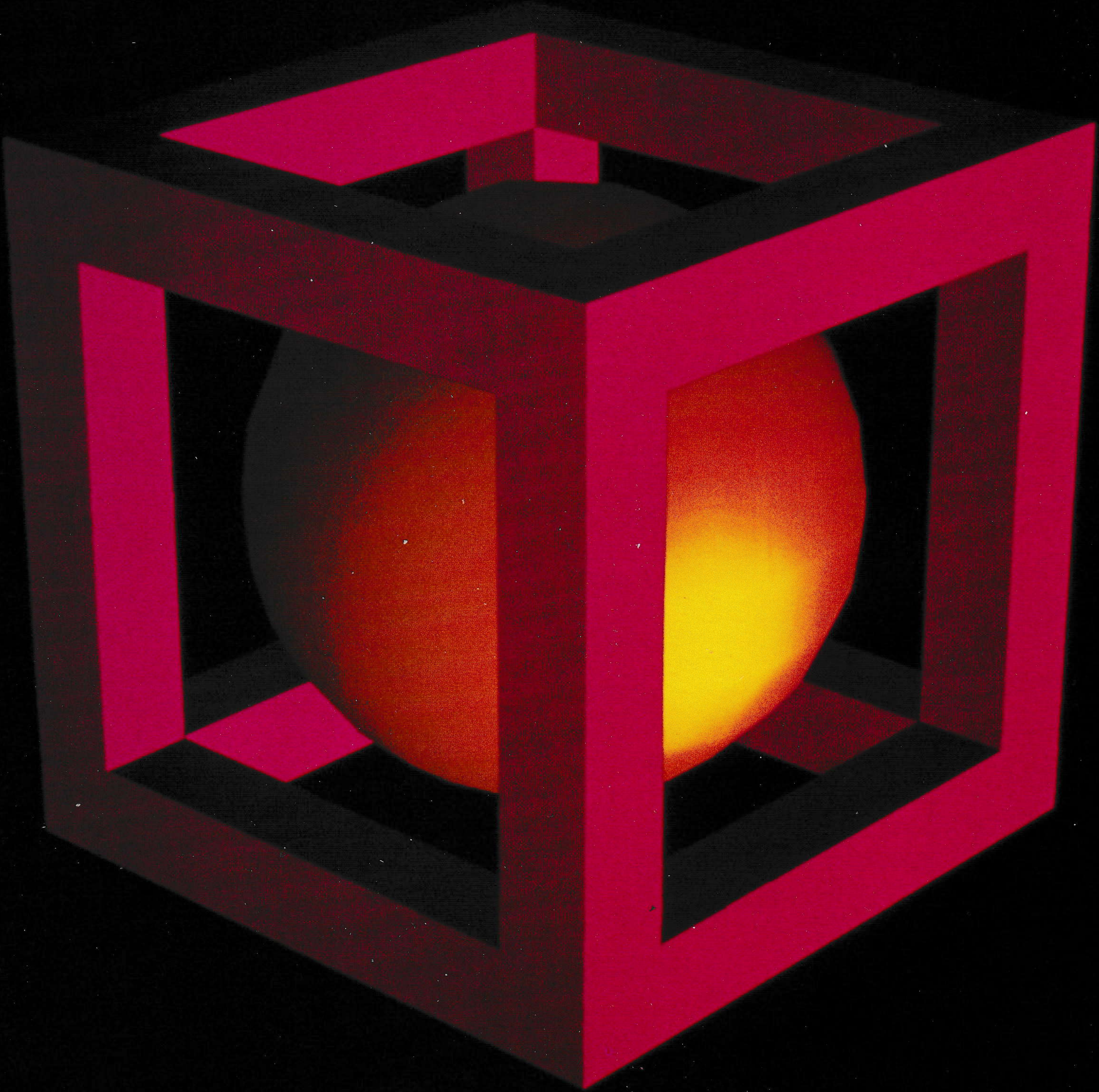
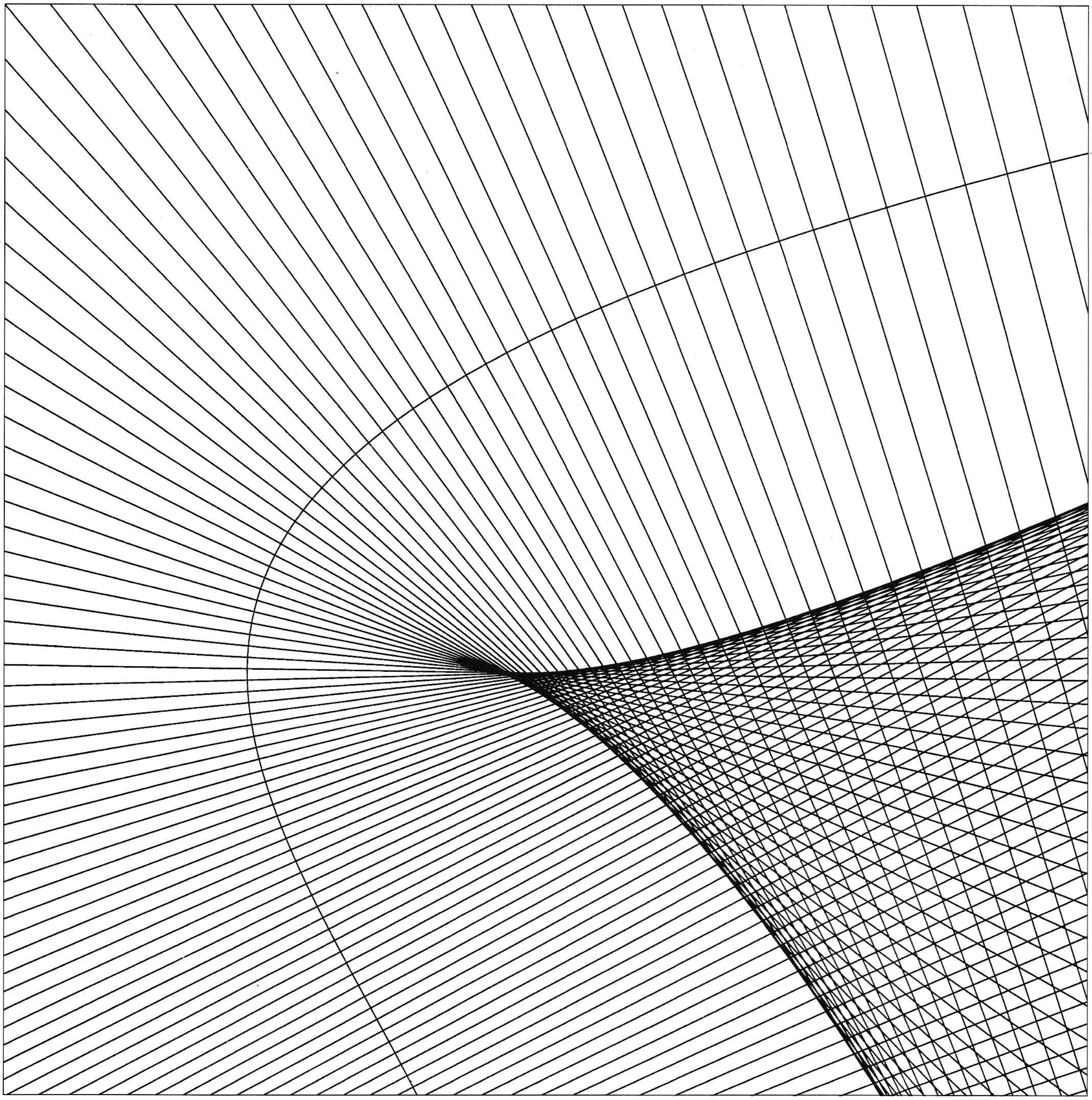
QUANTUM

JANUARY 1990



The student magazine of math and science



A smoothly varying family of lines in the plane almost always forms folds and cusps. This family of lines was generated as the set of normals to the curve that sweeps through. Although this figure is a construction in the plane, it gives a definite impression of three-dimensionality. In his article on developable surfaces, Dmitry Fuchs offers a three-dimensional interpretation arising from the bending of paper—see page 16.

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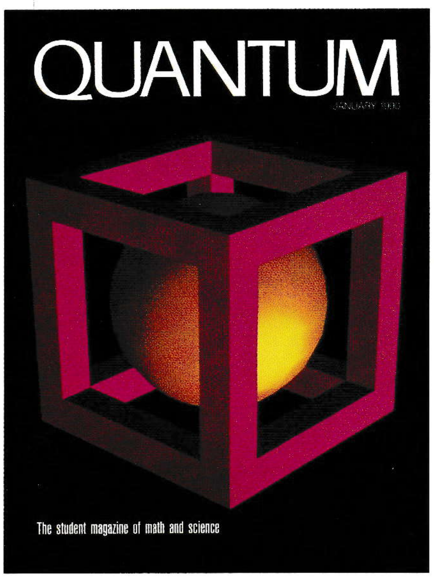

Cover art by Ron Scott

A computer was used to generate the sphere-within-a-cube on our cover. You won't need a computer, though, to solve the following problem based on that figure. Assume the edges of the cube are line segments of no thickness and the sphere touches the faces of the cube. Let the length of an edge be 2 units. Mentally place another sphere in one corner so that it touches the given sphere and the three faces of that corner. You could keep doing this infinitely many times. If the radius of the original sphere $r_1 = 1$, what does r_{1990} equal? If the diameter of the original sphere $d_1 = 2$, what is the sum of the diameters of *all* the spheres that can be packed into a corner? (Solution on page 55)

FEATURES

DEPARTMENTS

QUANTUM

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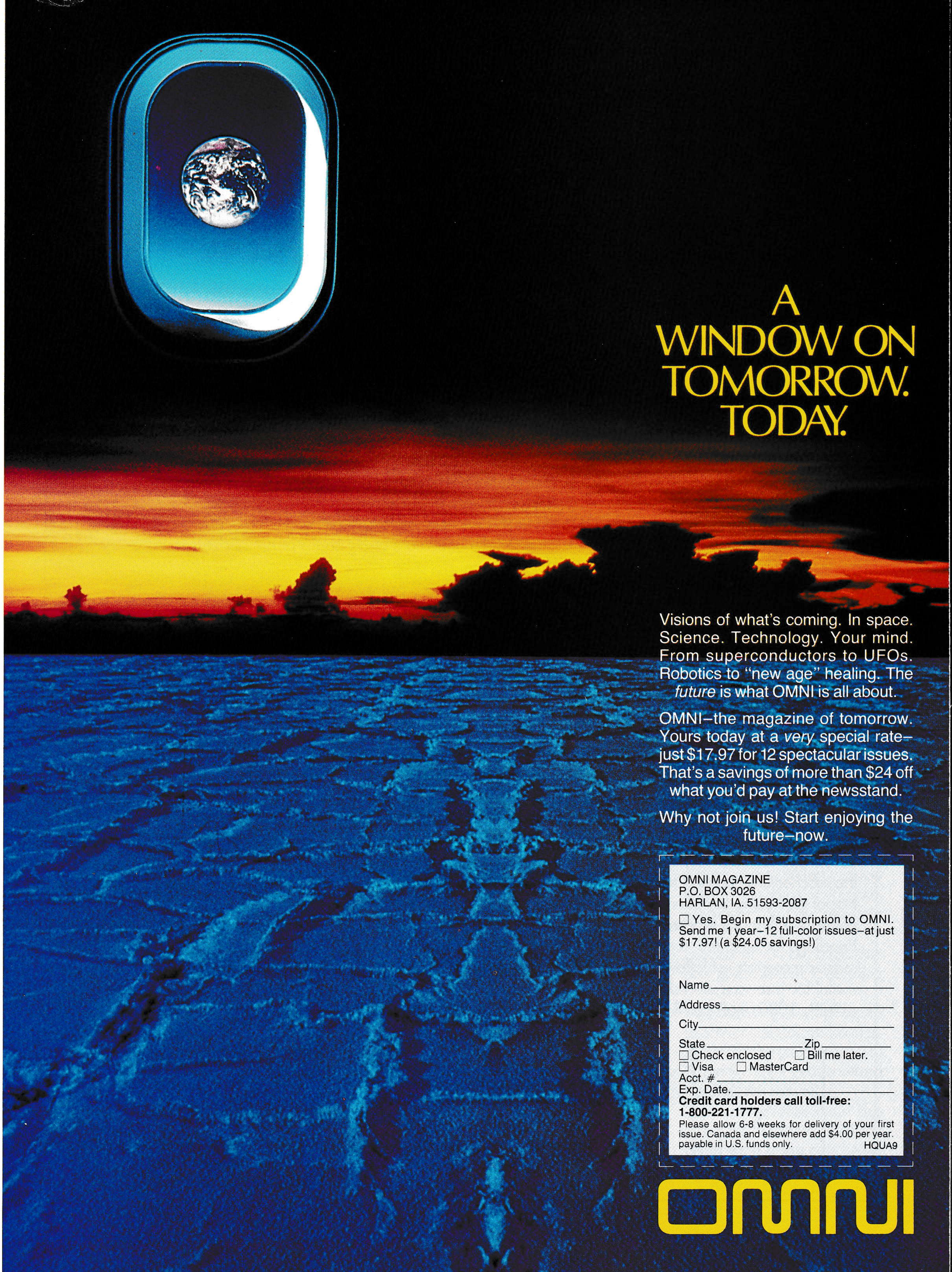
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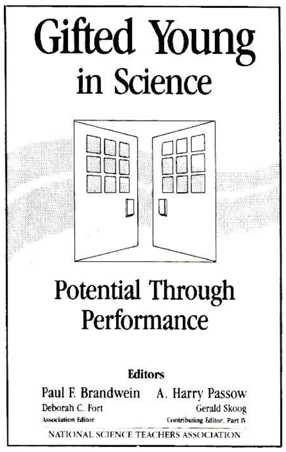
Learning Science

with the National Science Teachers Association

Theory...



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#PB-75/1, 1989, 422 pp. $24.00/hardback
#PB-75/2, 1989, 422 pp. $17.00/soft cover

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How do objects move? Isaac Newton really believed that an object moving in a straight line would continue with constant speed. Do your students? This manual was created to help teachers introduce the sometimes daunting subject of Newtonian mechanics to students in the middle grades. The 27 activities presented here use readily available materials to give students visual, aural, and tactile evidence to combat their misconceptions. And the teacher-created and tested modules are fun: Marble races, a tractor-pull using toy cars, fettucini carpentry, and film container cannons will make teachers and students look forward to class. Readings for teachers, a guide for workshop leaders, and a master materials list follow the activities, making this manual useful for inservice workshops.

#PB39, 1989, 168 pp. $16.50

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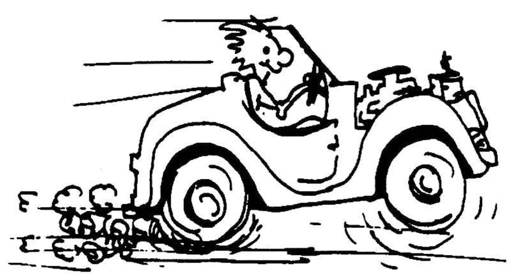
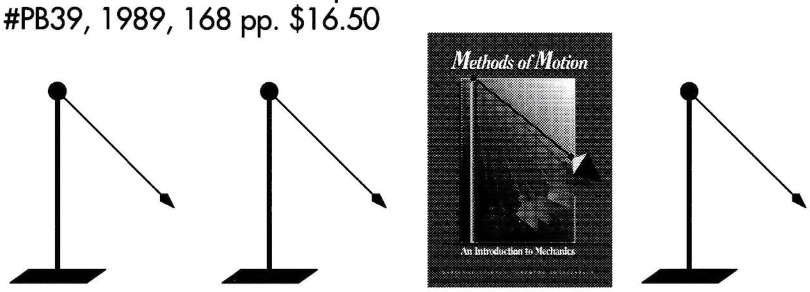




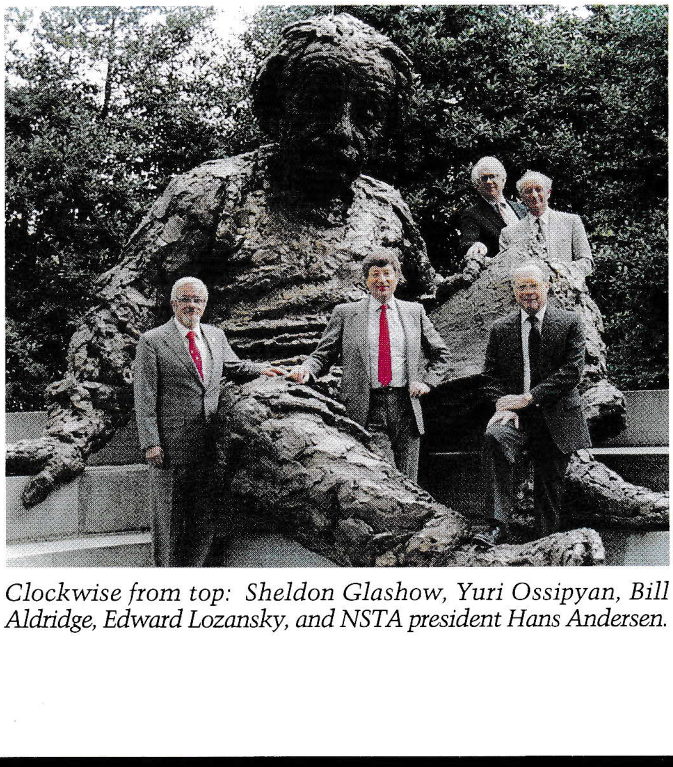
All orders of $25 or less must be prepaid. Orders over $25 must include a purchase order. All orders must include a postage and handling fee of $2. No credits or refunds for returns. Send order to: Publications Sales, NSTA, 1742 Connecticut Ave. NW, Washington, D.C. 20009.

