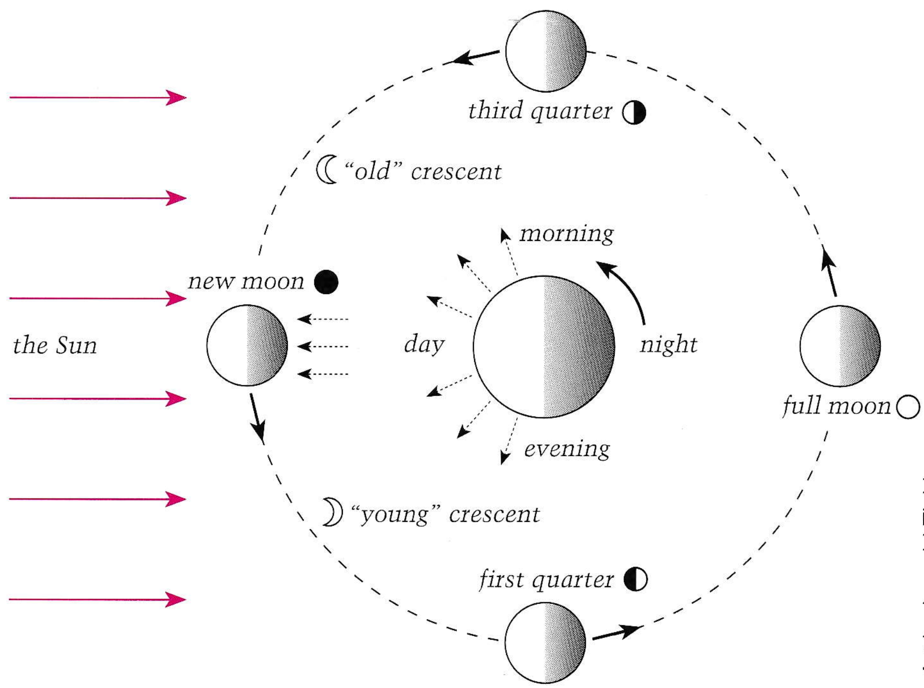
by Alexey Byalko

EVERYONE IS FAMILIAR WITH lunar radiation—that is, the sunlight reflected by the Moon's surface. But have you ever noticed the weak light given off by a new moon on a clear night? This "ashen" light can reliably be seen for only two or three nights around the time of a new moon, when the little lunar crescent is sufficiently narrow and its radiance doesn't keep us from seeing the weak light from the other part of the lunar disk. Under these conditions the disk gives off some light, and the entire disk is outlined against the black background of the sky. What causes this radiation?

As you know, every month (strictly speaking, every 29.5 days) the relative positions of the Sun, Earth, and Moon almost repeat themselves. The word "almost" is due to the fact that the Moon's orbit is tilted a bit (by just 6°) relative to the plane of the Earth's orbit and isn't exactly circular. However, this imprecision will not be important here.

Look at the figure: the Sun illuminates the Earth and the Moon, which revolves around the Earth (the rotation of the Earth and the revolution of the Moon proceed in the same direction). The Sun is far away, so it isn't pictured here, and the sunbeams are shown as parallel rays. One hemisphere of the Earth and of the Moon are illuminated—it is nighttime on the dark half of the Earth. Naturally, the Moon is better viewed at night, and if there are no clouds, light in the Earth's atmosphere will have practically no effect on our observations. Looking at the figure, you can see why every month the phases of the Moon change: new moon, first quarter, full moon, third quarter.

By the way, do you know how to determine quickly, just by looking at the Moon, whether a crescent moon is waxing or waning? (This is admittedly child's play, but the hardest questions to answer quickly



Art by Leonid Tishkov


Тишков 1994

and correctly are often those that have only two possible answers.) The Russians have a mnemonic device based on the Cyrillic alphabet: one of the lunar crescents looks like the letter "c," which could stand for старый ("old"); and if you add a vertical stroke to the other crescent, you get "р" for раний ("early").

To invent an analogous device in English, we need to find a word beginning with "c" that reminds us of "waning" (say, "crumbling") and a word starting with "p" that relates to "waxing" ("plumping"?). The physicist Lev Landau invented another mnemonic device: "If you feel like petting the Moon, it's young"—that is, waxing. (Clearly Landau wasn't left-handed!)

Keep in mind that these mnemonic devices are not universally applicable—they were invented by people in the northern hemisphere. In Australia, for example, the mnemonic devices would work in reverse (if one could get them to work); and in the tropics no such devices are suitable at all, because there the horns of the crescents point up and down. Still, there is a rule that can be used at any latitude on the Earth: if you see the Moon in the Morning, it is Waning; and when it shines in the Evening, the crescent is Enlarging. Here the capital letters form a kind of symmetry, as shown at the table below:

Morning	**E**vening
Wane	**E**nlarge

In the Russian variant, the first letters exactly coincide:

Утро	**В**ечер
Убывает	**В**озрастает

If you refer to the figure, you'll see why this occurs. The figure presents a view of the Earth–Moon system from the North Pole (from the North Star, really). You need to look at this figure in a mirror to see how it looks from the South Pole (or from the Southern Cross).

This figure also helps us understand that the extra illumination from the Moon (its "ashen light") is caused by sunlight reflected from the Earth. This radiance is particularly strong in the new moon phase, when the Moon is dark and when the entire globe of the Earth as seen from the Moon is illuminated by the Sun. Let's estimate how much weaker this ashen light is than the Moon's usual radiation.

To do such an estimate, we need to know how light is reflected by the Earth and Moon. Their surfaces diffuse the incident light, but not evenly in all directions. So in order to calculate the relative luminance of simultaneously observed lunar ashen light and the reflected sunlight of a thin crescent, we need to know how the diffused light propagates in different directions. This is not a simple problem. However, we can obtain the relative luminance quite easily when the Moon is full, because in both cases the light is diffused in a similar way (predominantly backward). Thus instead of luminance we can compare total amounts of light.

The fraction of sunlight reflected into space by a heavenly body is known as its albedo. The Earth's light is reflected by its atmosphere—by clouds in particular, which cover about a half of the Earth's surface. On average the Earth's albedo is about $A_E = 30\%$, although this value changes slightly depending on whether it is day or night over the Pacific Ocean, which occupies almost an entire hemisphere. The Moon, on the other hand, has no atmosphere, and its soil is rather dark, so it absorbs most of the incident light. On average the lunar albedo is $A_M = 8\%$ (when the Moon is full).

The illuminating power of lunar light coming to the Earth depends on the phases of the Moon. At full moon, the entire hemisphere illuminated by the Sun can be seen from the Earth; at the first and last quarters, only half of the illuminated hemisphere is seen; and at new moon, we see the Moon as dark—seen only by its ashen light.

The energy flux of the solar radiation at the distance of the Earth is $S_0 = 1{,}360\ \mathrm{W/m^2}$. Since the distance between the Earth and the Moon is far less than the distances between these two bodies and the Sun, we can assume that equal fluxes of solar light hit the Earth and the Moon. Let's estimate the total power of the sunlight reflected by the Moon and the Earth. If R_M is the radius of the Moon, then the Moon receives an illuminating power $S_0 \pi R_M^2$, and the corresponding reflected power will be

$$P_M = A_M S_0 \pi R_M^2.$$

Similarly the total power of the sunlight reflected by the Earth is

$$P_E = A_E S_0 \pi R_E^2.$$

Now let's consider the Earth as a point source that evenly radiates its reflected light into a hemisphere (there is only a small inaccuracy here). Then the energy flux striking the Moon is $S_1 = P_E/2\pi a_M^2$, where a_M is the distance from the Earth to the Moon. The total illuminating power of the lunar ashen light is then

$$P_{ME} = A_M S_1 \pi R_M^2 = \frac{A_M A_E S_0 \pi R_E^2 R_M^2}{2a_M^2}.$$

Comparing this value with the illuminating power of the Moon at full moon yields a simple formula:

$$\frac{P_{ME}}{P_M} = A_E \frac{R_E^2}{2a_M^2} = \frac{1}{24000}.$$

Since the geometry of the reflection is identical in both cases, the relation deduced for illuminating power will be correct for luminance as well: the ashen light of the Moon is weaker than its reflected sunlight by a factor of about 24,000.

Our eye is made in such a way that, if we squint, we can look briefly at the blazing Sun. We can also observe the sunlit Moon, whose illuminating power $(A_M R_M^2/2a_M^2)$ is weaker by a factor of 2.5 million. And we can even discern the ashen light of the Moon, which is weaker still by a factor of 24,000. Yet even this is far from the limit of the eye's sensitivity!

So why is it that we so rarely (once in a blue moon!) see the ashen light of the Moon? The background glow of the Earth's atmosphere prevents us from seeing this light clearly. If we make our observations in the morning or in the evening (not very late), the atmospheric light is the result of the diffusion of sunlight at high altitudes. In the dead of night the sky shines due to the lights in urban areas. The lunar crescent makes its own contribution: during the first or third quarter it is big enough to outshine the ashen light of the dark part of the Moon—the part not illuminated by the Sun. Also, the radiation of the sky is drastically increased by a light cloud cover or haze. So the ashen light of the Moon can be seen only on very clear nights and when the crescent is rather thin.

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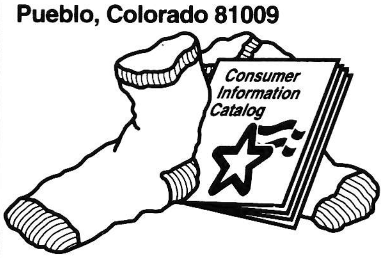

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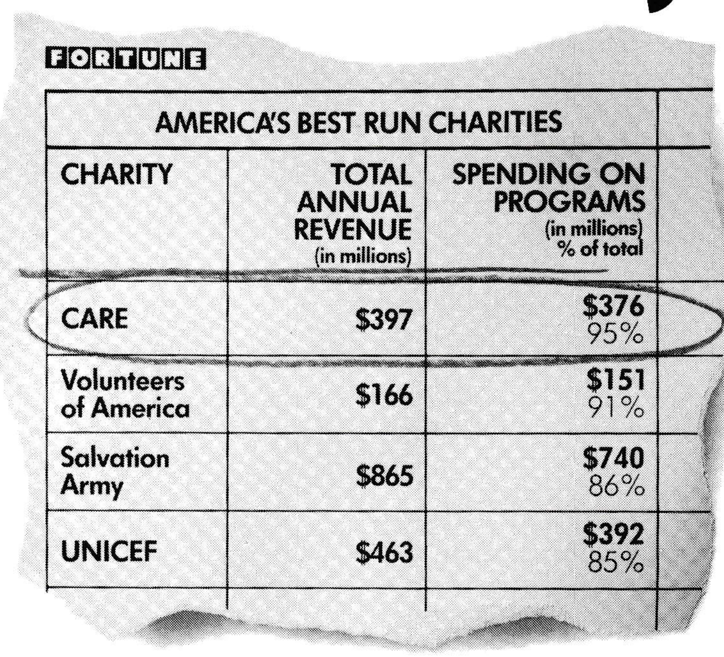

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HOROLOGICAL SURPRISES

Confessions of a clock lover

The cosmic consequences of switching hands

by V. M. Babović

MANY PEOPLE HAVE HOBbies, and their hobbies can be quite diverse. My hobby is clocks. Simply put, I adore everything that measures time. My friends know about my passion, and yet many of them are astonished to see my collection when they visit my apartment. I have wrist watches, pocket watches, wall clocks, alarm clocks, cuckoo clocks, factory punch clocks . . . all different sizes and designs. You can even find a variety of hourglasses on my shelves.

As you know, the minute hand of a clock is always longer than the hour hand. I don't really understand why. Sometimes a person only wants to know approximately what time it is. Wouldn't it be nice to have a wall clock whose longer hand points to the hours? Then you could easily see that the time is now, say, between ten and eleven o'clock, which may be all you need to know at the moment.

Well, I ordered a clock with just such a feature. The clock maker was a little confused by my request, but he did a professional job with the strange dial and delivered the device on time. I have placed this clock—whose long hand indicates hours and whose short hand indicates minutes—in a prominent place in my living room. I call it the *inversion clock*.