PUBLISHER'S PAGE

Welcome to Quantum!

All of us at the National Science Teachers Association are very excited about the premier issue of *Quantum* magazine. Working with the American Association of Physics Teachers and the National Council of Teachers of Mathematics, and in cooperation with the Soviet Academy of Sciences, we're looking forward to producing a magazine of the highest quality. We expect *Quantum* to provide interesting and stimulating material for the inquisitive, bright young people of our country. When *Quantum* contains equal amounts of material from both the US and USSR, we hope it will be published in both countries so that students around the world can share the challenges and pleasures of *Quantum*'s problems, articles, and other features.

Our new magazine is extremely fortunate to have three top-notch scholars as its editors-in-chief. Academician **Yuri Ossipyan** is a physicist, vice president of the Academy of Sciences of the USSR, and editor-in-chief of our sister publication *Kvant*, the Russian-language magazine of physics and math for secondary school students. He is also a member of Congress of People's Deputies, a Soviet legislative body similar in function to the US House of Representatives. Dr. **Sheldon Lee Glashow** is a professor of physics at Harvard University who has taught at summer science and math institutes for gifted high school students for the last several years. In 1979 he was awarded the Nobel Prize for physics. Dr. **William P. Thurston**, a professor of mathematics at Princeton University, is also committed to motivating academically gifted students. He has worked closely with the National Council of Teachers of Mathematics and currently serves as vice president of the American Mathematics Society. In 1982 he received the coveted Fields Medal for his achievements in mathematics.



Clockwise from top: Sheldon Glashow, Yuri Ossipyan, Bill Aldridge, Edward Lozansky, and NSTA president Hans Andersen.

The staff of *Quantum* looks forward to working closely with the members of the advisory boards, who are scientists, mathematicians, and teachers of the highest caliber. They'll provide support in selecting translations from *Kvant*, soliciting original material, and reviewing it for technical accuracy. From their vantage points closer to the "action," they'll also help us tailor the magazine to fit your needs and interests.

The picture below was taken at the celebrated statue of Einstein on the grounds of the National Academy of Sciences in Washington, DC. As the great physicist gazes at the celestial map at his feet, he seems to draw us into his "joy and amazement at the beauty and grandeur of this world of which man can just form a faint notion . . ."

The great Russian scientist and poet Mikhail Lomonosov, a chemist of international reputation who helped create Moscow University in 1755, was moved to utter similar sentiments by a display of the northern lights. "Nature, where are your laws? The dawn appears from the dark northern climes! Does not the sun there set up its throne? Are not the ice-bound seas emitting fire? Behold, a cold flame has covered us! Behold, the day has trod the earth at night!"

As he addresses nature in his ode, Lomonosov runs through a list of fairly technical questions: "What causes bright rays to vibrate in the night? Why does a thin flame strike the firmament? How does lightning without thunderclouds race from the earth to the zenith? How can it be that frozen steam generates a conflagration in the midst of winter?" He concludes by telling nature, "What you say about the things nearby is doubtful. So tell us, if you can, how vast is the universe? What lies beyond the smallest stars?"

We hope this magazine will inspire in you the sense that the pursuit of scientific truth is a grand and never-ending adventure.

Quantum is the just first of a number of cooperative projects undertaken by US and Soviet science and education organizations. We invite you to take part in some of the student exchanges planned, and we hope your teachers will participate in our teacher exchanges as well.

If you have comments, suggestions, or ideas about this premier issue of *Quantum*, please write to me. The next issue will be out in the spring, and we'll begin publishing quarterly in the fall of 1990.

—Bill G. Aldridge

LETTERS FROM THE EDITORS

IT GIVES ME great pleasure to introduce this first issue of *Quantum* magazine to the American reader. This publication is the result of cooperation between Soviet and American scientists and educators and is based on material from *Kvant*, a unique popular science monthly magazine that has circulated in the USSR for 20 years. During this period *Kvant* (which means "quantum" in Russian, as you must have guessed) has become the favorite magazine of all Soviet high school students interested in mathematics, physics, and science in general. Its current readership is over 200,000.

Reading *Kvant* or *Quantum* isn't always easy, but if you really like science, I'm sure you'll find it exciting. Of course, the magazine contains recreational material, lively illustrations, humor, and amusing anecdotes from the history of science. But its core and main source of interest are articles in physics and math that necessitate thinking, sometimes pretty hard thinking, to be understood. Experienced *Kvant* readers sometimes even resort to pencil and paper while reading the articles to work out equations or to make their own sketches. Certainly a lot of brainwork and some paperwork are required to solve the problems in our Problem Corner, intended for those of you who like Olympiads and other problem-solving competitions. But all this work doesn't go unrewarded—few experiences are as intensely exhilarating as the feeling "I've got it!" that comes as a flash when you've solved an intricate problem or grasped a profound idea.

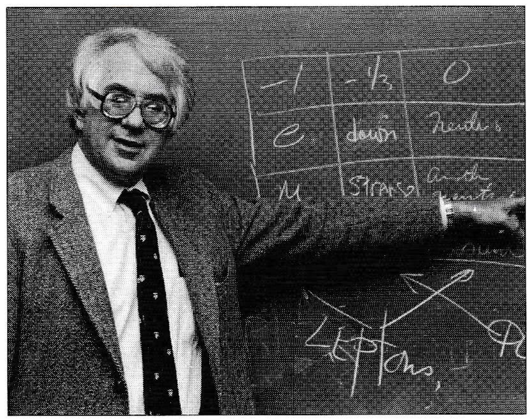
My American colleagues and I hope that in the future the cooperation between *Quantum* and *Kvant* will result in the simultaneous publication in English and Russian of issues with almost the same content so that Soviet and American high school students will be working on the same materials at the same time, competing peacefully and cooperating in the spirit of the present rapprochement between our two great countries. Whether you have already developed an interest in math or science, or have gathered from school courses that these subjects are boring (as I did in my early teens), I hope *Quantum* will help you discover the excitement inherent in mathematics and the natural sciences.

—Yuri Ossipyan

MATHEMATICS AND PHYSICS can be downright fun. I love to think about mathematical puzzles, quantum riddles, and the latest elementary particles, and I get paid for doing it! If you become a professional scientist or a teacher, you too may find it difficult to tell the difference between your hobby and your job—an enviable situation.

I wish *Quantum* had been around when I was a student—it would have made it a lot easier for me to have found fulfillment as a physicist. Although it has existed for some 20 years in its original Russian avatar, until now it has been unavailable to American students.

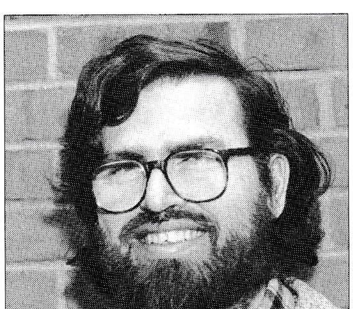
Almost every Soviet physicist I know either writes for *Quantum* or got into physics by reading the magazine and doing its problems. Hundreds of thousands of Soviet students are avid subscribers to the Russian version. Finally, an English-language edition has become available to you!



In this premier issue, we have translated some of the classic original Russian articles. In future issues, we plan to publish contributions from scientists throughout the world. Read *Quantum* and enjoy it, and let us know how we can make it even better.

—Sheldon Lee Glashow

AS A CHILD, I OFTEN HATED arithmetic and mathematics in school. Pages of exercises were tedious and dull. They weren't fun or challenging, they were just a chore. I'd find something else to entertain myself whenever I could. I doodled, I read books under the desk, I stared out the window and let my mind wander. Sometimes I tried to puzzle something out. Could you propel a sailboat with a big fan on the boat to blow on the sail? Might $\sqrt{2}$ eventually be periodic if you write it out in base 12 instead of base 10? How many ways are there to fold a map into sixteenths, in quarters each way?



When I became a mathematician I was surprised and heartened to find a community of people who also take pleasure in the kinds of things I enjoy, who like to really dig in and try to understand.

With the modern emphases on test scores, on "basics," on mathematics as a competitive sport, on getting "ahead" in math, and so on, it often seems that the diversity, richness, liveliness, and depth of mathematics has been pruned away from the school experience. Mathematics isn't a palm tree, with a single long straight trunk covered with scratchy formulas. It's a banyan tree, with many interconnected trunks and branches—a banyan tree that has grown to the size of a forest, inviting us to climb and explore. (If you've never seen a banyan tree, it's worth going out of your way for.)

I have great hopes that *Quantum* will open up a road to some of the breadth, wonder, and excitement of math and physics.

I'd like to post a warning, though, at the beginning of the road. Science writing, and math writing in particular, tends to be dense and full of hazardous turns and treacherous sand-pits. When I was a child I took pride in how many pages I read in an hour. In college I learned how foolish that was. When reading mathematics ten pages a day can be an extremely fast pace. Even one page a day can be quite fast. On the other hand, if you already understand something, you may get more by skimming than by reading every word. You need to be alert and suspicious; you need to question and think about what you're reading in your own way.

Quantum articles aren't written like articles in scientific journals, but some of the same reading habits still apply. Don't be afraid to stop in midparagraph or midsentence when something surprises or puzzles you. Speed isn't the issue. Don't assume something is obvious just because an author treats it that way. What you work out on the side, even though it takes much more time, will have immensely more value than what you read straight through.

—William P. Thurston

It's beautiful—but is it science?

There's something fishy about those waves

by Albert Stasenko

"DEEP AND FREEZING COLD IS the rippled blue surface of the river. Festively decorated boats with gaily colored sails glide along the river. Their elevated bows are decorated with dragon heads. The bright colors blaze in the sun. . . . The Vikings' voyage was long and dangerous." This is how V. P. Knyazeva describes Nikolay Rerikh's painting "Merchants from Overseas" (1901), which hangs in the Tretyakov Gallery in Moscow.

We'll make use of this striking poetical picture to ponder a few topics in physics—the law of conservation of energy, dimensional analysis, and statistical analysis of experimental results. What has the painting got to do with all that? Well, I guess we only need it to liven things up a bit.

Seriously, though, don't you think it's a challenge to try and find the speed at which the Vikings are travelling just by looking at the painting? Relative to the moving water and not the riverbanks, of course. And what is there in the painting to provide us with the necessary information? First of all, there is the wave formed on both sides of the bow upon which the dragon rears its head. And look at the circular waves moving on the water away from the boat. They look like circles all centered at some point of the plane containing the boat's waterline. Add to these the position of the yard and sail relative to the boat's symmetry plane.

Maybe you'll find something else that provides data on the direction and velocity of the boat, wind, and river. In the following sections, though, we'll concentrate on just two phenomena related to waves on water.

The wave at the bow

Why is the bow wave formed? Suppose the water is flowing symmetrically at velocity $\mathbf{V}$ (the velocity of the boat relative to the water) around a wedge (the bow of the boat) with vertical faces forming an angle of 2α (fig. 1). At the tip of the wedge we can split the vector into two components: one ($\mathbf{V}_{\parallel}$) parallel to one of the faces (sides of the boat) and the other ($\mathbf{V}_{\perp}$) perpendicular to it. So the flow of water relative to the side of the boat really combines two movements: it slips along the side at velocity $\mathbf{V}_{\parallel} = \mathbf{V}\cos\alpha$ and crawls up on it at velocity $\mathbf{V}_{\perp} = \mathbf{V}\sin\alpha$.

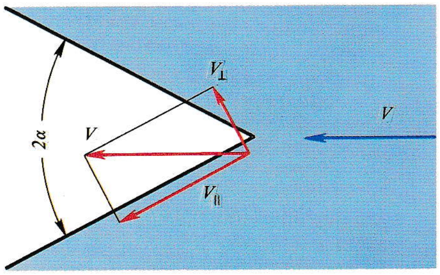


Figure 1

The perpendicular flow comprises several layers: the lower layers dive under the hull (fig. 2); the upper layer rises vertically, and we can easily estimate how high it reaches. Indeed, each water particle from the upper layer that possesses energy $m\mathbf{V}_{\perp}^2/2$ and abruptly changes direction can reach the height h where its potential energy mgh won't be greater than its kinetic energy:

$$mgh \le m\frac{\mathbf{V}_{\perp}^2}{2};$$

hence,

$$h \le \frac{\mathbf{V}_{\perp}^2}{2g}.$$

Therefore, by the law of conservation of energy and the principle of superposition of motion, we can estimate the boat's velocity: $\mathbf{V}_{\perp}^2 \ge 2gh$, which gives

$$\mathbf{V}^2 \ge \frac{2gh}{\sin^2\alpha}. \qquad (1)$$

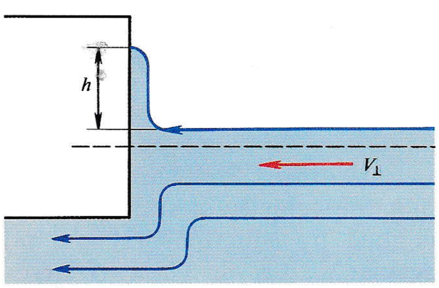


Figure 2

All we have left is to measure the angle α and the height h of the wave at the bow. The relevant computation will be carried out below, but first we'll consider the second available source of information on the boat's motion.

Surface waves

Surface waves are rather difficult to investigate, but we can find a great deal from very simple considerations based on the physical quantities involved in the phenomenon. First, we must elucidate the cause of the phenomenon. The wave is a traveling oscillation. And why does the surface of the water oscillate when its equilibrium state is disturbed? Any oscillation is the result of the interplay of two factors: inertia leading to displacement from the equilibrium state and the force restoring the equilibrium state.

If a "hump" appears on the water's surface, then a restoring force, such as the gravitational force $\mathbf{F}_g$ proportional to the acceleration of gravity g, can bring water particles back to their equilibrium state (fig. 3a). Falling down, the hump will by its own inertia drop below its equilibrium state, another crest will be forced out nearby, and so on. Consequently, a wave specified by velocity $\mathbf{u}$ and wavelength λ (the distance between humps) will move forward. In the case treated here, the density ρ of the oscillating water is a measure of its inertia.

Thus, the propagation of a wave on the surface of a liquid is evaluated in terms of the following quantities (with their units of measurement supplied): $\mathbf{u}$ (m/sec), λ (m), g (m/sec^2), ρ (kg/m^3).

How are they connected? For instance, how can we evaluate the velocity $\mathbf{u}$ of the wave in terms of the other quantities λ, g, ρ? Here the units used for the quantities given above will help us.