Art by Sergey Ivanov

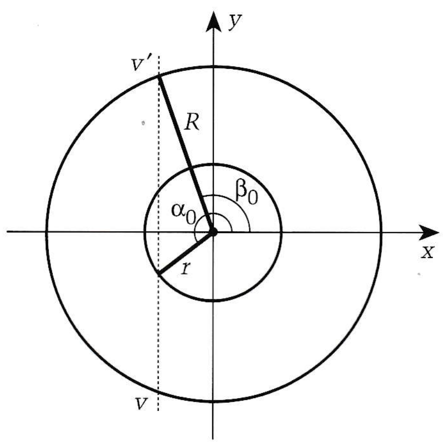

Figure 1

My inversion clock is a beautiful thing to look at. The bluish face, with the main shaft right in the middle, has been overlaid with a map of the world. Vertical lines indicate the Earth's time zones, and these lines will play a prominent role in the story I am about to tell.

I soon discovered that the inversion clock possesses some peculiar features unknown in the world of ordinary clocks. One day I noticed the tips of the hands were touching the same vertical line (vv' in figure 1). This fact piqued my curiosity, and from then on I would stare at the clock, estimating when this coincidence would occur again.

I figured out that, for a clock whose small (minute) hand has a length $r = 1$ cm and big (hour) hand has a length $R = 5$ cm, this event can occur only in the time interval $\Delta t \cong 46$ min around the hours of 12:00 and 6:00. Watching outside these two intervals is a waste of time—the vertical arrangement of the tips of the hands is impossible. Figure 2 shows why. The top of the hour hand goes from the left outermost position L to the right position D in the time interval

$$\Delta t = \frac{1}{\Omega}\left(\pi - 2\arccos\frac{r}{R}\right), \qquad (1)$$

where $\Omega = 2\pi/T_h$ is the angular velocity of the hour hand. Here T_h is its period of rotation. Since we have $T_h = 12$ h, equation (1) gives the quoted result of 46 min.

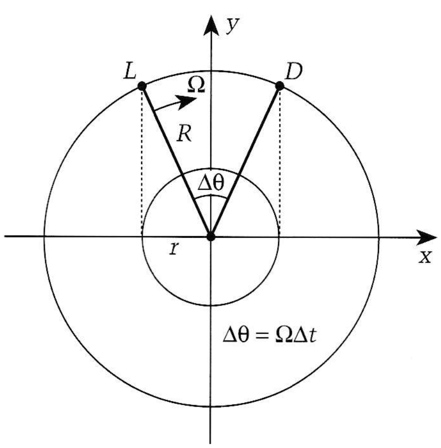

Figure 2

My next discovery came to me when I noticed that, in a cycle lasting for the interval Δt, only one vertical coincidence occurred, in contrast to the previous cycle, which consisted of three events. I wondered what law governed these events. Was it possible to have a cycle consisting of *two* events?

As I prepared to ponder these questions, and possibly others, I came up with a nice analogy. Full of excitement, I returned to my clock maker and ordered another inversion clock, but with the following specifications: $r = 1$ cm, $R = 9.6$ cm, $T_m = 365$ s, and $T_h = 10{,}759$ s. (Eventually I'll tell you why I chose these numbers, and what the analogy actually was.) The clock maker fulfilled my odd request, and I proudly hung this new clock in my office. This particular inversion clock I have named *Copernicus*.

When Copernicus began functioning, the angle of the smaller minute hand relative to the x-axis was $\alpha_0 = 200°$ (refer again to figure 1). The tip of the larger hour hand lies on the same vertical line vv', so

$$\beta_0 = \arccos\left(\frac{r}{R}\cos\alpha_0\right) = 95.6°.$$

The following function shows what happens after that:

$$f = R\cos(\beta_0 - \Omega t) - r\cos(\alpha_0 - \omega t). \qquad (2)$$

Here Ω and ω are the angular velocities $2\pi/T_h$ and $2\pi/T_m$, respectively. The term $R\cos(\beta_0 - \Omega t)$ is the projection (on the x-axis) of the R hand. The second term $r\cos(\alpha_0 - \omega t)$ is the projection of the r hand. Consequently, f is the difference of the two x-projections, and whenever $f = 0$, the tips are located one above the other. The first such moment is the initial position at $t_0 = 0$ s. From figure 3 we see that the next positions occur at $t_1 = 75$ s and $t_2 = 262$ s. So this first cycle contains three instances where the tips align vertically.

About 5,500 s will pass before we again have $f = 0$ (fig. 4a). The new cycle has that one point only—no others. Again we have to wait—this time until $t \cong 10{,}900$ s, when another cycle begins, again with only one event (fig. 4b).

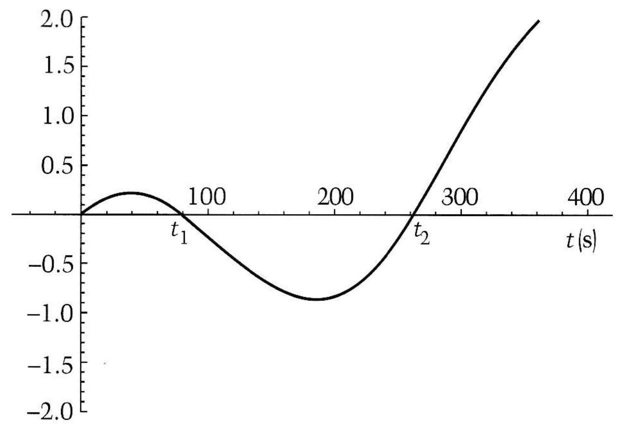

Figure 3

Finally, after about 16,000 s, a cycle begins that contains three events, as with the first one (fig. 4c). Thus the periodicity of this phenomenon is $T = T_h/2 \cong 5{,}380$ s. The "fine" structure of a cycle is determined by the second term in equation (2). Whether the time axis crosses the curve at one or three points depends on the initial conditions—that is, on the values of α_0 and β_0. There is also the possibility that the t-axis could appear as a tangent to a local extreme.

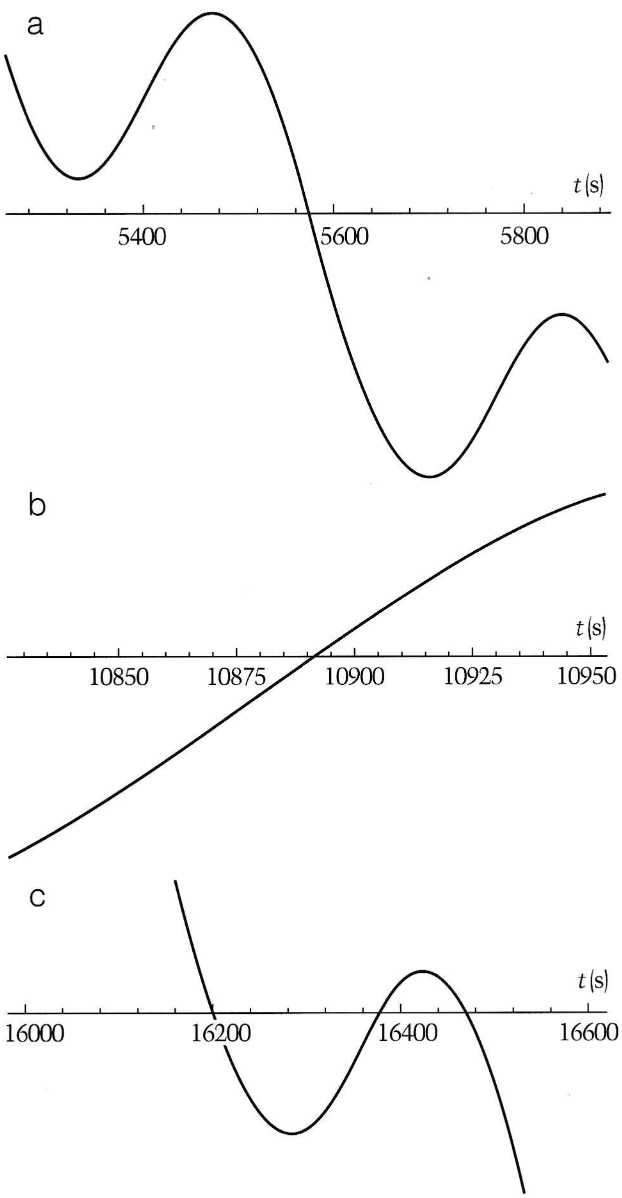

Figure 4

Well, the time has come for me to reveal the secret about Copernicus (the clock, not the person). Imagine that the length r represents the distance from the Earth to the Sun. The Earth moves around the Sun in a nearly circular path at an average distance of about 150 million kilometers—that is, $r \cong 1$ AU (astronomical unit). Let R be the distance between Saturn and the Sun—this is known to be 9.6 AU, on average. We'll designate $T_m = 365$ days (one Earth year) and $T_h = 10,759$ days (one Saturn year—about 29.5 Earth years). Then figure 1 can serve as a model showing the mutual positions of the two planets as they move around the Sun.

In addition to its moons, Saturn possesses a beautiful set of rings. The plane of Saturn's orbit around the Sun is approximately the same as the plane of the Earth's orbit. The inclination of the axis of rotation of each planet to the plane of the planet's orbit around the Sun is always constant. Therefore, the plane of Saturn's rings maintains its orientation in space (due to the law of conservation of angular momentum). In figure 1 the plane of the rings cuts the plane of the planets' orbits along the line vv'. This line must always be parallel to the y-axis, regardless of where Saturn moves (this is ensured by our choice of reference system).

Viewed from Earth through a telescope, Saturn's rings look like a stretched ellipse. When the two planets are on the same vertical plane, however, a relatively rare astronomical event occurs (it happens once every 15 years). The Earth crosses the plane of Saturn's extremely narrow rings (they're about 290,000 km in diameter and probably no more than 1.5 km thick). We see the ring system "edge on," as it were—that is, in profile. The ellipse degenerates—instead of seeing the rings in all their splendor, we can hardly make out a faint line. If viewing conditions are bad, we may not see anything at all. One might say that the celestial "emperor" has no clothes! And this lasts for several months.

Saturn suffered this embarrassment quite recently. During almost all of 1995 the rings were oriented in profile as viewed from Earth. The rings disappeared on the night of May 21–22, 1995. Actually, that night began a nine-month cycle of three disappearances, in qualitative agreement with figure 3. (We need only assert that every second there represents one day.) You can learn more about this phenomenon in an article that appeared in *Sky and Telescope* (May 1956, p. 60).

We'll have to wait another 15 years for the Earth to pass through the plane of Saturn's rings. It will be a single event (according to figure 4a). Around the year 2025, the Earth will again pass through the ring plane only once (fig. 4b). My grandson will be in a position, 43 years hence, to greet Saturn with its rings oriented edgewise. He'll see what I saw last year: a threefold disappearance. He'll read this article (with pleasure, I hope) and make any necessary corrections in figure 4c.

All these facts I gleaned from my Copernicus! Amazing, isn't it? A giant astronomical object has given meaning to my inversion clock. Just one more example of how a theory is born long before its fruitful application. And so, dear reader, never abandon your intellectual constructs, even when you can see no connection to reality . . .

V. M. Babović *is a professor of physics at Svetozar Marković University in Kragujevac, Yugoslavia. He is also faculty advisor of the Belerofont Society, a group of students interested in astronomy.*

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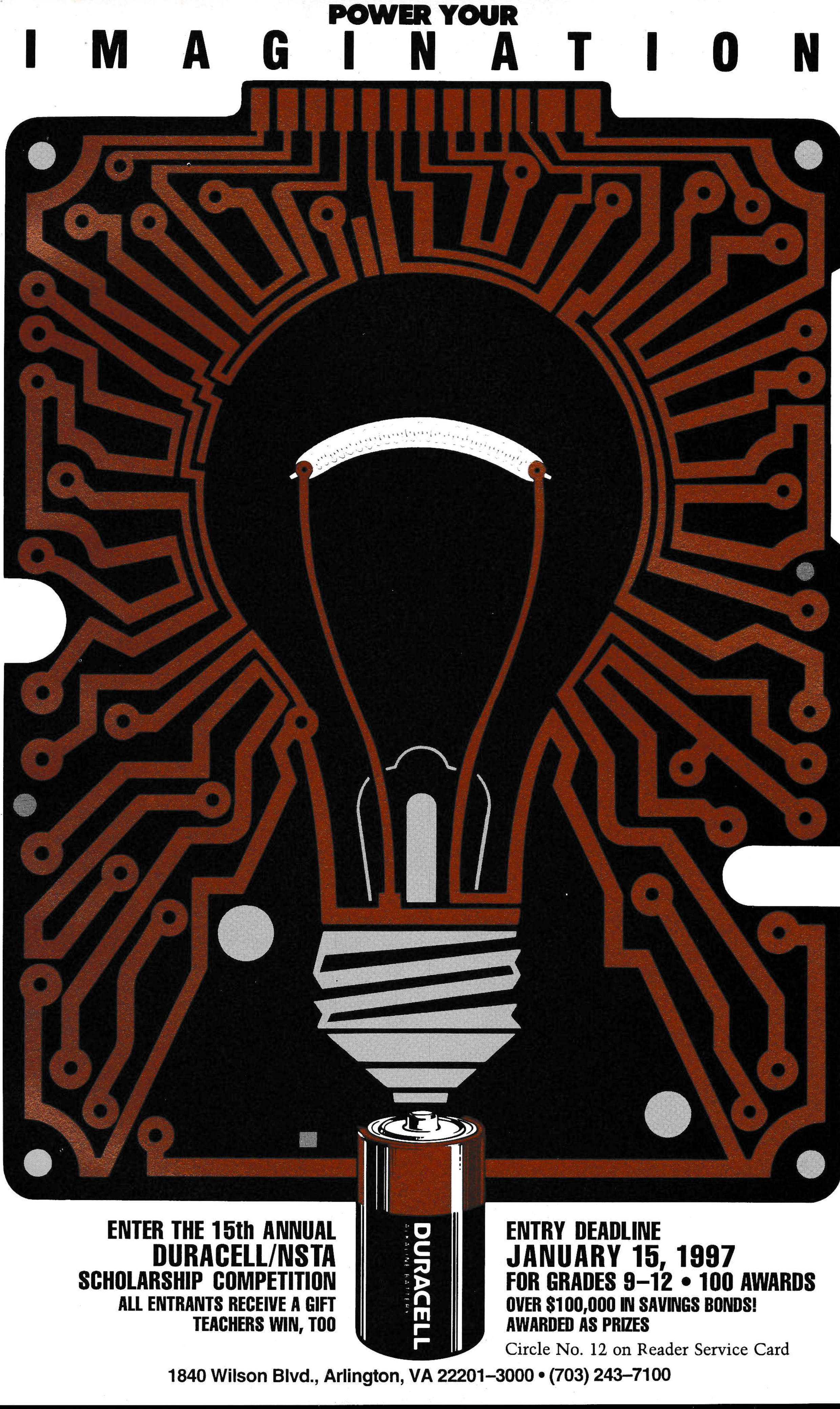

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AT THE BLACKBOARD

Merry-go-round kinematics

A dynamic game of cherry tossing

by Albert Stasenko

WHEN PITZIUS[1] WAS A LITtle boy he liked to invent new games. Once he asked his friend Frieda to throw cherries into his mouth. The distance between the two of them was L, the initial velocity v_0 of the cherry was directed at an angle α (the release angle of each toss)—everything just as you would expect in a physics problem in school. However, Pitzius placed himself at the center of a rotating merry-go-round. Let's study the trajectory of a flying cherry in two systems of reference: one fixed on the ground with Frieda standing on it (the L-system, or laboratory system of reference) and the other fixed on the merry-go-round (the R-system, or rotating system of reference). Pitzius and Frieda weren't keen on writing long, boring formulas—they preferred to draw the trajectory directly.

First of all, let's define the initial data more exactly. Let α be 60°, and let L, v_0, and ω_0 (where ω is the

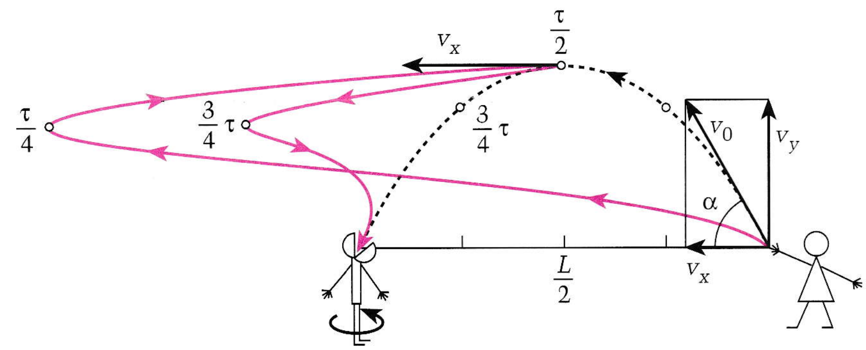

Figure 1

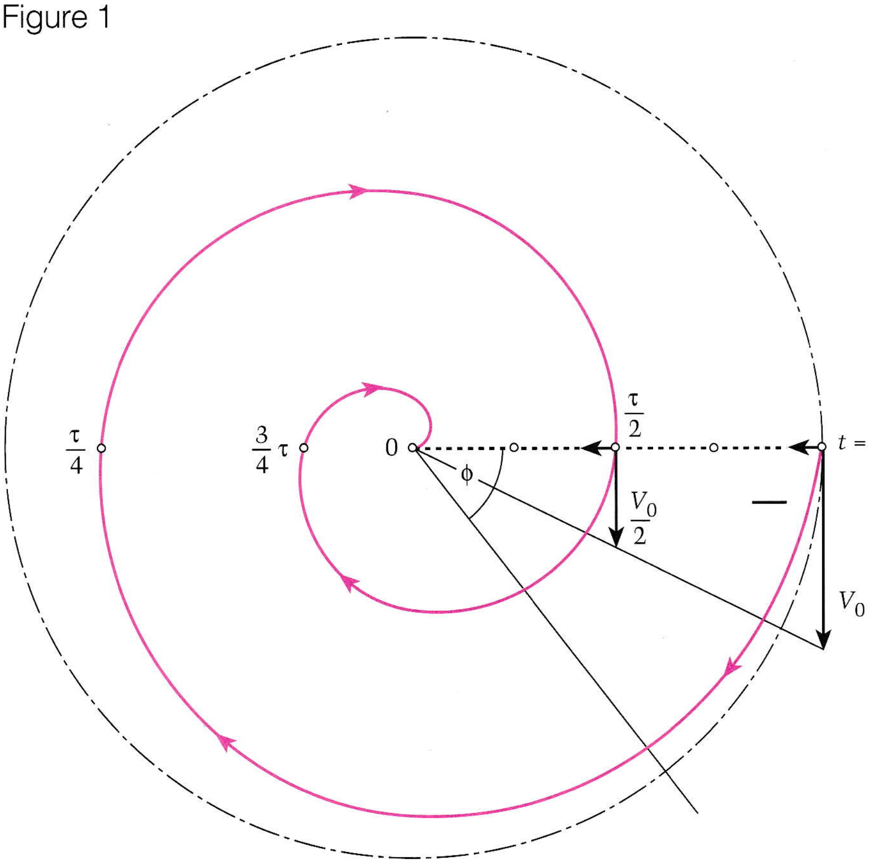

Figure 2

[1]Who is Pitzius, and why does he keep showing up in *Quantum*? (See brainteaser B168 in the March/April 1996 issue.) He has become the source of much bemused speculation among the US staff of the magazine. Do any of our readers recognize him? We'll give a copy of *Quantum Quandaries* to the first person who can tell us who Pitzius is and where he came from. (We'll verify the answer with our colleagues in Moscow, who supplied the original material in both instances.)—*Ed.*

angular velocity) be such that the merry-go-round makes two complete turns in the time τ that it takes for a cherry to arrive.

In the L-system the cherry's trajectory in the vertical plane is a parabola (the side view); in the horizontal plane, it's a straight line (see the broken black lines in figures 1 and 2). When a cherry is thrown ($t = 0$) the horizontal speed is $v_x = v_0 \cos \alpha$ and the vertical speed is $v_y = v_0 \sin \alpha$. (The air resistance is neglected.)

In the R-system Pitzius is at rest, but Frieda revolves in the direction opposite the platform's rotation. Her linear velocity is $V_0 = 2\pi L/T$, and the angular velocity is $\omega_0 = 2\pi/T$. Here $T = \tau/2$ is the period of the platform's rotation (the time necessary for a complete turn). Frieda's trajectory in the R-system is shown in figure 2 by the dot-and-dash line. Since the angular velocity is constant, her angular coordinate and that of the cherry (that is, the azimuth) in the R-system increase linearly with time: $\phi = \omega_0 t = 4\pi t/\tau$.

Let's mark some characteristic points along the cherry's trajectory, which can be found quite easily. For instance, it's clear that at the moment $t = T = \tau/2$ the cherry will be at its highest point at a distance $L/2$ from the axis of rotation. At this point the cherry's horizontal speed (which is constant during its flight) is $v_x = v_0 \cos \alpha$, its vertical speed is zero, and the linear velocity of revolution is $V_0/2$ (since the linear velocities of points on a solid platform are proportional to the distance from the axis of rotation). In time $\tau = 2T$ (two complete turns) the cherry will arrive at the axis, where its linear velocity of revolution is zero, its horizontal speed remains the same (v_x), and its vertical speed is now $-v_y$, which is equal in magnitude to the initial vertical velocity but directed downward.

At moments $\tau/4$ and $3\tau/4$ the cherry will be at the same height, but of course at different distances from the center.

Now that you know how to draw these characteristic points, you can picture the entire trajectory or program your computer to do it. No doubt the computer will be able to show you any projection of the trajectory. The author and the staff artist didn't use a mathematical graphing program to construct the red lines in figures 1 and 2, so you should take them with a grain of salt.

Note that in the L-system the force of gravity is the only force affecting the cherry (we neglected the air resistance, as you recall). However, in the rotating R-system the trajectory is rather complicated, so Pitzius was inclined to think that some extra forces were acting on the flying cherry in addition to mere gravity. His old wizard-teacher Ozz told him that these mysterious forces are known as "inertial."

Now, what if Pitzius throws the cherry pits back at Frieda with the same initial speed and at the same angle α? Surely you can sketch the trajectories in both systems of reference.

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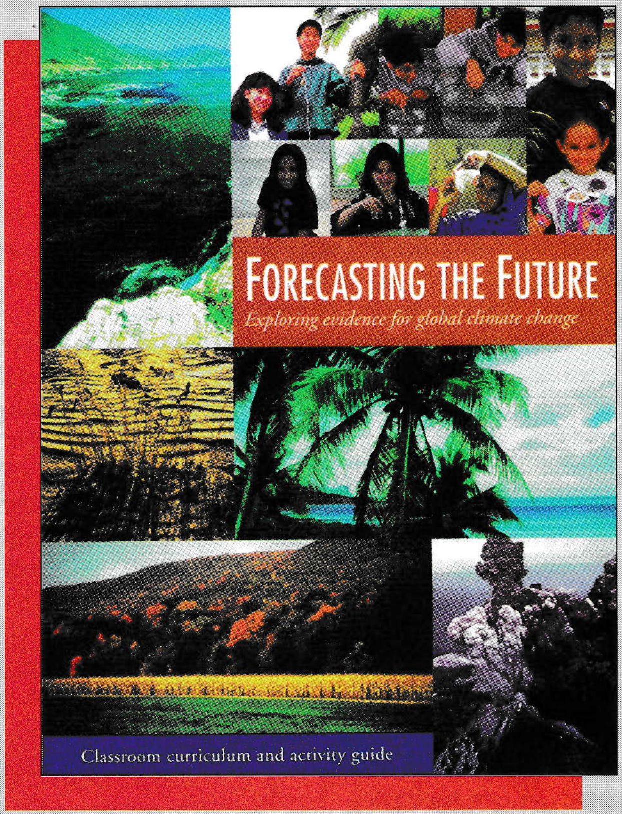

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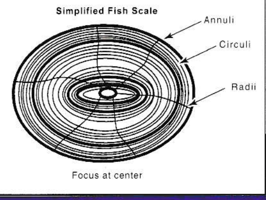