We can see that among the units for $\mathbf{u}$ there is sec^{-1}. Among the other three quantities only g contains the time unit (namely, sec^{-2}). Hence, $\mathbf{u} \sim g^{1/2}$, and it is obvious that g has played its role and can be of no further use to us—it has provided the required sec^{-1} for $\mathbf{u}$. But at the same time $g^{1/2}$ has given us m$^{1/2}$, whereas we need a "straight" m. So we must multiply $g^{1/2}$ by $\lambda^{1/2}$ to get everything in order—the quantity $(g\lambda)^{1/2}$ will have the necessary units m/sec. Thus, $\mathbf{u}$ (m/sec) $\sim$ [g (m/sec^2) λ (m)]$^{1/2}$.

Notice that we've obtained a proportion, not an equality, since any dimensionless factor k can stand in front of $(g\lambda)^{1/2}$—that is, $\mathbf{u} = k(g\lambda)^{1/2}$. We may have $k = 0.5$ or perhaps $k = 10$. At this juncture our dimensional analysis is powerless to help us, but it has already elucidated the main point—the physics of the phenomenon. The exact solution of the problem leads to $k = 1/(2\pi)^{1/2}$—that is,

$$\mathbf{u} = \sqrt{\frac{g\lambda}{2\pi}}. \qquad (2)$$

And where is ρ (kg/m^3)? It isn't there, simply because there is nowhere to put the kg unit given by ρ—it won't cancel out so as to provide $\mathbf{u}$ with only the units m/sec. This is clear from the physical point of view: both the weight of the hump, accelerating it downward, and the mass of the hump, determining its inertia, are proportional to ρ—hence, ρ cancels out and density does not appear in equation (2). (A similar phenomenon occurs in the case of a mathematical pendulum with string of length l whose period $t = 2\pi(\lambda/g)^{1/2}$ doesn't depend on the mass for the same reason as above.)

But if the waves become slight and ripple-like (later we'll specify what "slight" is), the hump is pulled back to its equilibrium state by another force, surface tension, which depends on the coefficient of surface tension σ (n/m). This is similar to pushing a taut rubber film with your finger. Because of the film's tension, a downward force $\mathbf{F}_\sigma \sim \sigma$ is exerted on your finger (fig. 3b). In our case, wave propagation can be described in terms of the quantities $\mathbf{u}_\sigma$(m/sec), λ (m), σ (n/m), ρ (kg/m^3)—$\mathbf{u}_\sigma$ denotes the velocity of waves due to surface tension, in contrast to the $\mathbf{u}$ in equation (2).

Now that we have some experience in the use of dimensional analysis, let's deduce the expression for the velocity of waves:

$$\mathbf{u}_\sigma\,(\text{m/sec}) \sim \sqrt{\frac{\sigma\,(\text{kg/sec}^2)}{\rho\,(\text{kg/m}^3) \times \lambda\,(\text{m})}}$$

(bearing in mind that 1 n = 1 kg m/sec^2). If we want to replace the proportionality sign with the equal sign, we have to take into account the missing nondimensional factor. According to the exact theory, it's equal to $(2\pi)^{1/2}$. Hence,

$$\mathbf{u}_\sigma = \sqrt{\frac{2\pi\sigma}{\lambda\rho}}. \qquad (3)$$

Isn't it remarkable that practically "free of charge"—without recourse to any physical theories—we have obtained the essential thing: the nature of the relationship between the relevant physical quantities! At this point it will be pertinent to recall the words of the great physicist Enrico Fermi: "Physics is no place for muddled thinking. . . . Those who really understand the nature of a phenomenon can obtain fundamental laws from dimensional thinking."

For ice-cold water, $\sigma \cong 80$ dynes/cm = 8×10^{-2} n/m, $\rho = 1$ g/cm^3 = 10^3 kg/m^3. Bearing this in mind, we can plot the velocity of surface waves defined by equations (2) and (3) against $\lambda^{1/2}$ (fig. 4). The two curves intersect when $\lambda = \lambda_\sigma \cong 2$ cm. So at

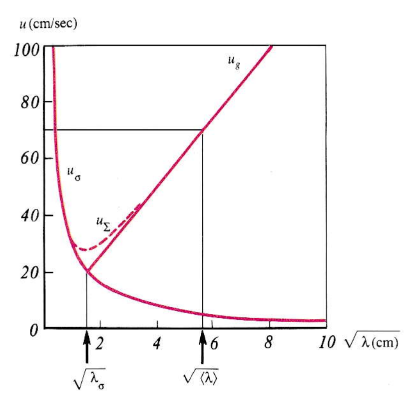
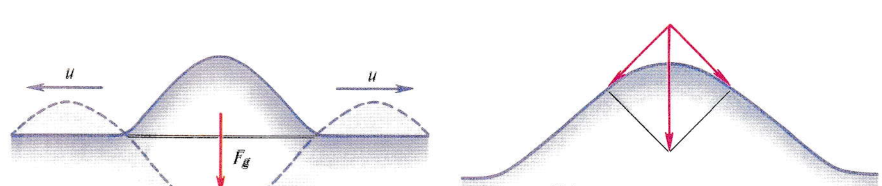


Figure 4



(a) (b)

Figure 3

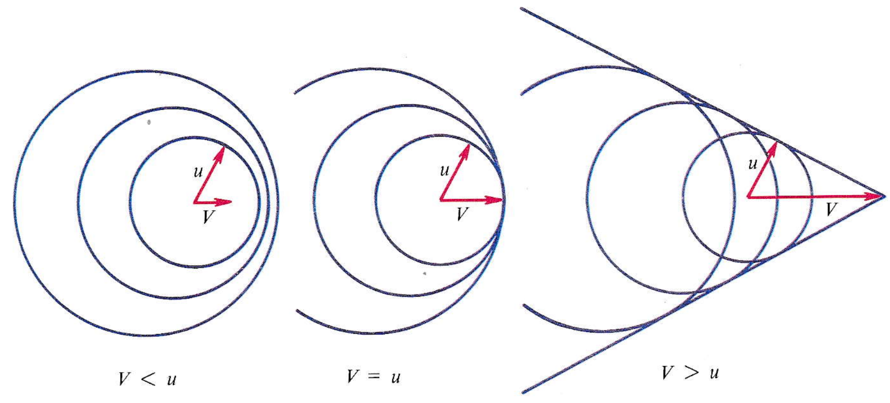


Figure 5

very small wavelengths ($\lambda << \lambda_\sigma$) wave velocity is determined by surface tension and at large wavelengths ($\lambda >> \lambda_\sigma$) by the pull of gravity. In the intermediate region ($\lambda \cong \lambda_\sigma$) the velocity of surface waves is determined by both gravity and surface tension. The expression for wave velocity becomes more complex: $\mathbf{u}_\Sigma = (\mathbf{u}^2 + \mathbf{u}_\sigma^2)^{1/2}$. The plot of $\mathbf{u}_\Sigma$ versus $\lambda^{1/2}$ is shown in figure 4 by the dotted line.

It often happens in physics that it's much simpler to treat certain limiting cases (λ approaches 0 and λ approaches infinity, in our case) than the intermediate region. Fortunately, the intermediate region is of no concern to us because we can safely state, just by glancing at the Rerikh painting, that the circle waves clearly have a wavelength greater than λ_σ. Hence, the velocity of such waves is determined by equation (2). To find it we must "measure" λ. The necessary computations will be performed below, but first we'll take another step.

When a water bug runs on water or a bullet flies through the air faster than sound, distinctive accompanying "breakers" appear. The greater the object's speed **V** compared to the speed **u** at which the disturbances caused by the object propagate, the closer these breakers are to the moving object. The three typical cases are shown in figure 5: if **V** < **u**, the waves outrun the object and are merely condensed in the direction of travel; if **V** = **u**, the wave crests crawl on one another at one point (in the direction of travel); if **V** > **u**, the object outruns the waves and breakers are formed. (Figure 6 illustrates the third case.)

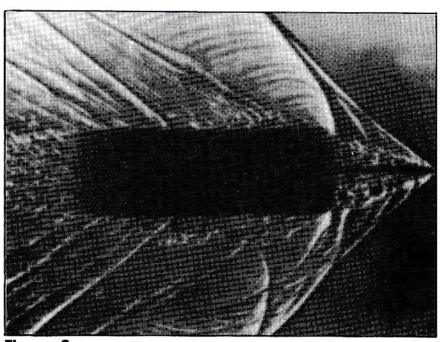


Figure 6. This photograph shows the breaker waves formed when a disk with a needle moves in a gas at a velocity greater than the speed of sound—that is, the speed of wave propagation in a gas.

Looking at the Rerikh painting, we see that it illustrates the first case. There's no evidence that the waves in the direction of travel are compressed, we have to conclude that the velocity of the boat is much less than that of the waves:

$$\mathbf{V} << \mathbf{u}. \qquad (4)$$

Therefore, judging by the pattern of the circular surface waves, the boat is practically standing still.

Now let's turn to computations. In order to perform them, we must "measure" the height h of the wave at the bow, the angle 2α, and the wavelength λ of the surface waves.

Measurements

We could try to measure the required quantities more accurately by taking into account perspective, projection, foreshortening, and so on. But this is all very difficult and time consuming. We would also be putting ourselves in a ridiculous position—treating a fairy-tale painting as if it were an experimental photograph is downright absurd. Besides, we don't know such things as the height of the bow , the size of the shields, and so on. There's one characteristic feature in the painting, however, that has apparently changed very little since the days of the Vikings: head size.

Before reading on, try to estimate h and λ yourself, if only with the help of this "standard." This will be your own personal "measurement." Keep in mind, though, that each onlooker will have an individual perception of the painting. Show this painting to your classmates and ask each one about the values of λ , h, and α. (It's best to conduct the poll such that each student doesn't hear the answers of the others. In this way the "readings" obtained will resemble the independent readings of a measuring device.) Record the answers given, then plot the values of h along the horizontal axis and, along the vertical axis, the number of students n who have put forward the value h.

I conducted such a poll among 30 colleagues and obtained the graphs shown in figures 7 and 8. It's interesting that most estimates of h and λ were multiples of 5 cm; some were multiples of 2–2.5 cm; but none were multiples of a smaller number. (For example, nobody put forward the value h = 27.1357891439 cm.) It was as if those polled were using a ruler with a scale that reads down to 2–2.5 cm.

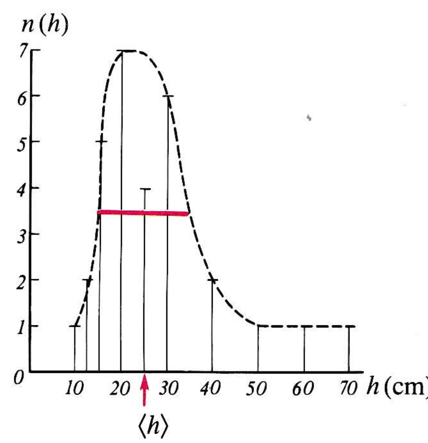


Figure 7

With the help of these graphs, we can calculate the averages $\langle h \rangle$ and $\langle \lambda \rangle$ and regard them as sufficiently fair.

The procedure for calculating the averages is standard. Each value h_i must be multiplied by the number of people n_i who suggested it; then all values of i must be added and divided by the total number of participants in the poll. As a result, we get

$$\langle h \rangle = \frac{\sum_{i=1}^{30} h_i n_i}{\sum_{i=1}^{30} n_i} = \frac{1}{30}\sum_{i=1}^{30} h_i n_i$$
$$= \frac{1}{30}[(1 \times 10) + (2 \times 12) + (5 \times 15) + (7 \times 20) + (4 \times 25) + (6 \times 30) + (2 \times 40) + (1 \times 50) + (1 \times 60) + (1 \times 70)]$$
$$= \frac{789}{30} \cong 26 \text{ cm} \cong 0.26 \text{ m}. \qquad (5)$$

Similarly,

$$\langle \lambda \rangle = \frac{1}{30}[(2 \times 10) + (3 \times 12) + (4 \times 15) + (1 \times 18) + (8 \times 20) + (2 \times 25) + (2 \times 30) + (1 \times 40) + (3 \times 50) + (1 \times 70) + (1 \times 90) + (1 \times 120) + (1 \times 150)]$$
$$\cong 34 \text{ cm} \cong 0.34 \text{ m}. \qquad (6)$$

These averages are indicated by red arrows in figures 7 and 8.

The accuracy of the measurements in our poll can be roughly estimated by the width of the smooth curve at half its height (the red lines in figures 7 and 8). A comprehensive theory of errors in physical experiments has been developed and is very similar to the one we've used. In an experiment, of course, the poll is conducted among measuring devices (not people).

And how about the angle α? Those polled agreed that 2α is between 90° and 180° (the bow of the boat in the painting is rather blunt). Hence,

$$\sin 45° < \sin\alpha < \sin 90°,$$
$$\frac{\sqrt{2}}{2} < \sin\alpha < 1. \qquad (7)$$

Now we've obtained all the data necessary to estimate the velocity of the boat and surface waves.

The Results

Substituting the values of h, λ, and $\sin\alpha$ given by equations (5), (6), and (7) into equations (1) and (2), we get

$$\mathbf{V} \geq \sqrt{\frac{2 \times 9.8 \text{ (m/sec}^2) \times 0.26 \text{ (m)}}{\sqrt{2}/2}} \cong 2.7 \text{ (m/sec)},$$
$$\geq \sqrt{\frac{2 \times 9.8 \text{ (m/sec}^2) \times 0.26 \text{ (m)}}{1}} \cong 2.3 \text{ (m/sec)}; \qquad (8)$$

$$\mathbf{u} = \sqrt{\frac{9.8 \text{ (m/sec}^2) \times 0.34 \text{ (m)}}{2\pi}} \cong 0.7 \text{ (m/sec)}. \qquad (9)$$

Now we come to a surprising result: equations (8) and (9) do not agree with equation (4). Indeed, according to the height of the bow wave, the velocity of the boat **V** is at least 2.3 m/sec; but if we base our judgment on the circular surface waves (which are almost concentric in the painting), the velocity of the boat must be considerably less than the velocity of these waves **u** = 0.7 m/sec, which is already less than **V**.

If you pay enough attention to detail, you'll discover physical inconsistencies in other works of art (which is probably due to the fact that art has its own aims and laws). You can still admire the works of art and derive aesthetic pleasure from them, but you can also use them as attractive illustrations when discussing the laws of physics with your little brother or sister. ◘

Albert Stasenko, *doctor of technical sciences, is a professor at the Moscow Physics and Technological Institute, where his specialty is aerodynamics and gas dynamics. He enjoys sailing in his spare time.*