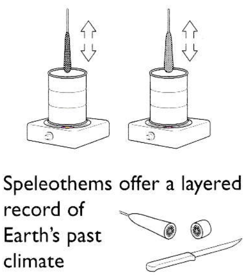

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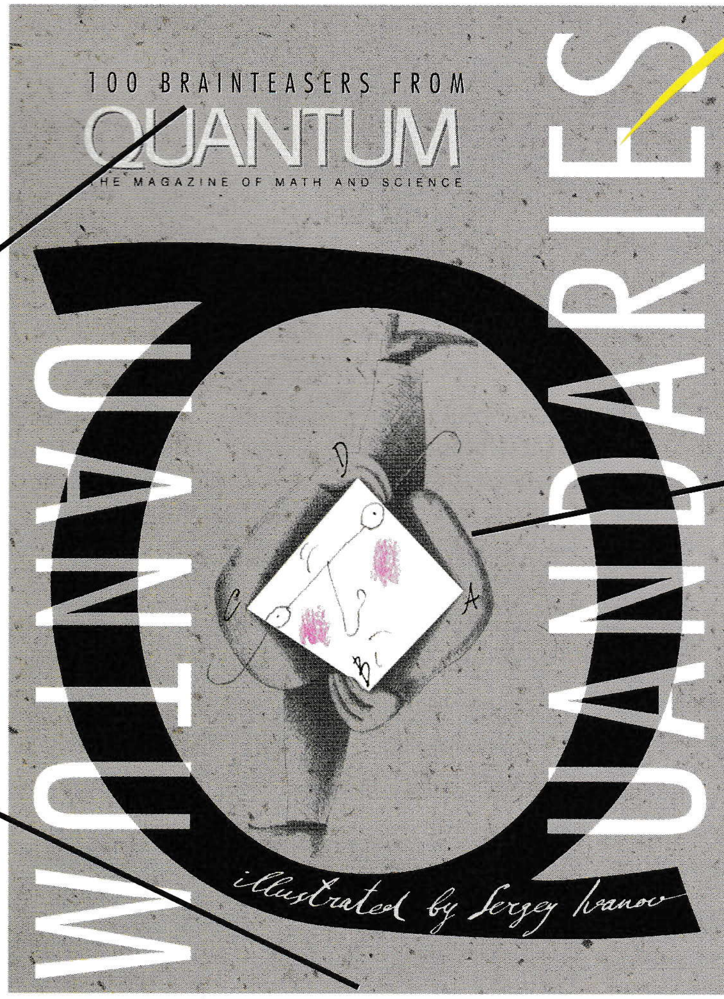

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Vikings and voltmeters

The 1996 International Physics Olympiad in Norway

by Dwight E. Neuenschwander

THIS PAST JUNE 30–JULY 7, five high school students of the United States Physics Team met with teams from 55 other nations, as Norway hosted the 1996 International Physics Olympiad (IPhO) at the University of Oslo. Team USA earned three gold and two bronze medals.

Norway is a land of deep fjords, rugged mountains, midnight sun in summer, and the aurora in winter. Seeing the dense forests of tall spruce trees and numerous mossy boulders, with the high ridges lost in mist, it is clear how people of imagination living here would create a folklore of trolls roaming in the twilight gloom. One recurring character in the Norwegian folk tales is the Ash Lad, the third and youngest son in a poor family. Despised by his elder brothers, the Ash Lad must poke about the ashes, doing all the dirty work. He is the one who, after his brothers have failed, cleverly outwits the troll, and through his persistence and good heart also wins the princess and half the kingdom. At the 1996 IPhO there were over 250 clever, persistent, good-hearted lasses and lads, who, like the Ash Lad, tried to conquer the problems before them. But in their case, the problems were presented not by sturdy trolls, but by tough Norwegian examiners who knew their business and would not be outwitted.

All five members of Team USA returned home from Oslo wearing medals:

Christopher Hirata (gold), Deerfield High School, Deerfield, Illinois

Andrew Houck (bronze), Manalapan High School, English Town, New Jersey

Paul Lujan (bronze), Lowell High School, San Francisco, California

Joon Pahk (gold), Thomas Jefferson High School, Alexandria, Virginia

Ari Turner (gold), Los Alamos High School, Los Alamos, New Mexico

It should be noted that Paul Lujan and Joon Pahk also earned medals in the 1995 IPhO, which was hosted by Australia. The 1996 IPhO organizers also announced a Special Award for "Youngest Medalist." That distinction was won by Christopher Hirata. Chris is 13 years old. In the total number of points accumulated, the

Photos courtesy of Bernard V. Khoury

The five members of the 1996 US Physics Olympiad Team who competed in Oslo (l to r): Chris Hirata, Ari Turner, Paul Lujan, Andrew Houck, and Sang-Joon Pahk.

USA placed third, behind China and Romania. In fourth place was Taiwan, followed closely by Russia, Vietnam, Germany, and Iran.

The 20-member US Physics Team, which the five members above represent, was coached by Jennifer Catelli, Hugh Haskell, Boris Korsunsky, Mary Mogge, Dwight Neuenschwander, and Eric Salter. The training camp was held at the University of Maryland physics department. The US involvement in the IPhO is administered by the American Association of Physics Teachers and the American Institute of Physics, and is sponsored by the ten AIP physics professional societies and by numerous corporations.


The Viking-inspired ship Odins Ravn, *part of a five-ship IPhO flotilla that cruised Oslo Fjord.*

From ram's horn to Kater's pendulum

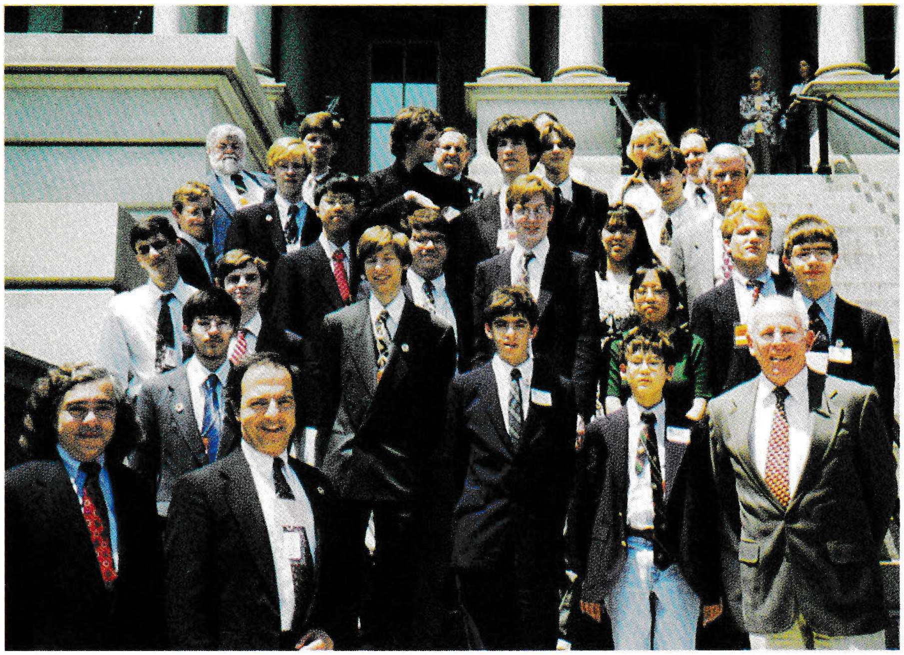
The opening ceremonies of the 1996 Olympiad were held in the Oslo City Hall, the site of of the Nobel Peace Prize presentations. Highlights of the IPhO opening included the teams marching in bearing their national flags, followed by music wonderfully performed by Odd Lund on traditional Norwegian instruments, including the *lur* and the ram's horn. Welcoming comments were presented by the mayor of Oslo and various dignitaries from the host university and the Norwegian Physical Society. Eivind Osnes, chair of the Olso University physics department, said: "Why do we university people take on this event for high school students? . . . It is a privilege to work with young people. As we train them to read the book of nature . . . we are also discovering ourselves."

The heart of any Physics Olympiad is the examination, which has two parts: a theoretical competition, with three problems marked at ten points maximum apiece, and an experimental competition, marked at twenty points maximum. The first problem set in the theoretical exam was a collection of independent problems on the mechanics of skiing, electric circuits, thermodynamics, and magnetism. Problem set 2 asked the students to consider several cases of the motion of an electron in a cylindrical capacitor that could have not only a radial electric field but an axial magnetic field as well. Problem set 3, the most difficult of the theoretical section, modeled the Earth as a water-covered sphere. The student was to calculate, as a function of longitude, the shape of the water's surface in the plane of the Moon's orbit, taking into account the Earth's rotation and the Moon's gravity. A numerical answer for the maximum difference between high and low tides was obtained.

In the first part of the experimental examination, the students measured the period of a physical pendulum consisting of a rod-and-nut arrangement. The threaded rod was suspended by a nut that rocked on a pair of knife blades, one on either side of the rod. The period was measured as a function of the suspending nut's position along the rod. By placing a second nut on the threaded rod, in the next step the student made a Kater's pendulum (having the same period for a second point of suspension) and used it to measure the local gravitational field g to high precision. In the second section of the experimental exam the students studied the optics of the timing system, which worked by reflecting infrared radiation from a concave cylindrical side of the pendulum's suspension nut. In the third set of measurements, using a Hall probe the students measured the field of a magnetic disc as a function of distance along its axis. Then setting the pendulum into oscillation over the disc, and making use of another small magnet embedded in the end of the rod, from their oscillation data and their value of g they measured the magnetic dipole moment of the disc.

Land of the master shipbuilders

During the closed-door coach's meetings where the exams were previewed and discussed, and after exam sessions while grade reviewing was underway, the students were treated to several excursions. They visited the Viking Ships Museum, which houses thousand-year-old ships from the era 850–1050. The skill in design and workmanship of the Viking shipbuilders of old is revealed in three excavated vessels. The Vikings were able to dominate the seas, and thus the economic and political life of their times, because of this advanced technology. Other excursions took various IPhO participants to the Akershus Castle and fortress, dating from 1299,



The twenty-member US Team along with (front row) Ernest Moniz, Associate Director for Science; Bernard V. Khoury, Executive Officer, AAPT; John Gibbons, Assistant to the President for Science and Technology; (back row, center) Jack Hehn.

overlooking Olso harbor; to the museums that housed the 19th-century polar expedition ship *Fram* and Thor Hyerdahl's *Kon-Tiki* and *Ra II*; the Vigeland Sculpture Park; the Holmenkollen Ski Jump built for the 1952 Winter Olympic Games; the Norsk Folkemuseum featuring ancient dwellings and stave churches collected from all over Norway and re-assembled in one setting; and trips to Hamar and Lillihammer, site of the 1994 Winter Olympics. After the competition was over, the entire IPhO assembly boarded five sailing ships for a cruise on Olso Fjord.

The closing ceremony was held on the original campus of the Universitetet i Oslo, located downtown between the Royal Palace and the Norwegian parliamentary building. The special speaker was Dr. Ivar Giaever of Norway, who shares the 1974 Nobel Prize in Physics with Brian Josephson and Leo Esaki for their work on quantum tunneling in superconductors. The audience was treated to additional performances by Odd Lund again on the *lur* and the ram's horn. He was joined by soprano Åshild Watne, who played harps of ancient Norwegian design and sang for us the songs of old Norway. Our musicians, dressed in traditional Norwegian style, took all of us in spirit into forest and fjord of times long ago, using the universal language of music.

"Best advocates of science"

After the closing ceremonies, everyone repaired to the Norsk Sjøfartmuseum (maritime museum) for the closing banquet. Surrounded by carved figureheads from sailing ships of the past, and beneath Charles Krohg's 1893 painting "Leif Eiriksson Sights America," Prof. Eivind Osnes of the Olso University physics department raised some fundamental issues that we seldom have time to discuss in the frenetic activity of the competition itself:

> We regard the Physics Olympiad as a part of our efforts to promote physics in our schools and—in a broader perspective—in our society on the whole.... At the closing of our Physics Olympiad, it may be worth contemplating how we actually are succeeding. ... This much can be said: We have to strike the right balance between knowledge and imagination. ... The biggest problem is not how to teach the good students. One may bore them, but if they have the motivation, they will always survive. It is much more difficult to kindle a desire for knowlege and imagination of physics among the great masses of students, so that physical knowledge and thinking in turn may penetrate our society. ... To facilitate such a development, I [suggest that] you involve yourself in the questions of the everyday life of our society, and not hide away as a remote scientist. We will be the best advocates of science through our dealings with other people.

At the end of the banquet, the program was opened to all participants for musical performances. The musical talents of many of the Physics Olympians are impressive in their own right. But an unexpected high point of the evening came when the teams from China and Taiwan, joined by the team of Singapore, made an impromptu choir of fifteen and sang in unison a song about working together. This event showed what the International Physics Olympiad is ultimately about. As everyone knows, the governments of mainland China and Taiwan have serious political differences, which they will have to iron out between themselves. But with the politicians out of the way, these young people at the IPhO showed how very little interest is held by intelligent, sincere people in pursuing such differences. Two or three decades hence, the students who participated in this IPhO will be among the influential leaders in their respective nation's scientific, educational, industrial, and political establishments. They will go into those positions with friends and colleagues from other nations. The IPhO exists for reasons that transcend cleverness in physics: the physics helps bring *people* together, toward ends that are larger than physics itself. *Takk, og farvel!*

Dwight E. Neuenschwander *is the academic director of the US Physics Team and the director of the Society of Physics Students.*

This article was adapted from one appearing in the *SPS Newsletter* and the *AAPT Announcer*.