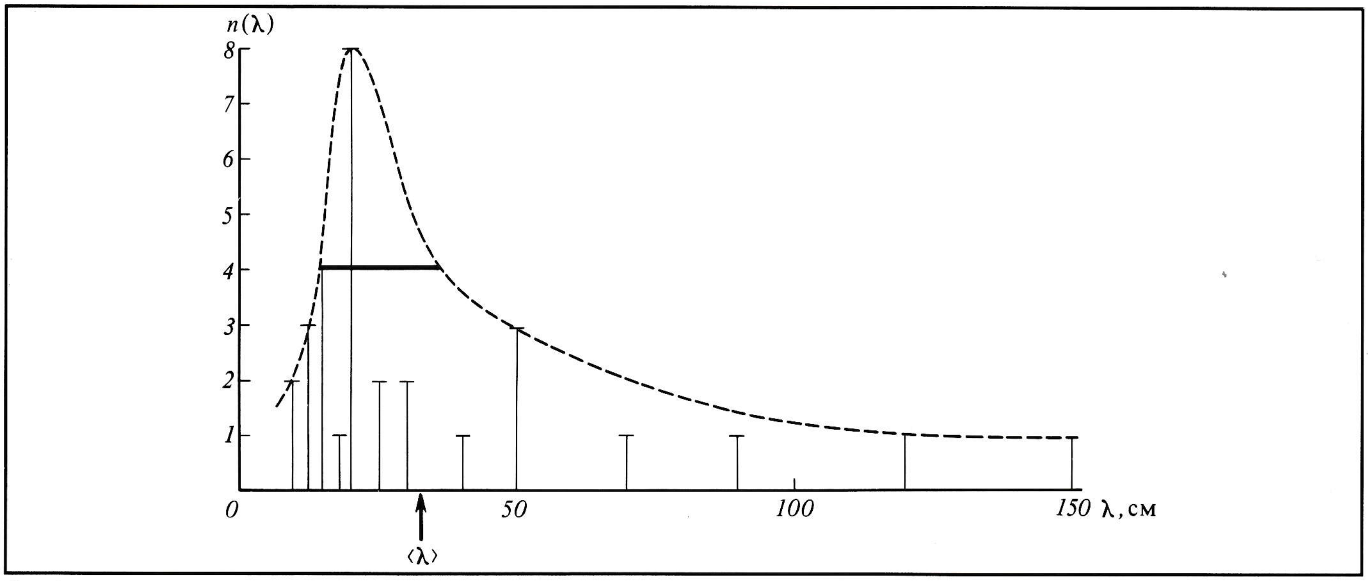


Figure 8

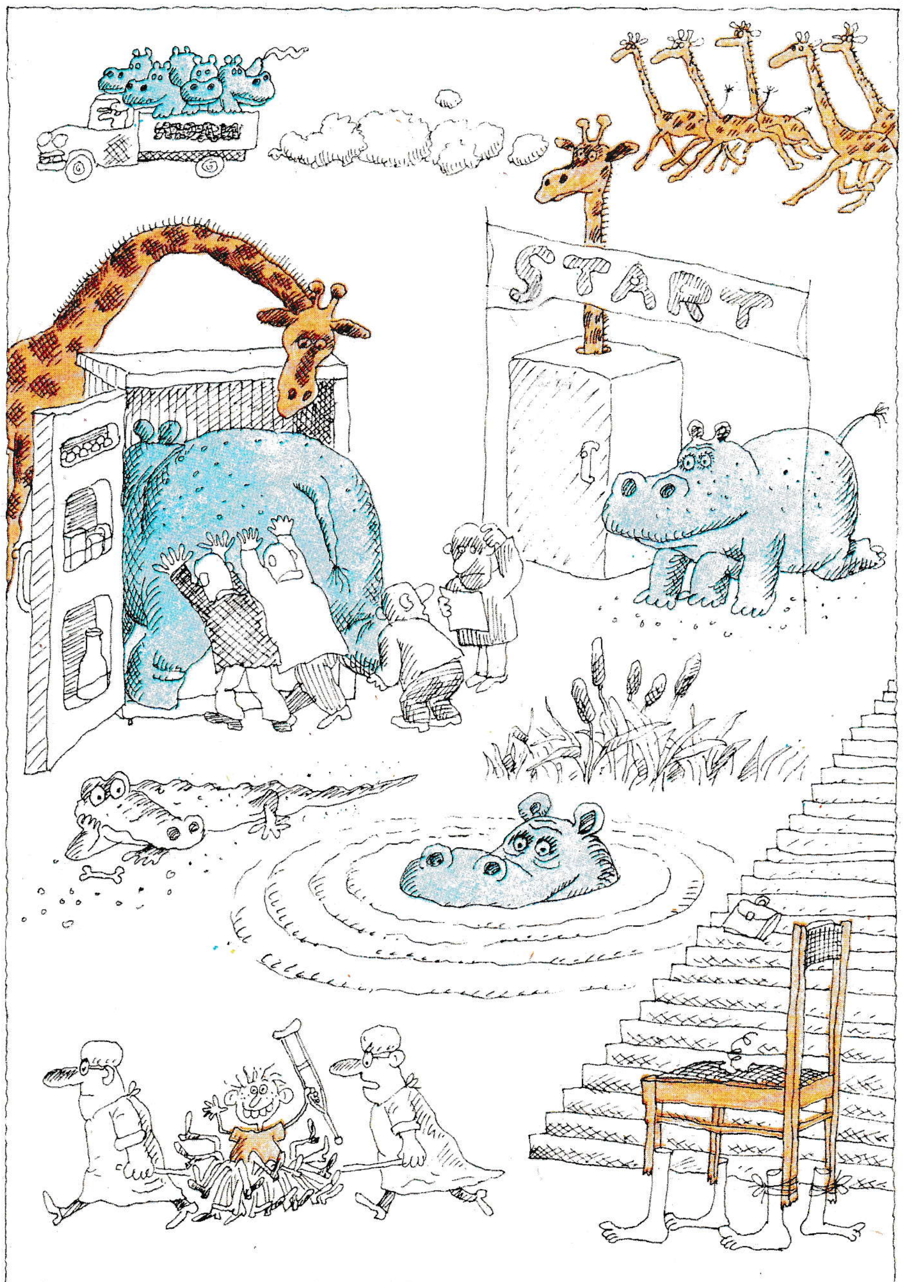
QUANTUM SMILES

It was just

One problem after another

—but I failed to see the links

by B. M. Bolotovsky



THE FIRST THING I WANT TO TELL YOU is that I have no idea where these problems came from. I heard some of them from Oleg Dolgov. Soviet readers of our magazine have seen him on TV—for a few years he was the captain of a team on the very popular quiz show "What? Where? When?" But even Dolgov, who can be considered an expert in things like this, doesn't know who the author is. In addition to the problems Dolgov told me, I'd heard other problems of this kind; but no matter how hard I tried, I couldn't find who made them up. Personally, I liked the problems so much I decided to publish them. I hope you enjoy them too, and I secretly hope that one of you will be able to help us find the authors. Or maybe the authors themselves will see their problems published and will step forward. And now, without further ado . . .

Problem 1. *How many operations are needed to put a hippopotamus into a refrigerator?*

After Dolgov had posed the problem, I fell to thinking . . . Dolgov came to the rescue. "I'll tell you how to solve the first problem. You have to perform three operations to put a hippopotamus into a refrigerator:

1. Open the refrigerator;
2. Put the hippopotamus in;
3. Close the refrigerator."

This solution helped me understand the meaning of the word "operation." Every action shown in the solution was actually an operation. Another problem followed.

Problem 2. *How many operations are needed to put a giraffe into the refrigerator?*

Having thought a little, I said, "More operations are probably needed to put the giraffe into the refrigerator than to put the hippopotamus there."

"Why?" Dolgov asked me.

"Because the refrigerator won't hold the giraffe. The giraffe must be folded up before it can be put in."

"There's no need to fold the poor

animal," Dolgov said. "The refrigerator's big enough. The giraffe can easily fit into it if it's empty."

"So it's enough to perform three operations, as before: open the refrigerator, put the giraffe in, and close the refrigerator."

"Wrong," Dolgov said. "Four operations are required." And he enumerated them:

1. Open the refrigerator;
2. Take the hippopotamus out;
3. Put the giraffe in;
4. Close the refrigerator.

Need I explain? I'd forgotten the refrigerator was full. The hippopotamus was still there after the first problem.

And the problems kept coming.

Problem 3. *The hippopotamus and the giraffe are a kilometer away from a river. Which of them will reach the riverbank first?*

When solving problems like this, it's useless to think. Nevertheless, I thought a little before saying, "The giraffe will get there first—its legs are longer."

"Wrong again," said Dolgov.

"What's the right answer then?"

"The hippopotamus will reach the river first."

"Obviously! Why?"

"Because the giraffe is still in the refrigerator."

I laughed and decided to keep trying my luck.

Problem 4. *How many hippopotamuses can a five-ton truck hold?*

I started to think again, but Dolgov didn't let me think for long.

"Don't waste your time. I'll tell you the right answer. It'll hold five tons of hippopotamuses—a full load. Solve the next problem yourself—and be quick!"

Problem 5. *How many giraffes can the five-ton truck hold?*

"Five tons of giraffes," I said, not quite sure of myself.

"Wrong! Not a single one."

"Why?"

"Because the truck is full of hippopotamuses."

Sure enough—after problem 4 the hippopotamuses were still in the truck. No one had taken them out.

I liked the problems so much that I memorized them and told them to my daughter Katya when I got home. She's in the sixth grade. To my great surprise, Katya solved the problems in no time, one after the other, and gave me another problem to solve.

Problem 6. *A boy fell down from the fourth step of a staircase and broke his leg. How many legs will the boy break if he falls from the fortieth step?*

Not sure of the answer, I said, "Forty steps . . . are ten times as many as four steps . . . so the boy should break ten legs. But that's probably wrong."

"It is," said my daughter.

"What's the right answer?"

"The boy will break only one leg."

"Why?"

"Because he already broke one and he only has two."

Then Katya and I decided it wasn't very good of us to pose the problem about the boy because we felt sorry for him—it's very painful to break a leg. So we changed the problem so that it was about a chair—a chair falls down the stairs and breaks its legs. We felt sorry for the chair too, though not as much as for the boy. Besides, the problem seemed very interesting to us. One chair isn't too much to sacrifice for a problem like this.

WELL, I'VE TOLD YOU all the "chain problems" I know. Do you know any others? If so, send them to *Quantum* and amuse your fellow readers.

The adventures of Hans Pfaall and Fatty Pyecraft

The deflating physics of two strange fictions

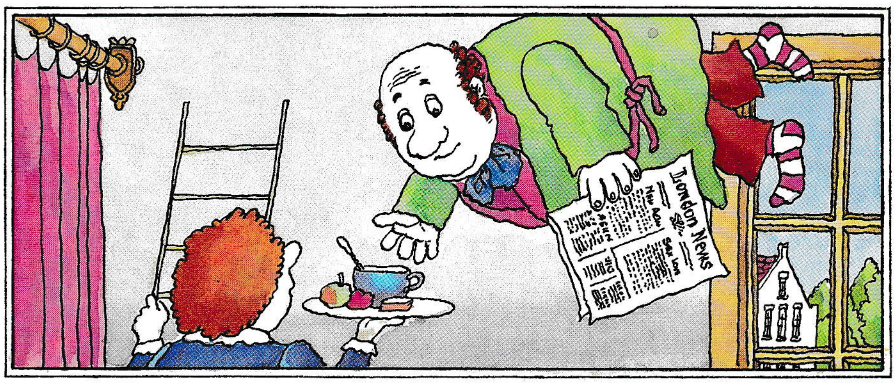
by V. Nevgod

ONE OF EDGAR ALLAN POE'S lesser-known "Tales of the Grotesque" is entitled "The Unparalleled Adventure of One Hans Pfaall." The hero of the tale makes a remarkable discovery—he obtains an extraordinary gas whose density is 37.4 times less than that of hydrogen. A balloon containing such a gas possesses an incredible lift force. With the help of this balloon, Poe tells us, Hans Pfaall succeeds in reaching the moon.

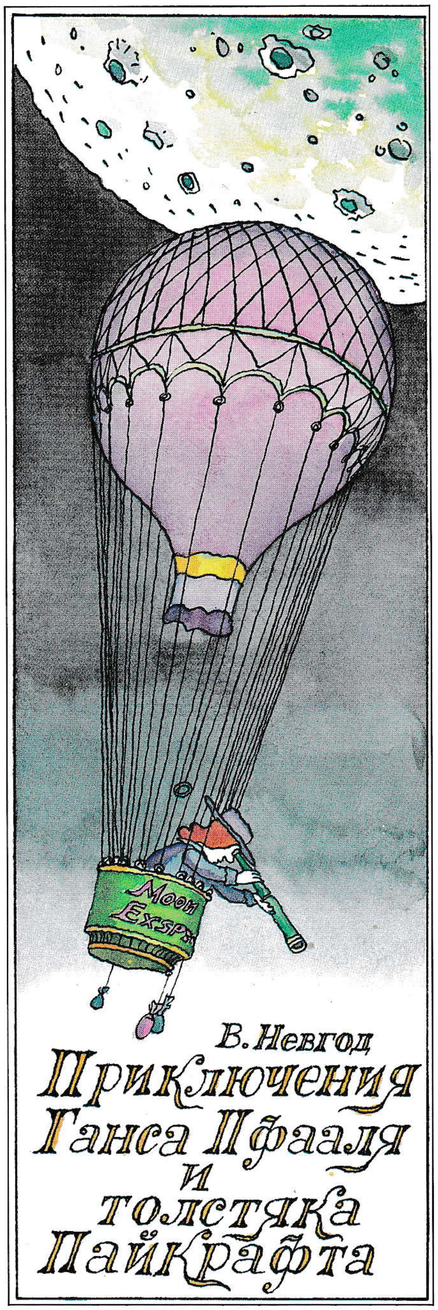
It's common knowledge that there is no gas in nature that's lighter than hydrogen—no proof of that will be given here. (But can you explain this fact?) Nevertheless, the following problem is of interest: if such a gas existed, by what factor would it increase the lift force of the balloon (as compared to hydrogen)?

Despite the simplicity of the problem, many people fail to give a correct answer at first. A "logical" conclusion would seem obvious: since the gas is lighter than hydrogen by a factor of 37.4, if follows that its lift force is greater by the same factor. Perhaps Poe counted on such a hasty conclusion by readers when he wrote his tale. Then again, it's equally possible he himself made that same error by following a seemingly logical pattern of thought.

Simple calculations, however, show that the gain in lift force is so small that it should be considered negligible. We'll prove this by finding the lift force of a hydrogen balloon and that of a balloon containing Hans Pfaall's gas.



Consider a balloon with a volume of 1 m^3. The density of air is 0.00129 g/cm^3; of hydrogen—0.00009 g/cm^3; and of Hans Pfaall's gas—0.0000024 g/cm^3. The lift force of a balloon is equal to the difference between the buoyancy force (equal to the weight of the air displaced by the balloon) and the weight of the gas contained in the balloon (the shell of the balloon being considered weightless). The lift force of a hydrogen balloon, then, is about 12 newtons; that of a balloon containing Hans Pfaall's gas is 12.9 newtons.



So the gain in lift force is only about 0.9 newton! The result is so insignificant that it wasn't worth Hans Pfaall's (or Edgar Allan Poe's) effort to discover the miraculous superlight gas and violate the laws of nature to boot. (To be fair, we should remember that the Mendeleyev periodic table had not yet been drawn up in Poe's lifetime.) What led to the blunder? The low weight of hydrogen.

Were it possible to obtain a gas lighter than hydrogen by a factor of 1,000, it wouldn't significantly help boost a balloon's lift force. The limit of such an increase is 0.9 newton—the weight of the hydrogen itself.

H. G. WELLS wrote a popular fantasy called "The Truth About Pyecraft." In it a funny-looking fat man, named Pyecraft, desperately wants to lose weight. He takes a mysterious Indian remedy and loses more than his excess weight—he loses *all* his weight! Day after day he floats under the ceiling of his study, not daring to go outdoors—he's afraid of floating away like a balloon. Eventually someone advises Pyecraft to order a special suit with lead sewn into the lining. Donning lead-soled boots in addition to his lead suit and carrying a bag of solid lead, he's able to go outside again like a normal person.

The obvious question pops up: how much lead does Pyecraft need to walk around? Let's perform a simple calculation. Suppose fatty Pyecraft weighs 1,000 newtons (his mass being 100 kg). His body volume would then be about 0.1 m^3. On becoming weightless, poor Pyecraft sort of turns into a strangely shaped balloon of the same volume. Its lift force would be only about 1.3 newtons!

So what's the real "truth about Pyecraft"? Even in his ordinary clothes, Pyecraft didn't have to float under the ceiling of his study—he could sit at his desk (though not too steadily) and could even walk carefully around the room (avoiding abrupt movements). He'd have no need of a lead suit or lead boots—he could go outside in his regular clothes, maybe with the addition of a heavy briefcase to protect him from a strong wind. He'd be in no danger of floating up into the upper atmosphere.

It turns out that Pyecraft's "airworthiness" and the problems he faced were greatly exaggerated. It's doubtful, of course, that H. G. Wells failed to notice this when writing the story. Most likely, he deliberately ignored the relevant data so he could depict more colorfully and eloquently the comic misadventures of the hapless Pyecraft. No doubt the author was convinced that his readers, carried away by the bizarre plot, wouldn't spot the exaggerations.

GEOMETRY WITH A TWIST

Bend this sheet

But do not fold, staple, or mutilate

by Dmitry Fuchs

TAKE A SHEET OF PAPER AND BEND IT without crumpling it. You get a piece of a surface whose shape depends on how you bend the sheet. Some possible shapes are shown in figure 1.

It's certainly not true, though, that any surface can be formed by bending a sheet of paper. For example, it's common knowledge that you can't bend a sheet of paper into a sphere—if you press a piece of paper onto a globe, folds necessarily appear. Of course, you can roll a sheet of paper into a tube or paper

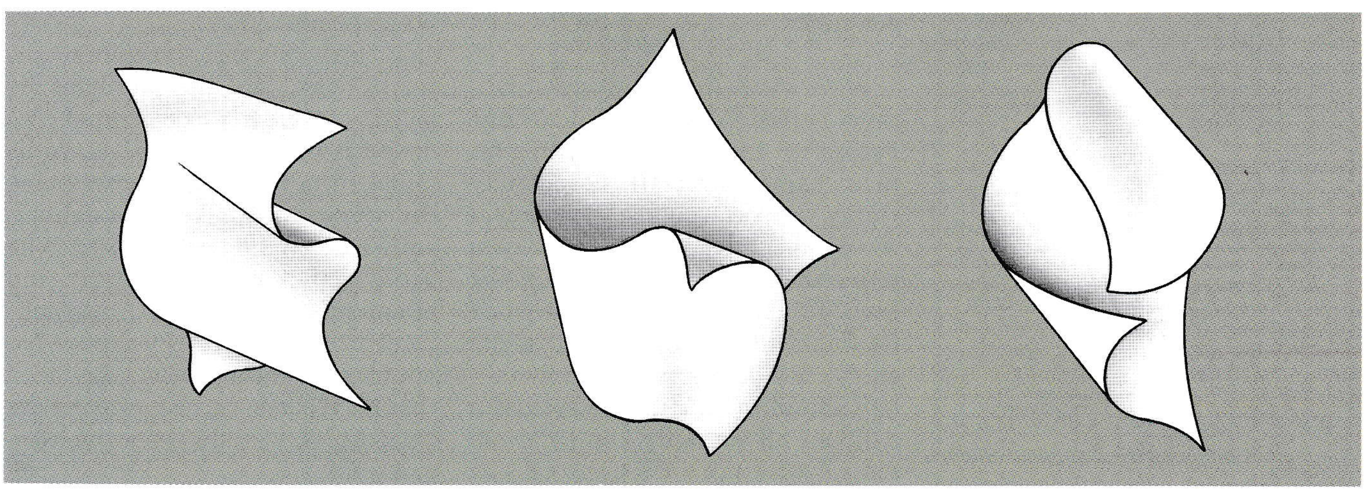


Figure 1

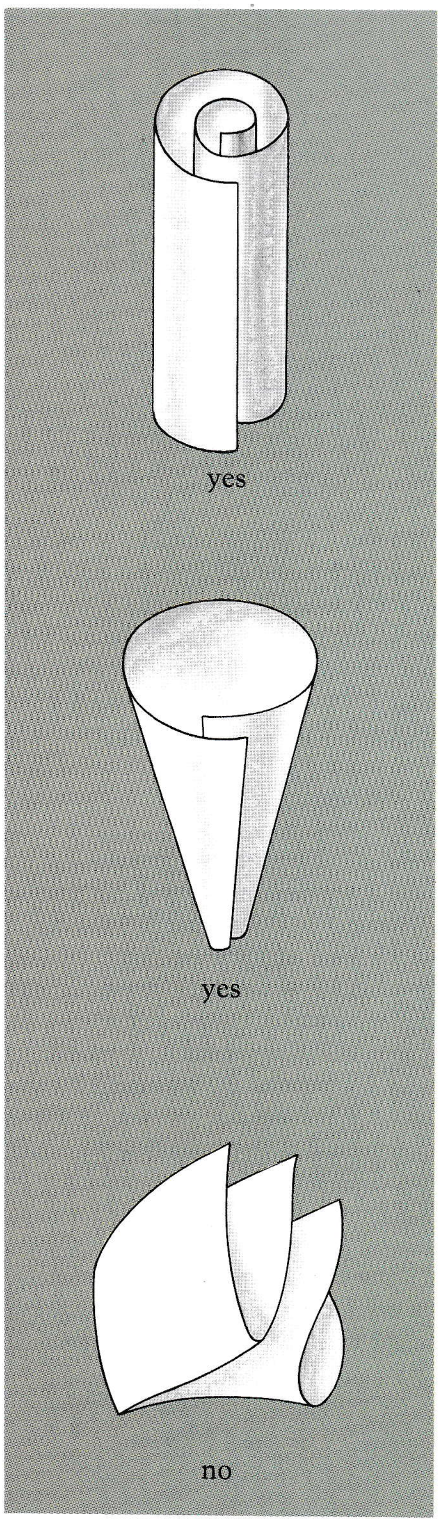


Figure 2

cone, but it's impossible to furl it up in four like a handkerchief without creating folds (fig. 2).

DEFINITION. *Surfaces that can be represented as bent sheets of paper are called developable surfaces.*

"That's not a definition at all, that's nonsense—nothing useful can come of it," the pedantic reader will say, and we have to agree. It's not that there's anything wrong with our definition. It's just that this finicky reader shouldn't bother to read on—we won't be offering any rigorous definitions or proofs here.

Let's just say that all our proofs, as in an ancient manuscript on geometry from India, will pretty much consist of one word: "Look!"

However, for the perceptive reader (not the pedantic one, who has closed the magazine by now) we can add that two essential physical properties of paper sheets are assumed here. The first is *unstretchability* (*incompressibility*), which means that a curve drawn on the sheet can change its shape when the paper is bent but must always preserve its length. The second property is *flexibility*, which means that there are no other constraints on the nature of the bending.

THE FACT THAT not all surfaces are developable is evident from the fact that every developable surface is a *ruled surface*. This means you can place a knitting needle anywhere on the surface so that the needle touches the paper along an entire line segment containing any chosen point. (Proof: try it, as in figure 3.) In other words, a developable surface consists of straight line segments that are contained entirely within it. These segments are called the *linear rulings* (or simply the rulings) of the surface.

If some point of a developable surface is an internal point of two different rulings, then a whole section of the surface near this point is flat (fig. 4). We'll exclude this case from consideration and require that no part of our surface, no matter how small, be a piece of a plane.

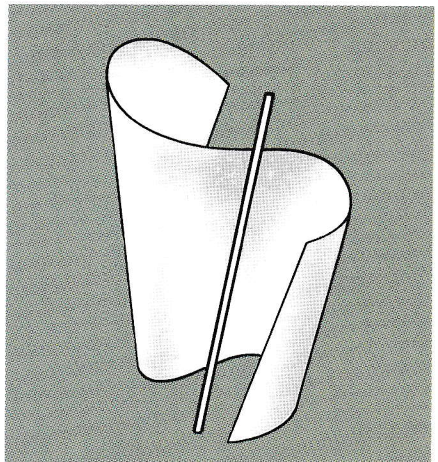


Figure 3

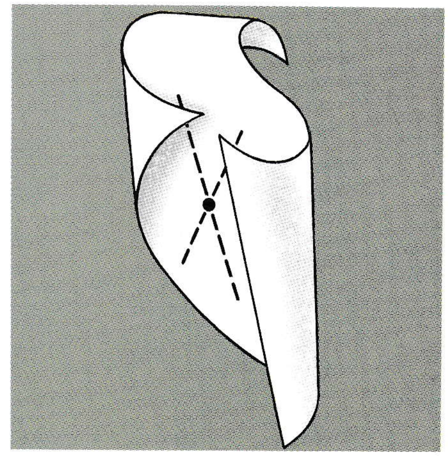


Figure 4

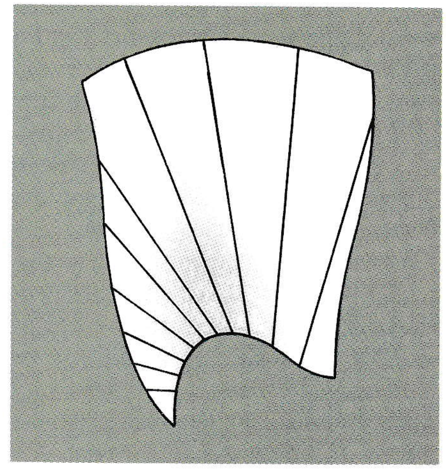


Figure 5

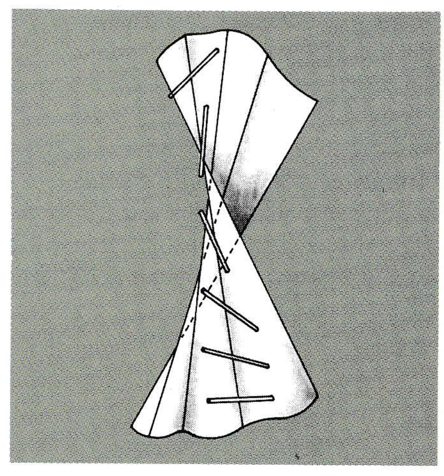


Figure 6

Therefore, exactly one ruling passes through each point of our surface. These rulings form a con-

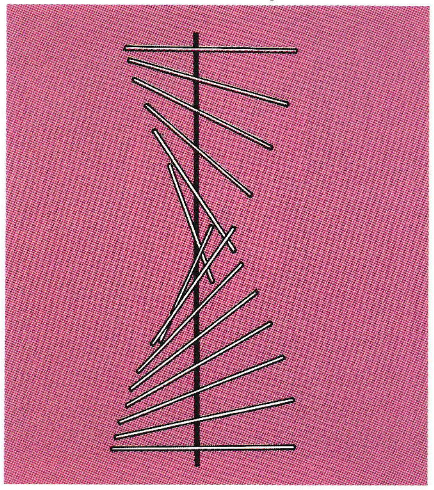


Figure 7

tinuous family of segments sweeping out the surface (fig. 5). Some of the segments can degenerate into points on the boundary of our piece of surface. (This last remark is aimed at the shadow of the pedant hovering in the background.)

Now you shouldn't think that every ruled surface is developable. There are plenty of ruled surfaces—a line segment moving in space sweeps out a ruled surface, for instance.

Let's take any ruled surface, make a linear ruling, and for each point of the ruling draw the line perpendicular to the ruling and tangent to the surface (fig. 6). We'll obtain a pattern similar to a Christmas garland (fig. 7)—our perpendiculars protrude haphazardly as they turn on the ruling. For any developable surface, however, all such perpendiculars must belong to the same plane. In other words, not only can you place a knitting needle on a developable surface at any given point, you can lay the flat of a ruler tangent to the surface along the same line as well (fig. 8). (Try it and see.) This property of developable surfaces is sufficient—that is, any surface possessing this property is necessarily developable.

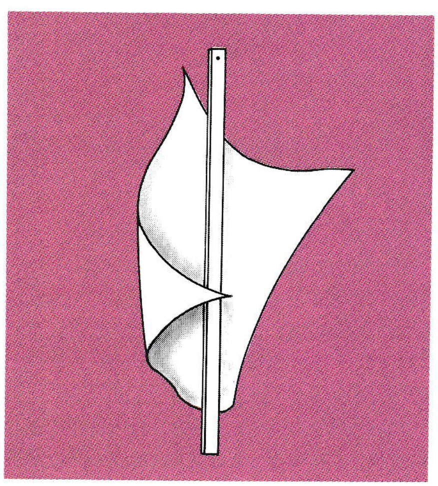


Figure 8

By the way, have you ever come across a hyperboloid of one sheet, or a hyperbolic paraboloid (fig. 9)? If not, no matter—we don't need them here. But if you're familiar with them, notice that they are *not* developable. Why not? Because a pair of rulings passes through each point of these surfaces, which is impossible for developable surfaces.

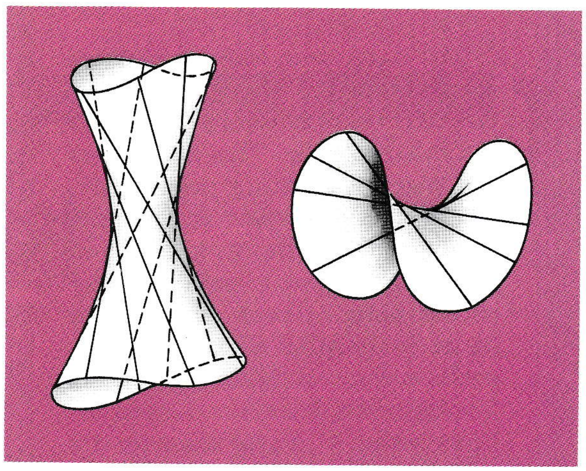


Figure 9

Look at figure 5 again. Recall that we're not dealing with an infinite surface but only a piece of it. (Indeed, a sheet of paper can't be infinite!) The piece is delineated by the rulings. What will happen if we try to extend them in one direction or another?

At first the question seems quite innocent. If you extend the rulings of the surface in figure 5 "up" toward where they fan out, nothing interesting will happen—the surface will just grow, becoming flatter and flatter (fig. 10).

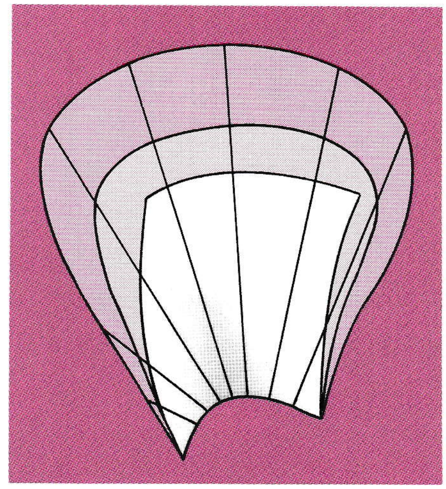


Figure 10

But what if you extend the rulings in the opposite direction? Take a sheet of paper, bend it approximately as shown in figure 5, and try to imagine the shape of the surface obtained by extending the rulings in the direction where they converge (fig. 11). Put the magazine aside and really think about it. Then come back and look at the answer.

The answer is this: the surface will no longer be smooth—a *cuspidal curve* will appear (fig. 12a). (If you guessed it on your own, you're a true geometer!) A cuspidal curve is a curve in whose neighborhood the planar section of the surface looks approximately as shown in figure 12b.

I promised to avoid proofs. It's not that I dislike proofs (that certainly isn't true). And it's not because the proofs of my statements,

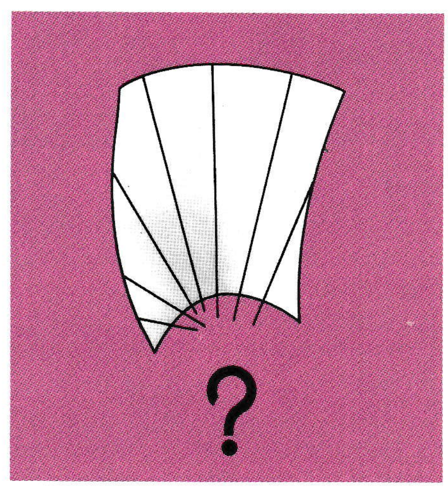


Figure 11

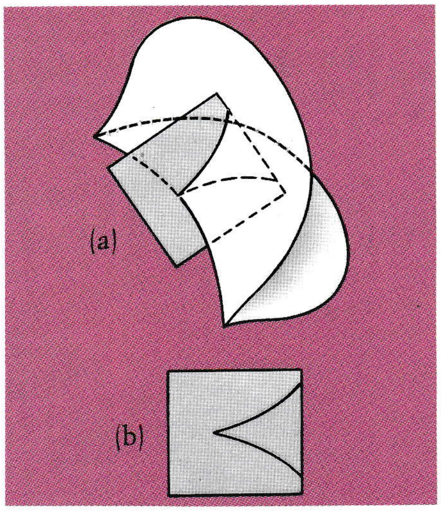


Figure 12

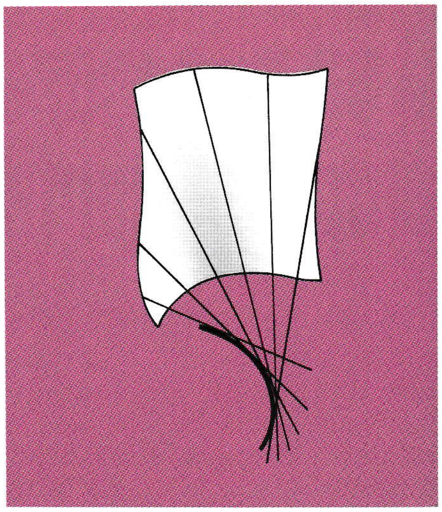


Figure 13

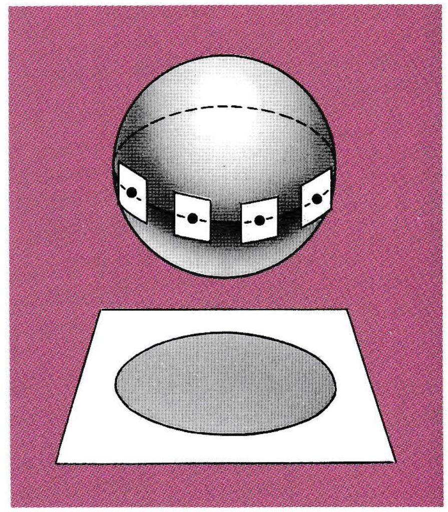


Figure 14

and of my last statement in particular, are so complex they can't be explained to a high school senior who knows a little calculus (that's true only to a certain extent). It's just that the proofs of the theorems in this article—at least the proofs that I know—involve playing around with formulas: defining the surface by an equation, expressing developability in terms of derivatives, and so on and so forth. I don't want to present a proof that isn't likely to clarify anything. But I still have to convince you that certain statements are correct even if I don't prove them.