Young US mathematicians excel in Bombay

A report on the International Mathematical Olympiad

COMPETING AGAINST TEAMS representing a record 75 countries, six American high school students won six medals and took second place at the 37th International Mathematical Olympiad (IMO) held in Bombay, India, from July 5 to 17, 1996.

The top 10 teams and their scores (out of a possible 252 points) were Romania (187), the United States (185), Hungary (167), Russia (162), the United Kingdom (161), China (160), Vietnam (155), South Korea (151), Iran (143), and Germany (137).

The IMO is a rigorous two-day competition consisting of problems that would challenge most professional mathematicians. In addition to comprehensive mathematical knowledge, success at the IMO requires exceptional mathematical creativity and inventiveness.

Here's a representative question from the 1996 IMO:

Let *ABCDEF* be a convex hexagon such that *AB* is parallel to *ED*, *BC* is parallel to *FE*, and *CD* is parallel to *AF*. Let R_A, R_C, R_E denote the circumradii of triangles *FAB*, *BCD*, *DEF*, respectively, and let *s* denote the semiperimeter of the hexagon. Prove that *s* does not exceed $RA + RC + RE$.

The US team was chosen on the basis of performance in the 25th annual USA Mathematical Olympiad held in May of this year. The training program was held at the University of Nebraska–Lincoln from June 5 to July 3.

The members of the US team were

Carl J. Bosley (Washburn High School, Topeka, Kansas)—gold medalist

Christopher C. Chang (Henry M. Gunn High School, Palo Alto, California)—gold medalist

Nathan G. Curtis (Thomas Jefferson High School for Science and Technology, Alexandria, Virginia)—silver medalist

Michael R. Korn (Mounds View High School, Arden Hills, Minnesota)—gold medalist

Carl A. Miller (Montgomery Blair High School, Silver Spring, Maryland)—silver medalist

Alexander H. Saltman (LBJ High School Science Academy, Austin, Texas)—gold medalist

The team's leader, Titu Andreescu of the Illinois Mathematics and Science Academy, was pleased with the team's performance. "The problems were very difficult and every team member performed to his best potential," he said. "We had an outstanding four-week training program preceding the competition, and our hard work paid off."

Accompanying the team were Kiran Kedlaya, a recent graduate of Harvard University, deputy of the team, and a former US IMO team member; and Walter E. Mientka, a professor at the University of Nebraska–Lincoln and the US team leader observer.

The USA Mathematical Olympiad Activities are sponsored by nine national associations in the mathematical sciences with arrangements made by the Math-

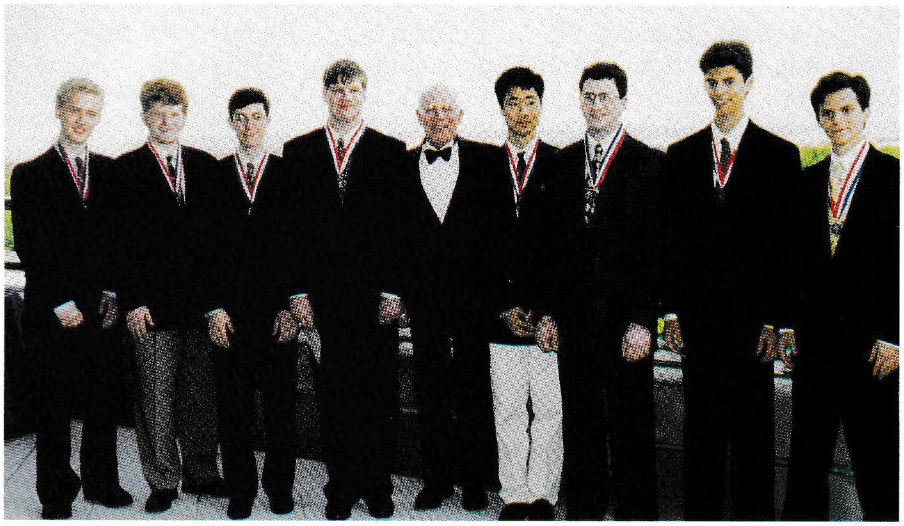


Photo by Robert Allen Strawn

The 1996 USA Mathematical Olympiad winners on the roof of the State Department headquarters in Washington, D.C., where an awards ceremony was held on June 3 (left to right): Carl A. Miller, Josh P. Nichols-Barrer, Michael R. Korn, Nathan G. Curtis, John H. Gibbons (Assistant to the President for Science and Technology), Christopher C. Chang, Carl J. Bosley, Daniel A. Stronger, Alexander H. Saltman.

ematical Association of America. Financial support is provided by the Army Research Office, the Office of Naval Research, Microsoft Corporation, and the Matilda R. Wilson Fund. The American Mathematics Competitions, which manages the examinations that select the USA Mathematical Olympiad winners and the IMO team members, is sponsored by the American Association of Pension Actuaries, the American Association of Two-Year Colleges, the American Mathematical Society, the Casualty Actuarial Society, the Mathematical Association of America, Mu Alpha Theta, the National Council of Teachers of Mathematics, and the Society of Actuaries.

Bulletin Board

First Step to Nobel Prize

Two students from the United States won diplomas and research stays at the Institute of Physics in Warsaw, Poland, on the strength of papers submitted in the international competition "First Step to Nobel Prize in Physics." Mani S. Mahjouri's paper was entitled "Simulation of Charged Particle Motion in Jupiter's Magnetosphere"; Uri Voskoboynik's work was on "Anamolous Field Dependence of the Blocking Temperature of Natural Horse-spleen Ferritin." In addition, two other American students won honorable mentions: Charles Tahan for his research paper "The El Niño Southern Oscillation—A Computational Model Utilizing Satellite Data," and Naomi S. Bates for her paper "Detection of High-Velocity Gas in Face-on Galaxies."

Submissions are now being accepted for the fifth annual competition. The general rules are as follows:

1. All secondary school students regardless of country, type of school, etc., are eligible for the competition. The only conditions are that the school cannot be considered a university college and the age of the participant must not exceed 20 years on March 13, 1997.

2. There are no restrictions on the subject matter of the papers, their level, methods applied, etc. The papers must, however, have a research character and deal with physics topics or topics directly related to physics.

3. Participants can submit more than one paper, but each paper should have only one author. The total volume (text, figures, captions, tables, references, etc.) of each paper should not exceed 25 normal typed pages (about 25,000 characters).

4. The papers will be refereed by the organizing committee and the best will be given awards. The number of awards is not limited. All awards will be considered equivalent. The authors of the prize-winning papers will be invited to the Institute of Physics for a one-month research stay (scheduled for November 1997). Expenses for winners while in Poland will be paid by the Institute of Physics; travel expenses to and from Poland are borne by the winners themselves.

5. In addition to the regular awards, the organizing committee may establish a number of honorable mentions. Participants who win honorable mentions receive diplomas, but they are not invited to the research stay.

6. Participants should send their papers in duplicate and in English only by March 31, 1997, to Dr. Waldemar Gorzkowski, Secretary General of the "First Step," Institute of Physics, Polish Academy of Sciences, al. Lotnikow 32/46, (PL) 02-668 Warszawa, POLAND.

7. **Important**: Each paper should contain the name, birth date, and home address of the author and the name and address of her/his school.

Additional information on the competition and on the proceedings of past competitions can be obtained from Dr. Waldemar Gorzkowski: phone (022) 435212, fax (022) 430926, e-mail gorzk@gamma1.ifpan.edu.pl. Current information on the competition and related topics can be also be obtained by anonymous FTP at ftp.ifpan.edu.pl in the subdirectory pub/competitions.

CyberTeaser dynamos

Some of the early entrants in the September/October CyberTeaser contest at our Web site encountered a "dynamometer" rather than the "spring scale" that ended up in the final wording of brainteaser B183 in this issue. But the exotic terminology didn't seem to faze them, and they weighed in with answers that were generally correct.

The following Web visitors were the first ten to submit an answer that satisfied our cyberjudge:

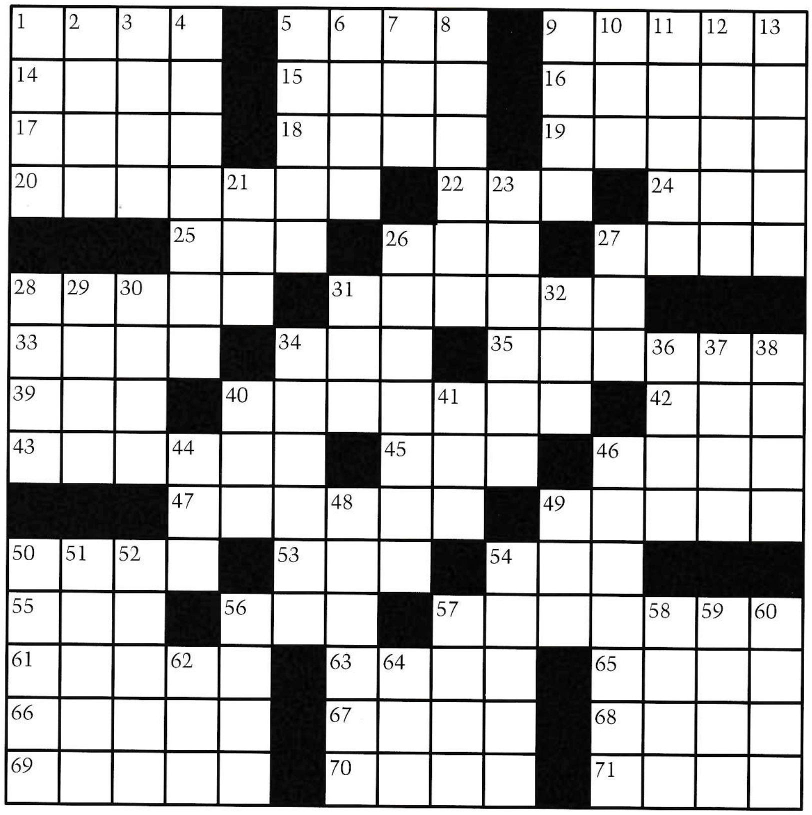
Xi-An Li (Middlebury, Vermont)
Keith Grizzell (Gainesville, Florida)
Roger Igor Khazan (Cherry Hill, New Jersey)
Robert Namestnik (Perth, Australia)
Nikolai Kukharkin (Princeton, New Jersey)
Gerhard Lenssen (Bernkastel-Kues, Germany)
Steven Massing (Wilmette, Illinois)
Jim Paris (Doylestown, Pennsylvania)
Ted Lau (Fenton, Michigan)
Francis Trudeau (Montreal, Quebec)

Congratulations to our winners, who will receive a *Quantum* button and a copy of this issue.

Everyone who submitted a correct answer before it was posted at our Web site was eligible to win a copy of our brainteaser collection *Quantum Quandaries*. Go to http://www.nsta.org/quantum to find out who won the book, and while you're there, try your hand at the new CyberTeaser!

Criss × cross science

by David R. Martin



Across

1 Portuguese neurologist Antonio ___ Moniz
5 44,522 (in base 16)
9 Finnish port
14 Playwright David ___
15 Anesthetized
16 Having spheres
17 Anthropologist ___ Hrdlicka
18 Spheres
19 Shelf
20 Anhydrous sodium carbonate
22 Choose
24 ___ connector (for coaxial cables)
25 Anger
26 Transistor-resistor logic
27 Electron pair acceptor
28 Indian dog
31 Element 27
33 Anger
34 Damage
35 Hungarian wind
39 Geologic unit of time
40 Magnetic field unit
42 Help
43 Gravitational force
45 ___ in the bag
46 Protected
47 Explosive mixture
49 Swollen root
50 Focusing device
53 Computer network abbr.
54 Trig. function
55 One
56 Keyboard letters
57 Type of stress
61 Tooth
63 Vetch
65 Appeal
66 Get ___
67 Latin abbr.
68 ___ and terminer
69 ___ of time
70 Eccentric engine parts
71 Distilled coals

Down

1 Geologic time periods
2 Festive
3 English chem. Frederick Augustus ___
4 Attached to a stem
5 Battery terminal
6 ___ mater (brain membrane)
7 ___ agar
8 Take up liquid
9 Electrical potential unit
10 100 square meters
11 703,932 (in base 16)
12 Italian Pres. (1962–64)
13 712,397 (in base 16)
21 Unrefined metal
23 Flat surfaces
26 Twisting deformation
27 Unit of pressure: abbr.
28 Sketched
29 Rabbit
30 Jap. novelist ___ Mori
31 Auto
32 Display light: abbr.
34 31A and 49D
36 60,075 (in base 16)
37 Widespread
38 Dammed German river
40 Unit of resistance
41 Circuit logic type
44 State of matter
46 Star's dark area
48 Procedural part af a plan
49 Element 50
50 Andean animals
51 ___ Gay
52 Synthetic fiber
54 Plant starters
56 Energy units
57 Neat
58 1977 Chem. Nobelist ___ Prigogine
59 Smile
60 Receptor organs
62 Type of gate
64 Abscisic acid: abbr.

SOLUTION IN THE NEXT ISSUE

SOLUTION TO THE JULY/AUGUST PUZZLE

B	A	S	E		A	Z	I	N	E		D	E	N	E
A	A	E	C		L	E	V	E	L		O	S	A	R
A	B	E	L		G	R	A	T	E		N	A	N	O
A	D	D	I	T	I	O	N		C	L	O	U	D	S
			P	A	T			S	T	E	R			
S	U	B	T	L	E		J	U	R	A	S	S	I	C
T	R	A	I	L		N	O	G	O	D		A	D	A
E	A	A	C		C	O	N	A	N		E	G	E	R
E	R	A		K	O	V	A	R		S	M	E	A	R
N	I	C	H	O	L	A	S		S	O	U	R	L	Y
			E	E	L	S			E	L	L			
S	T	E	L	L	A		K	E	R	O	S	I	N	E
A	E	B	I		G	L	O	V	E		I	D	E	E
B	E	A	U		E	A	T	E	N		O	E	T	A
A	D	A	M		N	O	O	N	E		N	A	S	A

ANSWERS, HINTS & SOLUTIONS

Math

M181

The answer is no. Suppose that, on the contrary, the eight numbers 1, 2, ..., 8 are the roots of the equation $f(g(h(x))) = 0$. For any quadratic polynomial $p(x)$ the equation $p(x_1) = p(x_2)$ holds if and only if $x_1 + x_2 = 2a$, where $x = a$ is the axis of the parabola $y = p(x)$. The numbers $h(1)$, $h(2)$, ..., $h(8)$ are roots of a fourth-degree polynomial, so at most four of these numbers are different. By the remark above, this is possible only if $h(1) = h(8)$, $h(2) = h(7)$, $h(3) = h(6)$, $h(4) = h(5)$ (we must form four pairs i, j such that $i + j$ is the same for all pairs). Since the vertex of the parabola $y = h(x)$ is at $x = 4.5$, the values $h(1)$, $h(2)$, $h(3)$, $h(4)$ form a monotonic sequence.

By a similar argument applied to $f(x)$ and its roots $g(h(1))$, $g(h(2))$, $g(h(3))$, and $g(h(4))$ we find that $h(1) + h(4) = h(2) + h(3)$.

Substituting $h(x) = Ax^2 + Bx + C$ into this equation, we get $17A = 13A$, or $A = 0$. But this is impossible, because in this case our initial equation would be of degree no greater than 4 and could not have eight roots.

M182

Let BC and DE be the diameters in question (fig. 1), O_1 and O_2 the centers of the given circles. Draw perpendicular bisectors to the diameters and denote by F their intersection point. We'll prove that F is equidistant from the endpoints of the diameters, which will solve the problem.

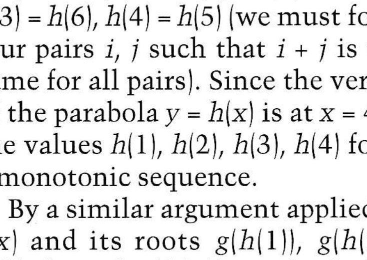


Figure 1

Since O_1A is perpendicular to the tangent to circle O_1 at A, we have $O_1A \perp DE$, so $O_1A \parallel FO_2$. Similarly, $O_2A \parallel FO_1$. This means that FO_1AO_2 is a parallelogram. Now the congruence of the right triangles BFO_1 and DFO_2 follows, because $FO_1 = O_2A = O_2D$ and $BO_1 = O_1A = FO_2$.

This implies the equality of their hypotenuses $FB = FD$. It remains to notice that $FC = FB$ and $FE = FD$ by construction.

M183

It is not hard to see that we can assume, without loss of generality, that the leading coefficient of $f(x)$ in both problems is positive.

(a) By this assumption, $f(x)$ is positive outside the segment $[x_1, x_2]$, where x_1 and x_2 are its smallest and largest roots. What is more, $f(x)$ grows indefinitely as x tends to $\pm\infty$, so for a large enough number d the value $f(x)$ will be greater than the absolute value of the minimum M of f on $[x_1, x_2]$ whenever the distance from x to the segment (that is, to every point on the segment) is greater than d. Take a positive integer l such that the distance between the segments $[x_1 - l, x_2 - l]$ and $[x_1, x_2]$ is greater than d—that is, $x_2 - l < x_1 - d$. Then $f(x) + f(x + l) > 0$ for all x, because wherever one of the terms in this sum is negative, the other is greater than $|M|$. It follows that

$$(f(x) + f(x + l)) + (f(x + 1) + f(x + l + 1)) + \dots + (f(x + l - 1) + f(x + 2l - 1)) = f(x) + f(x + 1) + \dots + f(x + 2l - 1) > 0$$

for any x, and we can take $k = 2l - 1$.

(b) The derivative of a polynomial of odd degree is a polynomial of even degree. Therefore, by virtue of problem (a), the derivative $f'(x)$ satisfies, for a certain k, and for any x, the condition

$$f'(x) + f'(x + 1) + \dots + f'(x + k) > 0.$$

Therefore, $f(x) + f(x + 1) + \dots + f(x + k)$ is a strictly increasing function; also, $f(x)$ is negative for large negative values of x and positive for large positive x, so it has exactly one zero. (S. Berlov, K. Kohas, V. Senderov)

M184

At the moment in question Sisyphus's total profit will be zero regardless of the order in which the stones were transferred. We'll prove this in three different ways.

The first proof. For brevity, any two stones in the same pile will be called *neighbors*. Then each stone moved gives Sisyphus a profit equal to the change in the number of pairs of neighbors. It remains to notice that at the moment in question the total change in the number of pairs of neighbors is zero.

The second proof uses the notion of an *invariant* (see "Some Things Never Change" in the September/October 1993 issue of *Quantum*). Here the value that doesn't change as the stones are moved is $A = ab + bc + ca + S$, where a, b, and c are the numbers of stones in the piles and S is Sisyphus's profit. Indeed, if a stone is carried from the pile with a stones to the pile with b stones, the value of A becomes equal to $A' = (a - 1)(b + 1) + (b + 1)c + c(a - 1) + S' = A + a - b - 1 + S' - S = A$, because $S' - S = b - (a - 1)$, where S' is the new value of his profit. Since the final value $ab + bc + ca$ is the same as it was at the start, Sisyphus's final profit is equal to the initial profit—that is, to zero.

We could just as well have used another invariant here—namely, $B = a^2 + b^2 + c^2 - 2S$.

The third proof. We can check that reversing the order of two moves retains the overall change in S'. Also, labeling the piles X, Y, and Z, the total change after the two moves $X \to Y$, $Y \to X$ or the three moves $X \to Y$, $Y \to Z$, $Z \to X$ is zero. Since all the stones returned to their original piles, the order of moves can be changed so that the moves will fall into pairs and triples of the form given above that bring zero profit to Sisyphus. (I. Izmestyev, D. Kuznetsov, I. Rubanov)

M185

The answer is yes. The sequence a_i can be defined recursively. Let $a_1 = 1$. If the terms $a_1, \ldots, a_{n-1}$ are already defined, we take the arithmetic means

$$p_k = \frac{1}{k}(a_1 + a_2 + \cdots a_k),\ 1 \le k \le n-1,$$

and put

$$a_n = \begin{cases} p_{n-1}, & \text{if } p_{n-1} \ne a_k, \\ & 1 \le k \le n-1, \\ p_{n-1} + n, & \text{if } p_{n-1} = a_s \text{ for} \\ & \text{certain } s, \\ & 1 \le s \le n-1. \end{cases}$$

Then $p_n = [(n-1)p_{n-1} + a_n]/n$ equals p_{n-1} in the first case $(a_n = p_{n-1})$ and $p_{n-1} + 1$ in the second case $(a_n = p_{n-1} + n)$. It follows that

(1) $a_1 + \ldots + a_n = np_n$ is divisible by n for all n;

(2) any positive integer m occurs in the sequence $\{a_i\}$: indeed, this is true for the sequence of the means $\{p_i\}$, because $0 \le p_{i+1} - p_i \le 1$, while $p_{i+2} - p_i \ge 1$ for all i; and if $m = p_n$, then, by definition, either m equals a_{n+1} or one of the numbers a_i, with $1 \le i \le n$;

(3) the numbers a_n are all different, because for any n and l, $n > l \ge 1$, either $a_n = p_{n-1} \ne a_l$ or $a_n = p_{n-1} + n > p_{l-1} + l \ge a_l$.

(O. Lyashko, A. Shapovalov)

Physics

P181

Deformation of the trampoline produces elastic forces. The resultant of these forces acting on the gymnast is directed upward. By the statement of the problem, the trampoline's sag is small compared to its size, so the resultant force obeys Hooke's law: $F = kx$, where F is the force acting on the gymnast, x is the trampoline's sag, and k is a proportionality factor. The maximum sag of the trampoline $x_{max} = h$, so the largest value of the tension is

$$F_{max} = kx_{max} = kh.$$

So we need k to estimate the ratio F_{max}/mg. Since we need only make a rough estimate, we assume that the trampoline is absolutely elastic and neglect the air resistance acting on the gymnast. With these simplifications we can use the law of conservation of energy: the decrease in the gymnast's potential energy is compensated by an equal increase in the trampoline's elastic energy—that is,

$$mg(h+H) = \frac{kh^2}{2}.$$

We took into account that at the moment when the trampoline's sag is h, the gymnast's velocity is zero. From here it follows that

$$k = \frac{2mg(h+H)}{h^2}.$$

Accordingly,

$$\frac{F_{max}}{mg} = \frac{kh}{mg} = 2\left(1 + \frac{H}{h}\right) = 26.$$

P182

The pressure under the curved spherical surface of a liquid differs from that of the gas over it by $\Delta P = 2\sigma/r$, where r is the sphere's radius. In the case where the walls of

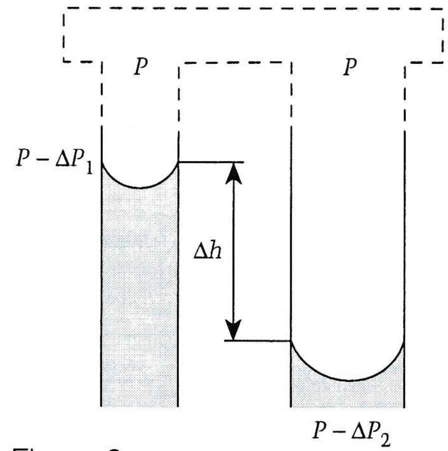


Figure 2

the tube are completely wetted with water, the meniscuses in both sides can be considered spheres of radii $r_1 = d_1/2$ and $r_2 = d_2/2$. Therefore, if a meniscus is not located at the end of the corresponding capillary, the air pressure over the water in the narrow side is greater than the pressure in the water under the surface by $\Delta P_1 = 4\sigma/d_1 = 2{,}800$ Pa, and in the thick side by $\Delta P_2 = 4\sigma/d_2 = 14{,}000$ Pa. So the difference in the levels of these meniscuses (fig. 2) is equal to

$$\Delta h = \frac{\Delta P_1}{\rho g} - \frac{\Delta P_2}{\rho g} \cong 14.3 \text{ cm}.$$

In our case $l < \Delta h$, which means that at least one of the meniscuses is located at the end of a side tube.