Take one more look at figure 5. As with any depiction of an object in space, that object is projected onto a plane (for example, onto this magazine page). Thus, we have a family of lines on the plane. Copy it on a separate sheet of paper and extend all the lines. You'll find that these lines accumulate around a certain curve to which they're all tangent (fig. 13). Now, a very simple theorem in calculus states that any continuous family of lines, unless they're all parallel or pass through a fixed point, will have such an "envelope." (A similar theorem is true for a family of curves.) So we have "caught" the cuspidal curve.

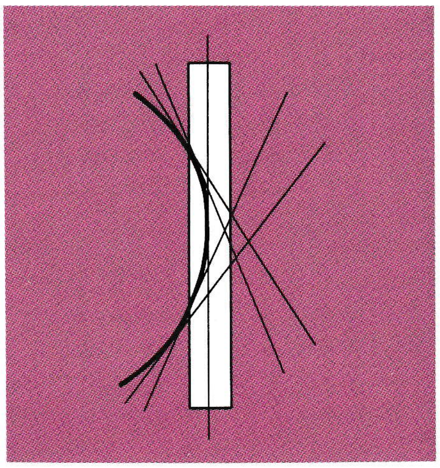


Figure 15

"But wait a minute," you (the perceptive reader) will object. "You can apply this argument (if it deserves that name) to any ruled surface. For example, the one-sheet hyperboloid (fig. 9) has a line to which the rulings projected onto the picture plane are tangent (the contour hyperbola), but there's no cuspidal curve on this surface."

You're right, as always. But I have an argument stashed away that will probably convince you. The curve you have in mind is the edge, or *visible contour*, of the depicted surface. At each point on this curve the plane tangent to the surface is perpendicular to the picture plane. (See figure 14, where the visible contour of a sphere is shown.) But for a developable surface, the tangent plane is one and the same at all

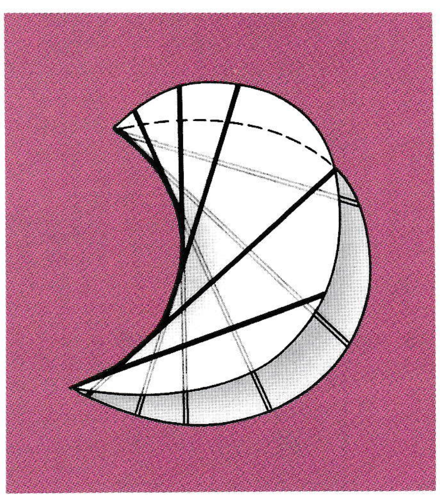


Figure 16

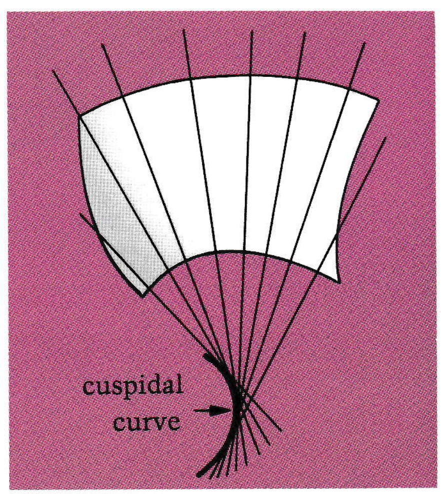


Figure 17

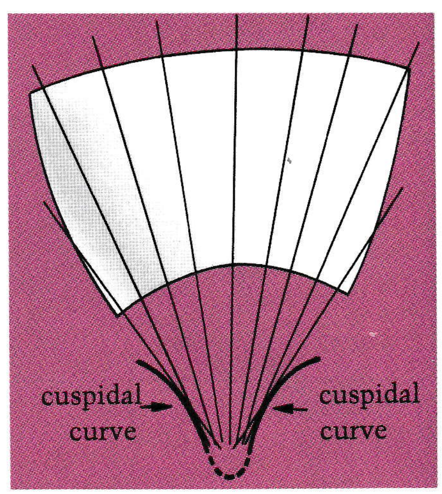


Figure 18

A word or two more about developable surfaces

(and a model to help visualize them)

"How paper bends" sounds at first like the most mundane (and innocent) of topics, but Dmitry Fuchs shows how intriguing (and tricky) an ordinary sheet of paper can be.

As I was reading his article I wasn't content to bend paper and imagine in my head what happens as the lines are extended, even though I knew something about developable surfaces from studying differential geometry. It's hard to visualize the cuspidal curves of a developable surface, let alone a swallowtail. I made some models and some computer pictures—maybe you'd like to try some for yourself.

The simplest spatial curve that doesn't lie in a plane is a helix (like the coils of a spring). Imagine a straight line segment tangent to the helix, touching it at the midpoint of the segment. Now imagine the surface swept out by the line segment as it moves along the helix. Fuchs shows that this surface has a sharp crease, or "cuspidal curve," along the helix. This is weird. How can a straight line sweep out a surface with a crease?

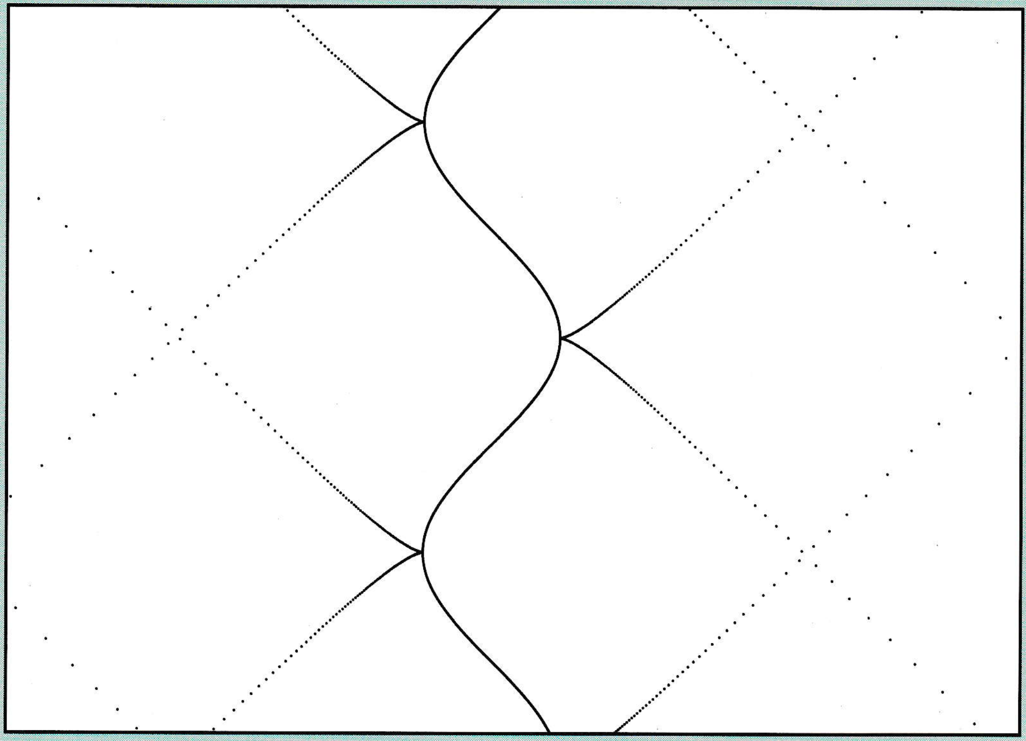
Computer pictures help a little. The figure below shows schematically what happens. The sinusoidal curve weaving up the center is the projection of the helix ($\cos t$, $\sin t$, t) onto the x–z plane. The dotted curves are a cross section of the surface swept out by the lines tangent to the helix. You can see the cusps along the helix. Can you visualize this? It really is strange.

A convincing demonstration can be made with paper. You need thin, straight sticks of some type to control the direction of the rulings. I used wooden skewers (like long toothpicks) from the local grocery store, but stiff wire like coat hangers or bicycle spokes should also work.

Cut out circles about size of a peanut butter jar lid from the middle of two sheets of paper. Throw away the circles and join the sheets along the edges of the holes. To do this, lay lengths of tape flat along the circle so that they overhang the hole. Cut slits from the edge of the tape up to the circle, fold the flaps of tape under, and squeeze together.

Cut a slit in both sheets from the circle out to an edge so you'll be able to open the model later to form a helix.

Now, one by one, lay the thin, straight sticks between the two sheets of paper so that they nestle against the circle where they're joined. Tape the clockwise end of each stick to the lower sheet of paper and tape the counterclockwise end to the upper sheet of paper. With about six or eight sticks the model will have reasonable stiffness.

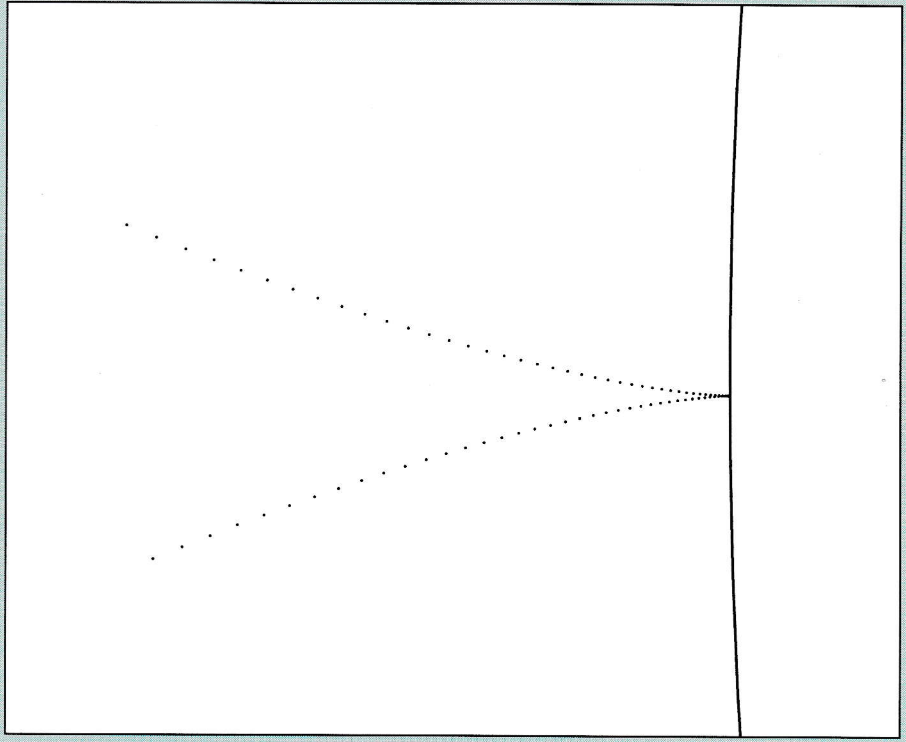


The sinusoidal curve in the center of the illustration at left is a helix (for example, a spring) as seen from the side. If you take a line tangent to the helix and move it along the helix so that it remains tangent, it sweeps out a surface. This surface is an example of a "developable surface," one that can be modeled by bending sheets of paper in three dimensions. The dotted curves on either side of the helix are cross sections of the surface. This surface interpenetrated itself and has a sharp cusp, or crease, where it meets the helix.

Now open the model so that the circle becomes a helix, the sticks become lines tangent to the helix, and the paper becomes a developable surface swept out by the tangent lines of the helix. You can stretch it so that the helix is very long or flatten it until the helix is very shallow. You'll see the cusp where the two sheets join. The cross section of the model is like the figure at left.

Despite all the sticks, the model still has considerable flexibility. If you know about the curvature of curves in space, the circle can be bent to the shape of any other curve with the same curvature.

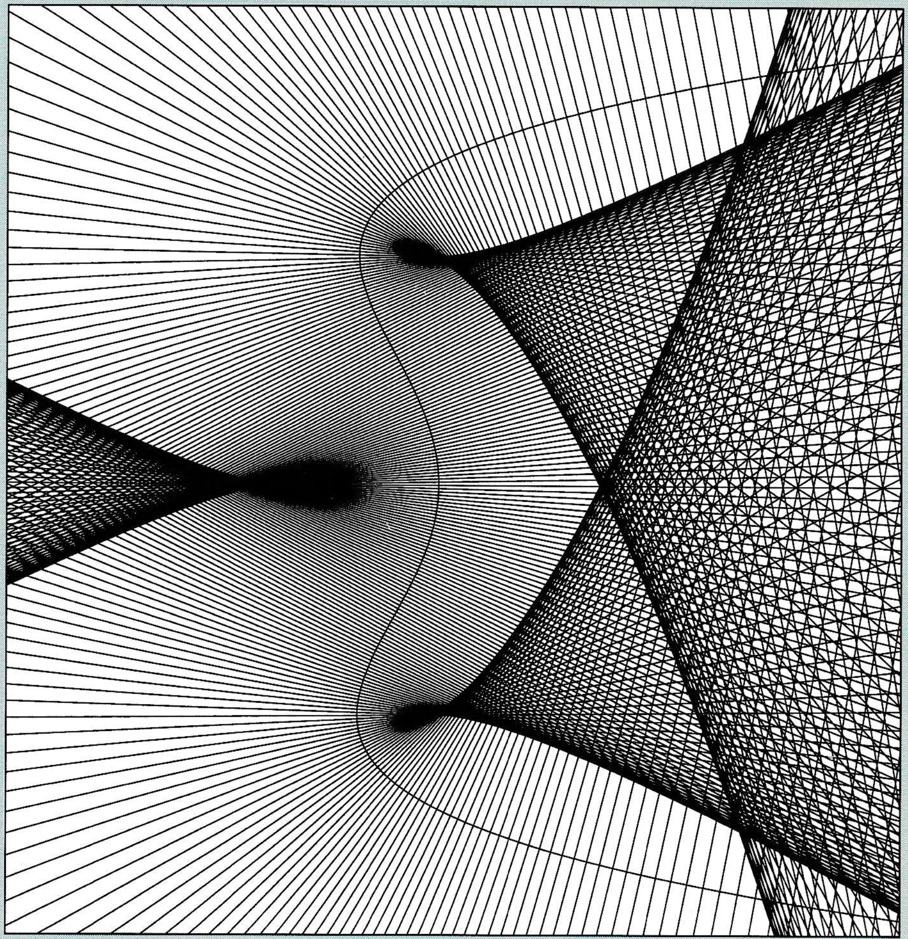
You can form additional turns of the helix, if you like, by joining more sheets of paper to the first two. You end up with a large paper screw. It will look nicest if you trim each sheet of paper to be a circle concentric with the hole and cut the wooden ribs so that they just reach the outer circle at both ends.



The figure above is a close-up of the figure on page 20, showing how the cusp becomes arbitrarily sharp as the scale gets finer.

JUST FOR THE RECORD, I tried a quick model of the swallowtail, but I'm not proud of the result. Try it for yourself! If you find a good construction technique, I'd like to hear about it.