When the ends of the Π-shaped tube are simultaneously in contact with the surface of the water, the increase in the water level in the narrow side results in an increase in the air pressure within the tube such that air will bubble out from the end of the thick side. Thus, the water in the narrow side will rise to the top to form a meniscus of radius $R_1 > r_1$. So the entire length l of the tube must be submerged in water.

At the lower end of the thick side the meniscus can be of radius $R_2 \ge r_2$. When $R_2 = r_2$, the air pressure in the submerged tube is maximum and equal to

$$P_{max} = P_{atm} = \rho g l + \frac{2\sigma}{r_2}.$$

The air continues to bubble until the mass of air decreases such that the air pressure is P_{max} or slightly less. A meniscus of radius $R_2 \geq r_2$ in the thick side will be formed at the lower end of the capillary—that is, at a depth l.

If the ends of the tube do not touch the surface of the water simultaneously, there will be either less bubbling or no bubbling at all. In this case the answer may be different.

P183

Assume that the circuit is connected to the source at time $t = 0$. In this case capacitor C_2 will be rather quickly charged via diode D_1 to a voltage equal to the amplitude V_0 of the voltage source. This capacitor receives a charge

$$Q_0 = V_0 C_2.$$

In subsequent time intervals the diodes D_1 and D_2 are closed (that is, no current will flow through them) because the potentials at their cathodes are always higher than those at the anodes, and the diodes are ideal—that is, their resistance in the closed state is infinity. The scheme works as if the diodes are switched off. Thus, the charge Q_0 is conserved. In this case the alternating current flows through the capacitors connected in series:

$$i = I_0 \sin\omega t = \left(V_0 \omega \frac{C_1 C_2}{C_1 + C_2} \right) \sin\omega t,$$

where I_0 is the current's amplitude. The alternating current produces a varying voltage across each capacitor:

$$\begin{aligned} v_{c1\sim} &= \frac{I_0}{\omega C_1} \cos\omega t \\ &= V_0 \frac{C_2}{C_1 + C_2} \cos\omega t, \\ v_{c2\sim} &= \frac{I_0}{\omega C_2} \cos\omega t \\ &= V_0 \frac{C_1}{C_1 + C_2} \cos\omega t. \end{aligned}$$

In addition to the alternating current, there will be a constant voltage component across each capacitor resulting from the charging of capacitor C_2 to voltage V_0. We can determine this constant component of the voltage V by means of the law of charge conservation:

$$C_2 V_0 = C_1 V + C_2 V.$$

From this we get

$$V = \frac{C_2 V_0}{C_1 + C_2}.$$

Thus, the total voltage across the capacitors is

$$\begin{aligned} v_{c1} &= v_{c1\sim} - V \\ &= V_0 \frac{C_2}{C_1 + C_2} \cos\omega t - \frac{C_2 V_0}{C_1 + C_2}, \\ v_{c2} &= v_{c2\sim} + V \\ &= V_0 \frac{C_1}{C_1 + C_2} \cos\omega t + \frac{C_2 V_0}{C_1 + C_2}. \end{aligned}$$

The graphs of these functions in the case where $C_2 = 2C_1$ are given in fig. 3.

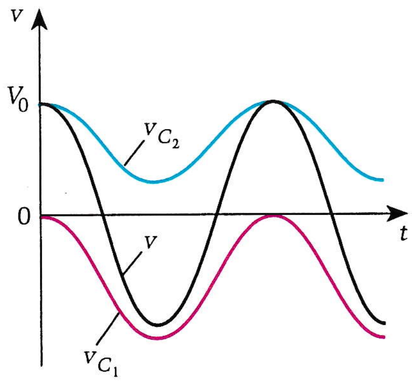


Figure 3

Note that if the connection to the voltage source is made at time $t \neq 0$, the circuit will settle to the steady-state regime after a certain time, when diode D_1 opens and capacitor C_2 charges to voltage V_0.

P184

As the magnetic field changes with time, an emf $\mathscr{E}$ is induced in the loop. The induced current is

$$I = \frac{\mathscr{E}}{R},$$

where R is the resistance in the loop. This resistance is $R = \rho(l/s)$, where l is the length of the wire and s is its cross-sectional area. Clearly $m = dls$. So

$$s = \frac{m}{ld}$$

and

$$R = \rho \frac{d}{m} l^2$$

—that is, the loop's resistance is proportional to the square of its length.

The emf induced in the loop is

$$\mathscr{E} = \frac{\Delta\Phi}{\Delta t} = \frac{\Delta B}{\Delta t} S = kS,$$

where S is area contained by the loop. The larger the area S, the higher the emf $\mathscr{E}$. For a given loop length the largest area is encompassed by a circle. Thus the emf is greatest when the loop is a circle. Denote its radius as r. Then

$$S = \pi r^2$$

and

$$\mathscr{E} = k\pi r^2.$$

In this case,

$$l = 2\pi r$$

and

$$R = \rho \frac{4\pi^2 r^2 d}{m}.$$

Finally,

$$I = \frac{\mathscr{E}}{R} = \frac{km}{4\pi\rho d}.$$

This is maximum current in the loop that we seek.

P185

The two half-lenses form an optical system consisting of two lenses of focal length F (lens L_1) and $2F$ (lens L_2). The optical axes of these lenses coincide. By definition, brightness depends on the flux of light striking the screen and on the area of the spot on it. For a given location of the light source, each lens forms its own spot on the screen due to the difference in their focal lengths. Nevertheless, the magnitudes of the light fluxes passing through the half-lenses are identical, because their areas are equal;

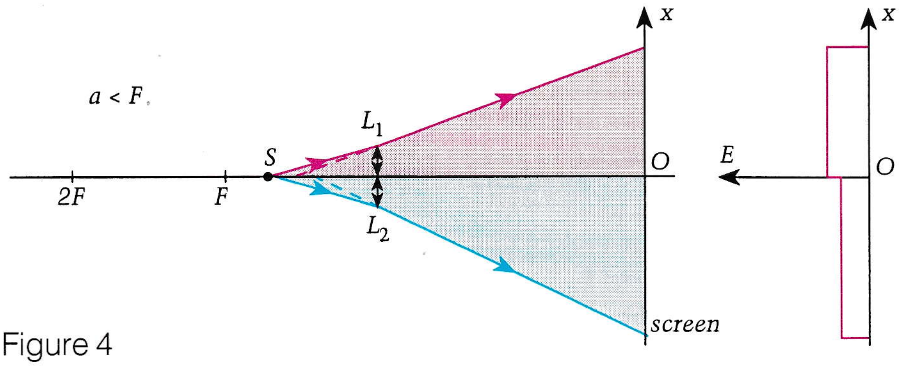


Figure 4

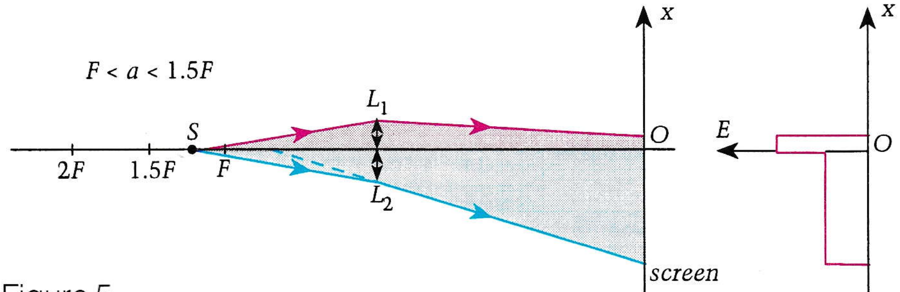


Figure 5

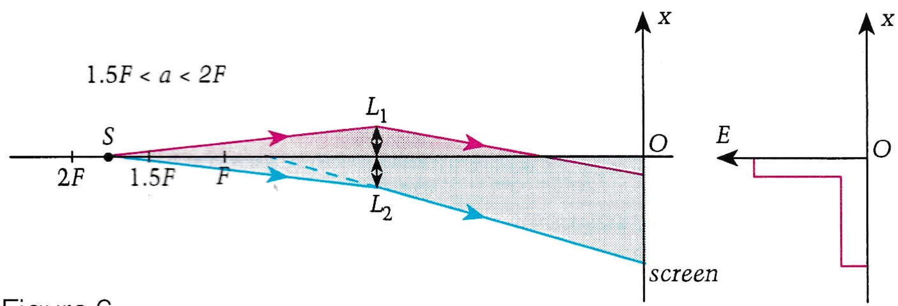


Figure 6

consequently the solid angles subtending the incident fluxes are also equal. Thus the brightness of the screen is determined by the area of the image. It's clear that when the distance to the object changes, the position of the image shifts, so the area of the spot on the screen varies as well as the corresponding brightness of the image on the screen.

The lens formula yields the image distances

$$f_1 = \frac{Fa}{a-F}$$

and

$$f_2 = \frac{2Fa}{a-2F}.$$

Let's consider the cases corresponding to different locations of the light source. If $a < F$, both f_1 and f_2 are negative—that is, both images are virtual. The incident rays are shown in figure 4. These rays do not cross, and each lens illuminates its part of the screen to form a spot with a certain brightness. (Recall that geometrical optics considers only narrow beams along the axis, so a spot of light on the screen is assumed to be small and its illumination homogeneous.) The graph in the right-hand part of figure 4 shows how the brightness depends on the distance x from the optic axis.

When $F < a < 2F$, the first image is real ($f_1 > 0$), while the second is virtual ($f_2 < 0$). The brightness also depends on the value of f_1—that is, whether it is larger or smaller than the distance to the screen $2a$. These two cases correspond to $F < a < 1.5F$ and $1.5F < a < 2F$. The paths of the rays and the brightness graph for these cases are shown in figures 5 and 6.

Finally, when $a > 2F$ both images are real. In addition, $f_1 < 2a$, but f_2 can be larger or smaller than $2a$. If $2F < a < 3F$, then $f_2 > 2a$. The answer is similar to that shown in fig. 6. When $a > 3F$, then $f_2 < 2a$, and the paths of the rays and brightness graph are similar to those shown in figure 5. Convince yourself that this is correct.

Brainteasers

B181

The father's unkind remark is unfortunately correct. Denote by c, h, and g the portion of the haystack eaten by the cow, horse, and goat, respectively, in a month. Then, according to what the son said, $h + g = 1$, $\frac{3}{4}(g + r) = 1$, and $\frac{1}{3}(c + h) = 1$, or $h + g = 1$, $g + c = 4/3$, $c + h = 3$. Adding the first two equations and subtracting the third from the result, we get $2g = -2/3$, which is, of course, impossible.

B182

The answer is a, which becomes obvious after we shift the triangle downward by a (fig. 7).

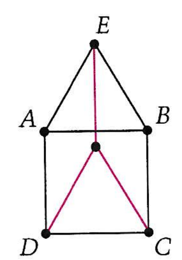


Figure 7

B183

Attach the string with the weight to the scale and lower the weight slowly into each of the vessels. In pure kerosene, the reading on the scale won't change as the weight goes down; but in the other vessel the reading will jump at the boundary between the two liquids: water is denser than kerosene, so the buoyant force increases abruptly at this point.

B184

One such required arrangement of points is shown in figure 8. Four points form a square *ABCD* with side length 1; two other points are placed just under and to the right of the center.

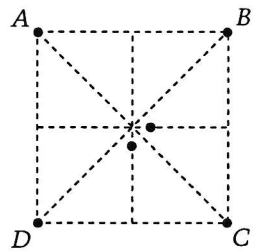


Figure 8

Consider the four squares whose diagonals are the sides of the square *ABCD*. It is tedious, but not difficult, to check that any five of our six points are covered by two squares of this size. On the other hand, there are only two ways to cover the four points *A*, *B*, *C*, *D* with two circles of diameter 1: we must either take the circles with diameters *AB* and *CD*, or *AD* and *BC*. In either case one of the central points is left uncovered.

B185

Divide the hall into 25 squares with 2×2 chairs in each as in figure 9. All officials in each of these squares are neighbors to one another, so at most two of them (truly the "highest-paid") can consider themselves highly paid. This limits the number of these fortunate persons to at most fifty. The figure shows a distribution of the salaries (in some arbitrary currency) that allows fifty officials to consider themselves highly paid.