—William P. Thurston



The family of lines at right is the set of normals to the curve $(t^4 - t^2, t)$ *weaving through the center of the figure. You can see the folds where the lines are tangent and the cusps where the folds reverse direction.*

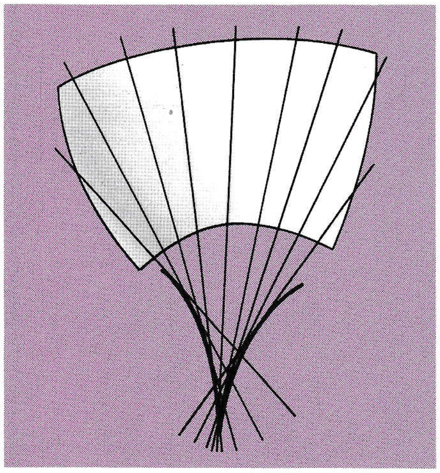


Figure 19

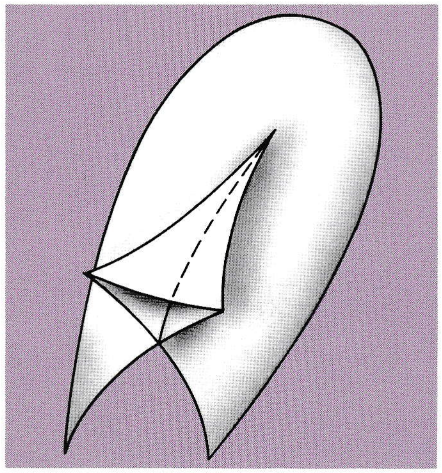


Figure 20

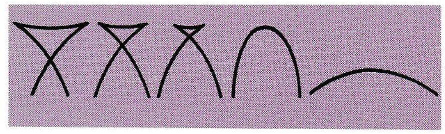


Figure 21

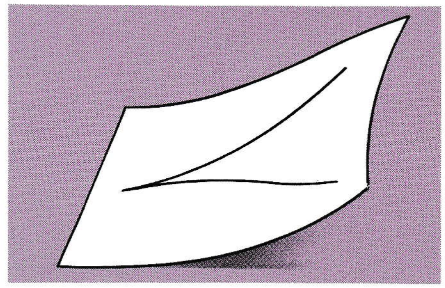


Figure 22

points on the ruling (remember the ruler lying flat on the surface!), and it's obviously not perpendicular to the picture plane at points that don't belong to the hypothetical cuspidal curve. This means that the tangent plane isn't perpendicular to the picture plane at the points on this curve either—it's positioned as shown in figure 15. This undeniably proves that our curve is a cuspidal curve and a visible contour isn't.

We can look at this from another angle. A developable surface (whose rulings aren't parallel and don't all pass through one point) consists of straight lines tangent to one curve—the cuspidal curve. So you can construct a developable surface by taking a spatial curve (with no flat stretches) and drawing all its tangents (fig. 16). These tangents will sweep out a developable surface, and the original curve will be its cuspidal curve. Any noncylindrical or nonconical developable surface can be obtained in this way (Euler's theorem).

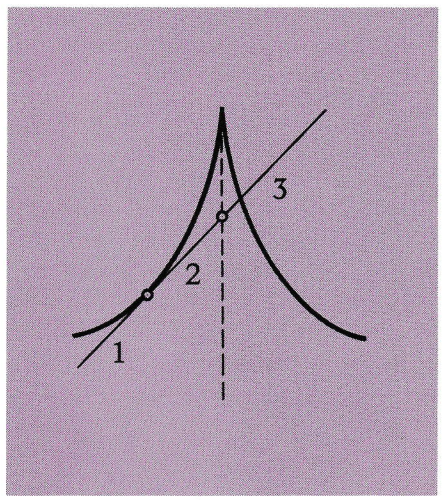


Figure 23

Is that all? No, as you'll soon see. Let's mentally enlarge the developable surface until we're able to walk along it, and let's choose a path perpendicular to the rulings (fig. 17). Since rulings are tangent to the cuspidal curve, we'll either quickly approach or quickly move away from it. What might the transition between the two states consist of? Look at figure 18: a sheet of paper, rulings, and two segments of the cuspidal curve. But what's between the segments? A smooth curve like the dotted line in the figure? No, that would be too incredible—a curve like that can't be tangent to the rulings at every point. So only one possibility is left: *the cuspidal curve itself must have a cusp* (fig. 19).

How is the surface near this inconceivable point structured? Let's start with a picture. The surface itself is shown in figure 20. In addition to a cuspidal curve, it must have a self-intersection line. Figure 21 shows several parallel planes cutting through this surface. To

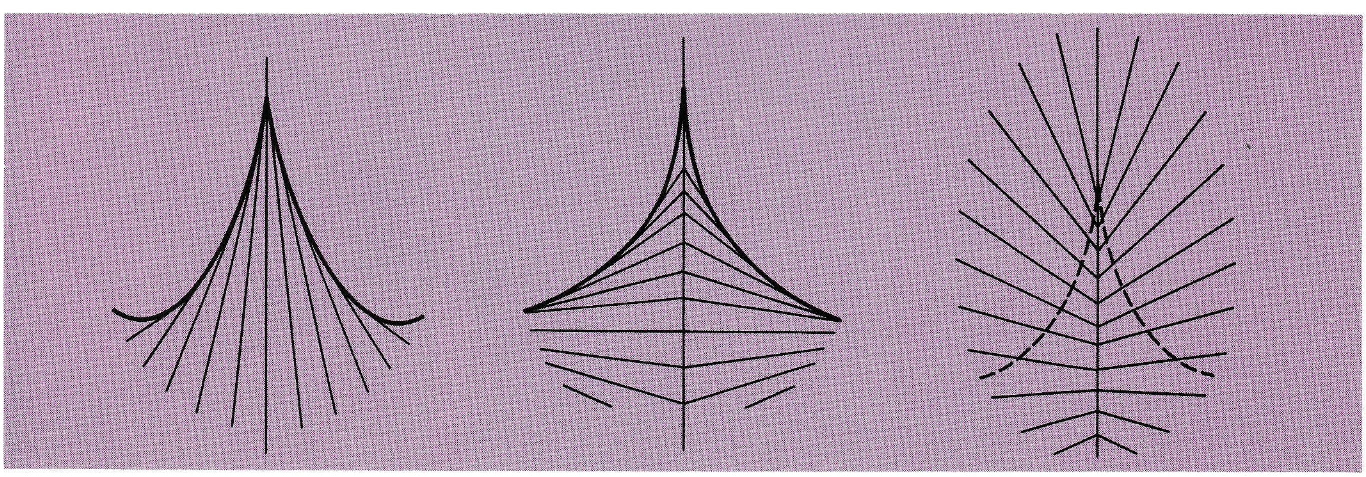


Figure 24

convince ourselves (to some extent, at least) that the surface really does look like that, we'll do what we did with the Euler theorem—we'll take the hypothetical cuspidal curve and draw all its tangents.

To construct a "typical" spatial curve with a cusp, take a planar curve with such a point and bend its plane a bit (fig. 22). Now draw all the tangents to this curve and look at the result from above. Divide each tangent into three parts as shown in figure 23. Now draw the first, second, and third parts of all the tangents separately (fig. 24). In the first picture we get a slightly flexed upper membrane stretched between the two branches of the cuspidal curve. In the second picture a two-piece joint spanning the branches of the cuspidal curve and the self-intersection line will appear. And finally in the third picture we'll get the rest of our surface. (Notice that the figures drawn in the second and third pictures have a "fracture" along the self-intersection line.) The entire surface is called a *swallowtail*—it really looks like one, doesn't it?

So we see that an arbitrarily bent (but not crumpled) sheet of paper, after infinite extension of its linear rulings, turns into a surface with a cuspidal curve that itself has at least one cusp. Near each cusp this surface looks like a swallowtail and has self-intersections.

Would you have imagined that a garden-variety sheet of paper has such interesting geometrical ramifications? Now you know why the swallow is perched at the beginning of this article!

In conclusion I'll add a few words about the swallowtail itself. This surface appears quite frequently in three-dimensional geometry, and many natural problems in calculus and mechanics are related to it. However, the picture (though not the name, which was not coined until the 1960s by the famous French mathematician R. Thom) first appeared on the pages of 19th-century algebra textbooks in the following context.

Consider the equation $x^n + a_1x^{n-2} + a_2x^{n-3} + \ldots + a_{n-1} = 0$. The number of solutions can vary from zero to n. For example, the equation $x^3 + ax + b = 0$ can have 1, 2, or 3 solutions. In order to find the actual number, we have to draw the discriminant curve $4a^3 + 27b^2 = 0$ on a plane with coordinates (a,b) (fig. 25). If the point (a,b) belongs to the interior of the shaded domain, the equation has 3 solutions. If it lies on its boundary (exception for the cusp), the equation has 2 solutions. In the remaining cases it has 1 solution.

An analogous problem for the fourth degree equation $x^4 + ax^2 + bx + c = 0$ leads to a swallowtail, similar to the one in figure 20, located in space with coordinates (a,b,c). If the points (a,b,c) lie

– inside the trihedral "box," the equation has 4 solutions;

– on its boundary, except for the cuspidal curve and the self-intersection line, it has 3 solutions;

– on the lines, except the vertex, it has 2 solutions—the same number as for any point lying above the surface (and not in the box);

– on the whole surface, except for the boundary of the box but including the vertex, it has 1 solution;

– below the surface, there are no solutions.

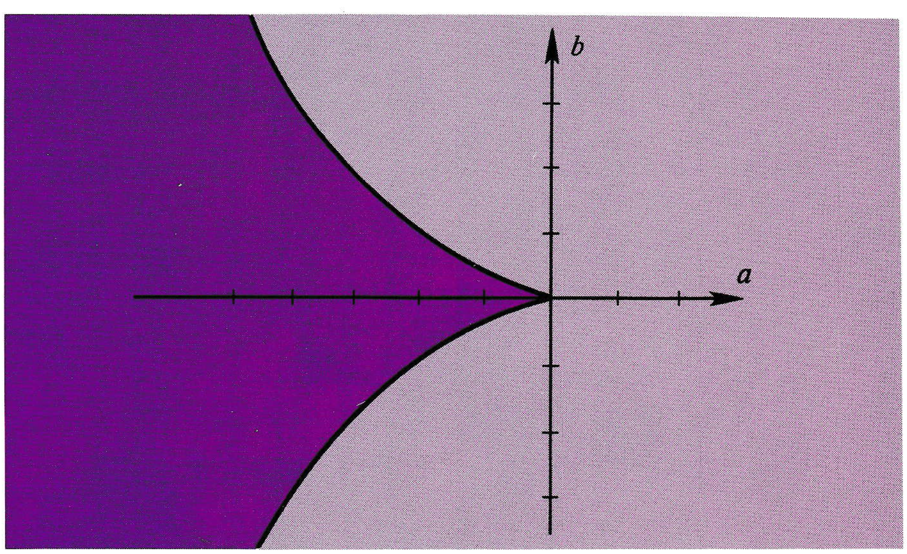


Figure 25

Dmitry Fuchs *is a graduate of Moscow University, where he now works as a leading research fellow in I. M. Gelfand's math and biology laboratory. Fuchs has published a dozen books and over 100 research papers in algebraic topology. He has written many articles for* Kvant *and has been very active in math Olympiads (he is a former winner himself). In his free time Fuchs enjoys poetry and painting.*

+1
Nishan

Pigeons in every pigeonhole

. . . and New Yorkers, and chess pieces . . .

by Alexander Soifer and Edward Lozansky

THE PIGEONHOLE PRINCIPLE, ALSO known as the Dirichlet principle after the famous mathematician Peter Gustav Dirichlet (1805–1859), plays a special role in mathematics as well as in problems at mathematical competitions. The principle is very simple: If $kn + 1$ pigeons (k and n are positive integers) are placed in n pigeonholes, then at least one of the holes contains at least $k + 1$ pigeons. It's very easy to prove this.

Assume that there are no holes that contain at least $k + 1$ pigeons. Then

the 1st hole contains $\leq k$ pigeons
the 2nd hole contains $\leq k$ pigeons
$\vdots$
the nth hole contains $\leq k$ pigeons
the total number of pigeons $\leq k \times n$.

This contradicts the given fact that there are $kn + 1$ pigeons. Therefore, there is a hole that contains at least $k + 1$ pigeons.

This simple principle works wonders. It's amazing how easy it is to understand this idea yet how difficult sometimes it is to discover that this idea can be applied! After all, you, the problem solver, have to create pigeonholes and pigeons. The areas of application of the principle include number theory, combinatorics, and geometry.

Let's take a look at some problems, from easy ones to those that may be not so easy for many of you.

1. New York City has 7,100,000 residents. The maximum number of hairs that can grow on a human head is 500,000. Prove that there are at least 15 residents of New York City with the same number of hairs.

Solution: Let's set up 500,001 pigonholes labeled by integers 0 to 500,000 and put residents of New York into the holes labeled by the number of hairs on their heads. Because 7,100,000 > 14 x 500,001 + 1, we conclude by the pigeonhole principle that there is a pigeonhole with at least 14 + 1 "pigeons"—that is, there are at least 15 residents of New York with the same number of hairs.

2. Given n integers, prove that one of them is a multiple of n, or some of them add up to a multiple of n. (From the Third Annual Colorado Mathematical Olympiad, 1986)

Solution: Denote the given integers by $a_1, a_2, \ldots, a_n$. Define:

$$S_1 = a_1$$
$$S_2 = a_1 + a_2$$
$$\vdots$$
$$S_n = a_1 + a_2 + \ldots + a_n.$$

If one of the numbers $S_1, S_2, \ldots, S_n$ is a multiple of n, we're done. Assume now that none of the numbers $S_1, S_2, \ldots, S_n$ is a multiple of n. Then all possible remainders upon the division of these numbers by n are $1, 2, \ldots, n - 1$—that is, we get more numbers (n, which are our "pigeons") than possible remainders ($n - 1$, which are our "pigeonholes"). Therefore, among the numbers $S_1, S_2, \ldots, S_n$ there exist two numbers—say, S_k and S_{k+t}—that give the same remainders upon division by n.

We are done, because

(1) $S_{k+t} - S_k$ is a multiple of n;
(2) $S_{k+t} - S_k = a_{k+1} + a_{k+2} + \ldots + a_{k+t}$.

In other words, we found some of the given numbers—namely, $a_{k+1}, a_{k+2}, \ldots, a_{k+t}$, whose sum is a multiple of n.

3. Given a real number r, prove that among its first 99 multiples $r, 2r, \ldots, 99r$ there is at least one that differs from an integer by not more than 1/100.

Solution: Let's roll the number line on a roller with a circumference equal to 1 (fig. 1). All integers will coincide on the roller with zero. Now we divide the circumference into 100 arcs of equal length (fig. 2). If at least one of the given multiples kr lies on one of the arcs (99/100,0) or (0,1/100), then we are done—kr differs from an integer by not more than 1/100.

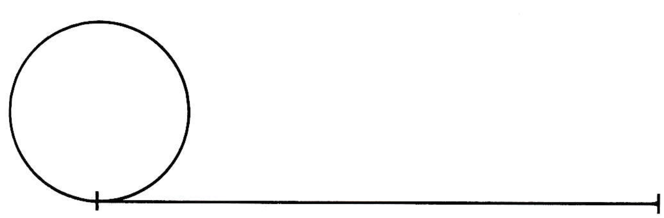


Figure 1

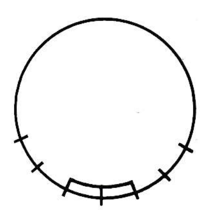


Figure 2

Assume now that none of the multiples kr, $r = 1, 2, \ldots, 99$, lies on the two arcs above. We have 99 pigeons (numbers $r, 2r, \ldots, 99r$) in $100 - 2 = 98$ pigeonholes (the remaining 98 arcs). Therefore, by the pigeonhole principle, at least two of the multiples—say, kr and tr $(k > t)$—lie on the same arc of length 1/100. All there is left to notice is (a) $kr - tr = (k - t)r$ is one of the given 99 multiples; (b) $kr - tr$ lies on one of the arcs (99/100,0) or (0, 1/100), which contradicts our assumption.

4. Given nine points in a triangle (interior plus perimeter) of area 1, prove that three of them form a triangle of area not exceeding 1/4.

Solution: Midlines partition the given triangle into four congruent triangles of area 1/4 (fig. 3). These congruent triangles are our pigeonholes, and the given points are our pigeons. Now nine pigeons are sitting in four pigeonholes. Since $9 = 2 \times 4 + 1$, there is at least one pigeonhole containing at least three pigeons.

If you feel that nine points are excessive to guarantee the result in problem 4, you're quite right. But in order to prove the stronger statement in

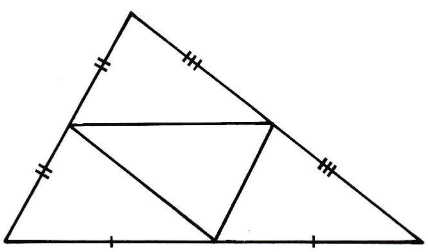


Figure 3

problem 5, we need to allow the pigeonholes to be different in size and shape.