2	1	2	1	2	1	2	1	2	1
3	1	3	1	3	1	3	1	3	1
4	1	4	1	4	1	4	1	4	1
5	1	5	1	5	1	5	1	5	1
6	1	6	1	6	1	6	1	6	1
7	1	7	1	7	1	7	1	7	1
8	1	8	1	8	1	8	1	8	1
9	1	9	1	9	1	9	1	9	1
10	1	10	1	10	1	10	1	10	1
11	1	11	1	11	1	11	1	11	1

Figure 9

Kaleidoscope

1. The choice of reference frame is determined by its practicality.

2. A definite answer can't be given without choosing a reference system.

3. (a) Circle; b) spiral.

4. If the raindrops are falling vertically relative to the Earth, the train is moving to the right. If the rain is coming down at an angle (relative to the Earth), it will be necessary to calculate the relative velocities of the drops and the train. It's possible that the train isn't moving at all.

5. The sign of the acceleration depends on the choice of coordinate axis and doesn't change during the stone's flight.

6. Yes, if at least one of the sources of smoke is moving (fig. 10).

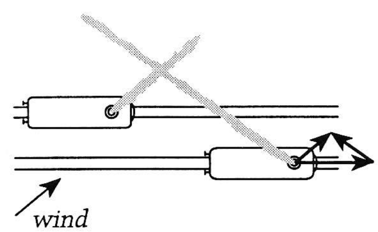


Figure 10

7. It's possible at supersonic speeds if both the plane and the projectile are moving in the same direction (in this case their relative velocity will be close to zero).

8. The velocity is directed downward and is equal to 1.5 m/s.

9. $\mathbf{v}_{rel} = \mathbf{v}_0$.

10. Yes, they can (fig. 11). The velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ are given in the stationary system of reference, while $\mathbf{v}_A'$ and $\mathbf{v}_B'$ are given in a system moving with velocity $\mathbf{u}$. Since the vectors $\mathbf{v}_A'$ and $\mathbf{v}_B'$ are not parallel, the lines representing the trajectories of points *A* and *B* in the *x'y'*-system will intersect.

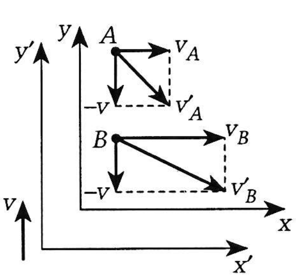


Figure 11

11. The rower must perform the same amount of work in both cases.

12. The greater a plane's velocity relative to the air, the greater its lift. By taking off and landing into the wind, the necessary relative velocity is gained at less velocity relative to the ground, so it is safer and more economical than taking off and landing with the wind.

13. The wind increases a plane's speed in the first half of the route and decreases it by the same amount in the second half. This means that the wind helps the plane during a shorter time than it impedes the plane. Thus the flight time will be greater because of the wind.

14. For observer *A* the surroundings are rotating with the same angular velocity as the platform about its axis, but in the opposite direction. Since observer *B* is located twice as far from *O*, *B*'s linear velocity due to rotation will be two times greater—that is, 2 m/s.

15. The kinetic energy of a body depends on the choice of reference frame. For example, if the boy shoots opposite to the train's motion, the bullet's kinetic energy relative to the ground will be zero.

16. The Sun accelerates the Earth and the pendulum equally. Since their relative acceleration is zero, the pendulum doesn't deviate from the local vertical.

17. The moving bands are not material bodies, so the theory of relativity sets no limitations on their speed.

18. The speed of the observed light is always 300,000 km/s. However, the color of the incident light differs from that of the light radiated by a quasar.

Microexperiment

The relative velocity of the trains is greater than the velocity of your train relative to the Earth.

Resistance

1. 8/15.

2. Substitute $(n-k)\binom{n}{k} = n\binom{n-1}{k}$ into equation (1). This relation can be derived, for instance, from the explicit formula $\binom{n}{k} = n!/[k!(n-k)!]$.

3. 7/12.

4. (a) $(m-1)/m$; (b) 1/2; (c) $2/m$; (d) 5/12; (e) 5/9.

Multidimensional cube

1. $n2^{n-1}$.

2. The general equation is proved exactly as its particular case, equation (1). The answer is $F_{n,k} = \binom{n}{k} \cdot 2^{n-k}$.

3. $\sqrt{n-k}$.

Borsuk's problem

1. The easiest solution is to take the five points (1, 0, 0, 0, 0), (0, 1, 0, 0, 0), ..., (0, 0, 0, 0, 1) in five-dimensional space: they are the vertices of a four-dimensional simplex embedded in this space. In four-dimensional space you can take the points (0, 0, 0, 0), (1, 1, 0, 0), (1, 0, 1, 0), (0, 1, 1, 0), and $(1/2, 1/2, 1/4, \sqrt{5}/2)$.

2. Take the graph in figure 5 in the article "Resistance in the Multidimensional Cube" and turn it vertically.

3. From the formula for $\binom{n}{k}$ it follows that

$$\frac{\binom{n}{k+1}}{\binom{n}{k}} = \frac{n-k}{k+1} = \frac{n+1}{k+1} - 1.$$

So $\binom{n}{k}$, as a function of k, increases for $k < (n-1)/2$ and decreases for $k > (n-1)/2$.

4. The proof essentially follows the solutions to the "best in their own ways" problem in the article, but with a more accurate estimate.

5. Simply replace the numbers in the solution as they are replaced in the statement.

6. Associate the 12 points in the "protruding edges" problem with the guests at the party and the 66 edges with the guests at the reception. Give to each group A of 6 guests at the party a cake of the same sort as was given to the 36-guest group $f(A)$ at the reception whose corresponding edges are exactly those that protrude from the set of points corresponding to A. Then the required property follows immediately.

7. There are $m/2$ elements in each of the subsets A in question and equally many outside each of these subsets. So the number of pairs with exactly one element in A is $(m/2)^2$.

9. We have

$$\frac{\binom{m}{m/2}}{2\binom{m-1}{m/4-1}} = \frac{m!(m/4-1)!(3m/4)!}{2((m/2)!)^2(m-1)!} = 2\frac{(m/4)!(3m/4)!}{((m/2)!)^2}.$$

Applying Stirling's formula to the last expression and canceling various terms, we will find that it is approximately equal to

$$2\frac{\sqrt{3/4}\cdot 2^m \cdot 3^{3m/4}}{1/2\cdot 4^{m/4} \cdot 4^{3m/4}} = \frac{\sqrt{3}\cdot 3^{3m/4}}{2^m} = \sqrt{3}\left(\frac{27}{16}\right)^{m/4} > \frac{m(m-1)}{2}+1$$

for large enough m.

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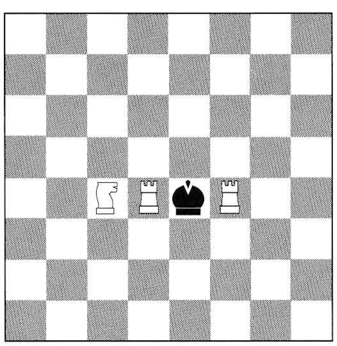
Chess puzzles and real chess

Sometimes the two worlds intersect!

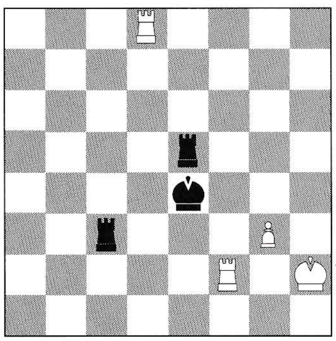
by Yevgeny Gik

THE TITLE OF THIS ARTICLE may seem somewhat strange to you. Puzzles and chess problems are a special genre that has nothing to do with an actual game of real chess. Right? Well . . .

Once Sam Loyd, a famous creator of chess problems and a "grandmaster of recreational mathematics," declared that he had found a way to checkmate a solitary king at the center of the chessboard with his two rooks and a knight, without his king's support. Chess-lovers flew into a rage, but when Loyd showed the solution to his puzzle, they had a good laugh:

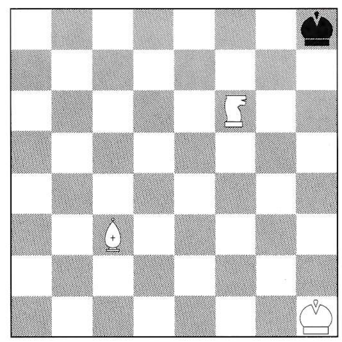


I suppose you might think that such a checkmate is possible only in a puzzle. But here's an incident that really happened at a chess tournament in Dushanbe (the capital of Tadzhikistan), as related by grandmaster O. Sabitov. In one of the endgames, the following position emerged:



In serious time trouble, white reached for the rook on f2. Black, noticing that after 1. Re2+ the king would be unable to retreat without losing the rook (1. ... Kf5 2. Rf8+ Ke6 3. Re8+), grabbed the rook on c3 to keep the king out of check. And check indeed followed, but from another square: 1. Rf4+. Completely flustered, black didn't pay attention to the dramatic change in the position and moved the prepared 1. ... Re3!? with lightning speed. The flag on white's clock was about to tumble and, forgetting about the check announced in the previous move, white attacked the enemy king from the opposite side by 2. Rd4!!, creating a unique position unprecedented in the entire history of chess.

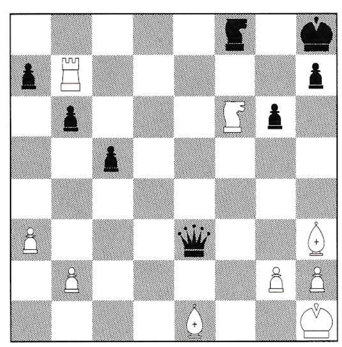
Here's another nice problem:



Mate in 1 move

For mate in one move, the white knight must . . . well, rear up! (It *is* a horse, after all!) After this "maneuver," squares g8 and h7 remain under its control (because rather than leave f6 it merely rises above it); in the meantime, the white bishop c3 checkmates.

This fantastic idea can bring up quite realistic associations. Once, analyzing his game with A. Gipslis, grandmaster E. Gufeld arrived at this position:



At first Gufeld reckoned that white, which he played, lose: materially, the odds are in his favor, but the bishop is under attack and can't retreat without leaving the first rank unprotected (Qc1+). But he recalled the trick with the rearing knight and found a remarkable way out: 1. Re7! Q:e7 2. Bc3!. Compare this position with the puzzle and you'll understand that there is little difference between them. The knight is ready to shoot up into the air and black can't prevent the loss of the queen (after 3. Nd5) or mate (3. Ng4+ Kg8 4. Nh6×).

Problem. The knight is on a1. Can it trace the entire chessboard without visiting the same square twice and finish its trip at h8?

The square the knight rests on changes color with each move. The initial square a1 is black, so after the 63rd move the knight will find itself on a white square. But h8 is black, so the task is impossible.

There are lots of puzzles of this sort. Are they related to the actual game? Yes, they are, and sometimes very directly.

Consider this position:

White: Kh8, Nb2, ph7.

Black: Kf7.

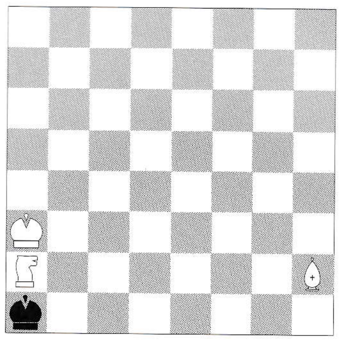
How do evaluate the chances here? To win, white must release the king. If white goes first, this is possible: 1. Nd3 Kf8 2. Ne5, and so on. But if it is black's turn to go, the white king can't break free from the corner: 1.... Kf8 2. Nd3 Kf7 3. Ne5+ Kf8. It is white's turn and white has to give away square f7 to the black king. So it's a draw.

There is a useful rule for this sort of position: *the weaker part achieves a draw if it can place the king (by its move) on a square of the same color as that of the square with the opponent's knight.*

Exercises

The first problem is serious and the second is a puzzle.

1. White mate in 5 moves:



2. There are two kings only on the board: white on a6, black on a8. Where must the white queen be placed so that white *cannot* mate in one move?

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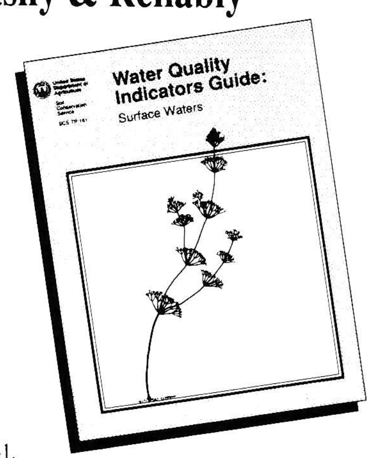

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