5. Given seven points in a triangle of area 1, prove that three of them form a triangle of area not exceeding 1/4.

Solution: Since $7 = 2 \times 3 + 1$, it would be nice to have three pigeonholes—then at least one of them would have at least three pigeons! Okay, let's draw only two midlines in the given triangle (fig. 4). We get three pigeonholes. At least one of them contains at least three pigeons. If one of the triangles contains three given points, we're done.

If the parallelogram contains three given points, then all we have left to prove is a simple lemma: *The maximum area of a triangle inscribed in a parallelogram of area 1/2 is equal to 1/4.* We leave the proof of this lemma to you.

We can almost hear you asking: "Well, is seven the smallest number of points guaranteeing the result in problems 4 and 5?" The answer is no. The smallest number is . . . But no—*you* try and find it on your own.

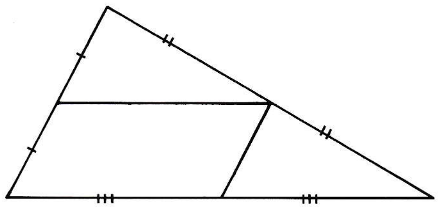


Figure 4

6. All vertices of a convex pentagon lie on the intersections of a grid. Prove that the pentagon (interior plus perimeter) contains at least one more intersection of the grid.

Solution: Let's introduce the coordinate system on the grid (fig. 5). To each vertex M of the pentagon with coordinates (x,y) we assign the ordered pair of the remainders upon division of coordinates (x,y) by 2. There are only four possible outcomes of this operation: (0,0), (0,1), (1,0), and (1,1). These are our pigeonholes. Since we have five pigeons (the vertices of the pentagon), there are two vertices, M_1 and M_2, that give the same pair of remainders. In order to complete the proof, all there is left to notice is that the midpoint of the segment M_1M_2 has integral coordinates and lies on the interior or perimeter of the pentagon.

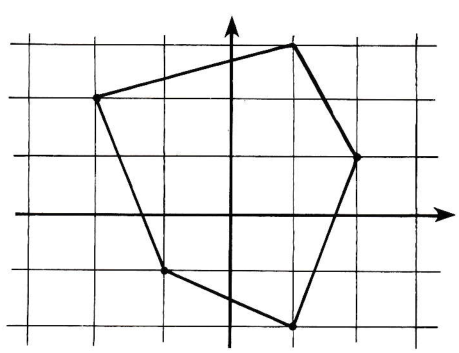


Figure 5

7. Forty-one rooks are placed on a 10 x 10 chessboard. Prove that you can choose five of them that are not attacking each other. (We say that one rook "attacks" another if they are in the same row or column of the chessboard.)

Solution: Let's make a cylinder out of the chessboard by gluing together two opposite sides of the board and color the cylinder diagonally in 10 colors (fig. 6).

Now we have $41 = 4 \times 10 + 1$ pigeons (rooks) in 10 pigeonholes (one-color diagonals). Therefore, there is at least one hole containing at least 5 pigeons. But the 5 rooks located on the same one-color diagonal do not attack each other!

We'd like to thank Boris Dubrov, a student at Minsk High School No. 107 in the USSR, for his valuable contributions. We thank you, our readers, for your interest and wish you happy sailing through the following problems.

Additional problems

1. A three-dimensional space is painted in three colors. Prove that there are two points exactly one mile apart painted in the same color.

continued on page 32

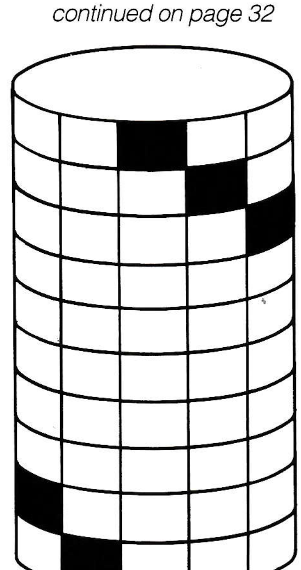
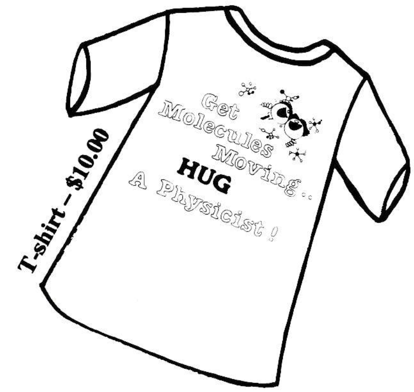


Figure 6

QUANTUM is being launched by the National Science Teachers Association (NSTA) in cooperation with the **American Association of Physics Teachers** (AAPT) and the National Council of Teachers of Mathematics (NCTM). A board of editors, including physicists from AAPT and mathematicians from NCTM, will provide technical review for *Quantum*. Jack Wilson, Executive Officer of the **American Association of Physics Teachers**, and the Association's members and staff want to thank everyone who has worked so hard to make this project a reality. The

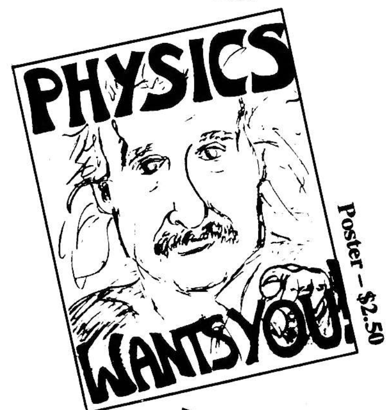
American Association of Physics Teachers

is an organization dedicated to improving physics education, includes student members, and invites you to join! Here is some of what AAPT has to offer to you:

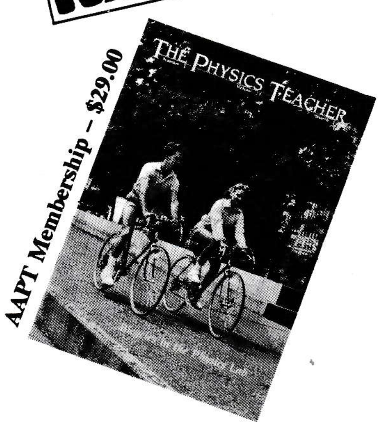


- **The International Physics Olympiad**
  Through tests given at your school, 20 high-school students are chosen to come to the University of Maryland at College Park for intensive training and more testing. Five of those students are selected to attend the Olympiad, which is usually held overseas. In 1989, the United States had its first gold-medal winner, Steven Gubser of Cherry Creek High School in Colorado. Steven was one of 150 students from 30 different countries.

- **Soviet-United States Exchange Program**
  The increasing freedom allowed Soviet citizens was reflected in the student-exchange program that took place in the summer of 1989. Fifteen Soviet students came to the United States to study and learn about our culture, while seventeen American students went to the Soviet Union. These students were chosen by testing and through teacher recommendations.



- **Contests**
  Several contests and awards programs for students are publicized in AAPT's news magazine, the Announcer *(which is sent to all members). One award, the Metrologic High School Physics Contest, is administered by AAPT (sponsored by Metrologic). This year, 30 student winners will receive a laser for their school.*



- **Books, Audiovisuals, Posters, and T-shirts**
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Kith and kin

Friendly numbers and twin primes

Time and again we've all had to factor numbers. Let's look at a number and the sum of all its divisors that are distinct from the number itself. If the sum is greater than the original number, this number is called *excessive*; if less—*insufficient*; and if the two are equal, the number is termed *perfect*. (For example, $6 = 1 + 2 + 3$ is the smallest perfect number. Find another.)

Pythagoras said: "My friend is he who is my second 'I,' like the numbers 220 and 284." These two numbers are remarkable in that the sum of the divisors of each of them is equal to the other number. They were called *friendly*.

It's generally thought that Fermat (1601–1665) discovered the friendly numbers 17,296 and 18,416 in 1636. But recently the following lines were found in a treatise written centuries earlier by the Moroccan scholar Ibn al-Banna: "The numbers 17,296 and 18,416 are friendly .... Allah is omniscient."

Long before Ibn al-Banna another Arabian mathematician, Ibn Kurra (836–901) formu-

Long before Ibn al-Banna another Arabian mathematician, Ibn Kurra (836–901), formulated a method for finding certain friendly numbers: If three numbers $p = (3 \times 2^{n-1}) - 1$, $q = (3 \times 2^{n}) - 1$, and $r = (9 \times 2^{2n-1}) - 1$ are prime, then the numbers $A = 2^{n}pq$ and $B = 2^{n}r$ will be friendly.

With $n = 2$ and $n = 4$, the numbers found by Pythagoras and Ibn al-Banna are produced. With $n = 7$, the numbers 9,363,584 and 9,437,056 are produced. These were found in 1638 by René Descartes (1596–1650), who didn't know Ibn Kurra's theorem. These three instances exhaust the values of $n \leq$ 20,000 with which Ibn Kurra's method will yield friendly numbers.

A =90,236,465,306,233,130,665,155,201,592,687,
078,644,413,045,485,690,038,961,540,360,
536,371,993,258,287,019,185,759,580,345,
274,700,499,275,323,129,070,333,233,826,
784,067,560,738,920,615,666,452,384,945

B =86,259,376,650,143,596,387,690,953,818,787,
166,659,714,840,888,357,774,281,383,581,
683,102,264,665,913,329,533,162,256,868,
364,964,774,727,067,384,973,129,580,885,
368,384,109,913,214,991,276,380,031,055

A and *B* are the largest known friendly numbers.

Many authors after Ibn Kurra studied friendly numbers but failed to discover anything substantial. Their works contain recipes like this: "In order to achieve reciprocal love, write the numbers 220 and 284 down on something, giving the smaller to the object of your love and eating the larger one yourself."

Leonhard Euler (1707–1783) was the first after Descartes to find new friendly numbers. He discovered 59 pairs in all, among them pairs of odd numbers: $3^2 \times 7 \times 13 \times 5 \times 7$ and $3^4 \times 5 \times 11 \times 29 \times 89$, as well as $3^2 \times 7 \times 13 \times 107$ and $3^4 \times 5 \times 11 \times 2{,}699$. He proposed five methods of finding friendly numbers.

At present, about 1,100 pairs of friendly numbers are known. They were found either by ingenious methods or (recently) by simple computer sorting. It's interesting that the computer has added very few numbers to this list—mathematicians had already found most of them.

Ancient Greek mathematicians were very interested in *prime numbers*—numbers having exactly two divisors. Among them they singled out pairs of primes they called number-twins—prime numbers differing by 2 (3 and 5, 5 and 7, 11 and 13, 17 and 19, and so on).

Although it is yet to be proven that there are infinitely many *twin primes*, the number of them in a given stretch $[x; x + a]$ of natural numbers can be estimated by using the formula $N = Ca/(\ln x)^2$, where $C =$ 1.3203236316.... The table at right shows how precise this method is.

Interval	By the formula	In actuality
10^8 to $(10^8 + 1.5 \times 10^5)$	584	601
10^9 to $(10^9 + 1.5 \times 10^5)$	461	466
10^{10} to $(10^{10} + 1.5 \times 10^5)$	374	389
10^{11} to $(10^{11} + 1.5 \times 10^5)$	309	276
10^{12} to $(10^{12} + 1.5 \times 10^5)$	259	276
10^{13} to $(10^{13} + 1.5 \times 10^5)$	211	208
10^{14} to $(10^{14} + 1.5 \times 10^5)$	191	186
10^{15} to $(10^{15} + 1.5 \times 10^5)$	166	161

In his recent book, *Archimedes' Revenge*, Paul Hoffman reports that the Chinese mathematician Chen Jing-run proved in 1966 that there are infinitely many pairs of numbers that differ by two in which the first is a prime and the second is either a prime or the product of two primes. Such a product is said to be almost prime—which "attests," Hoffman writes, "to the irrepressible optimism of mathematicians as well as to the intractibility of bona fide prime numbers." As for friendliness, Hoffman notes that modern mathematicians have extended the notion to sets of three. For example: 103, 340, 640; 123, 228, 768; 124, 015, 008. Here's another friendly triplet: 1,945,330, 728,960; 2,324,196,638,720; 2,615,631,953,920. "But such numbers," Hoffman laments, "do not look friendly to me." They are extremely hard to find and have an atrocious number of divisors—959, 959, and 479, respectively, for the second triplet above.

Holding up under pressure

Bridges of stone, concrete—and paper

by Alexander Borovoy

RIGIDITY IS ONE OF THE MOST important properties of any engineered structure. Obviously, the structure must not change shape from the force of its own weight or under the influence of the external load it must carry.

Actually, it's impossible to avoid deformation altogether, but an engineer must choose the material and the types of components so as to keep deformation within a certain calculated limit. This ever-present problem for engineers is related to requirements to make the structure as simple, inexpensive, and lightweight as possible, using the least amount of materials. In the aircraft industry, for example, the balance between maximal rigidity and minimal mass is vitally important, so engineers try to accommodate these mutually exclusive requirements by finding a solution that is in fact the optimal compromise.

Nature is the greatest of all inventors. Long ago it solved a lot of problems that still draw the attention of engineers, and in the course of natural selection it created such masterpieces as the bird's feather, the bamboo stem, the hollow bones of land animals, and so on. Notice that all these structures have one feature in common—they're all tubular.

Why are structures based on the hollow cylinder so widespread? To find an answer to this question, we'll begin with a very simple experiment.

THIS EXPERIMENT is known in the literature as the "Umov experiment," named after the eminent Russian physicist N. A. Umov (1846–1915). A professor at Moscow University for almost 20 years, his most important papers deal with the problem of energy transfer. In fact, he first introduced the concept of energy flow and energy density.

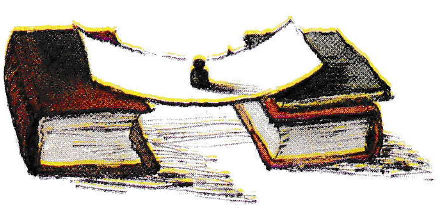
The Umov experiment, otherwise known as "the strength of the tube," can be performed at home with the simplest materials. Cut two rectangular sheets of thick paper, approximately 20 cm long and 8 cm wide. Next, using two piles of books as props (fig. 1), place one of the sheets on them and load it with small weights (if none are available, you can use coins). You see that under a small weight the sheet, which we can visualize as a kind of bridge, will sag quite a bit (fig. 1a).

It turns out, though, that the rigidity of our structure can be enhanced tenfold by a very simple method. Let's roll the second sheet into a cylinder, or tube, and wrap a thread around it so that it doesn't unfurl. You'll see that the tube doesn't bend significantly under the same weight. Only a substantially greater weight makes it deflect appreciably (fig. 1b).

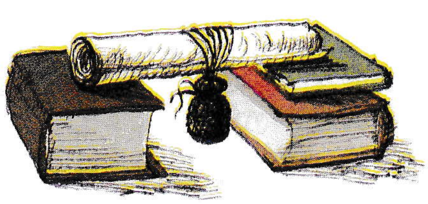
It's worth noting that we have obtained a simple method for testing structures. For example, we may evaluate the rigidity of bird feathers by carrying out an experiment similar to that with the paper cylinder. The author did it and found that a goose feather 10 cm long could withstand the load of a 0.5-kg weight. Next, we may apply the Umov method to so-called "sections" (fig. 2)—angle pieces, T- and I-sections, and corrugated sheets, which we can make of identical sheets of paper—and verify that they all have far greater rigidity than the initial sheet of paper. You may carry out a series of Umov experiments yourself, comparing the magnitude of deflection measured for different loads (or obtaining the same deflection by loading structures with different weights).

NOW WE NEED to understand, at least qualitatively, what determines the structure's rigidity with respect to deflection.

Let's do another experiment. We'll need a rectangular bar of rubber or other elastic material 10 cm long with a cross section of approximately 1–2 cm^2. Draw a grid of longitudinal and transverse straight lines (fig. 3a) and bend the bar. The grid deforms (fig. 3b) so that the transverse lines remain straight but are no longer parallel, while the longitudinal lines bend. It's easy to see that the bar's material is subject to stress on one side and strain on the other. But there's a longitudi-



(a)



(b)

Figure 1

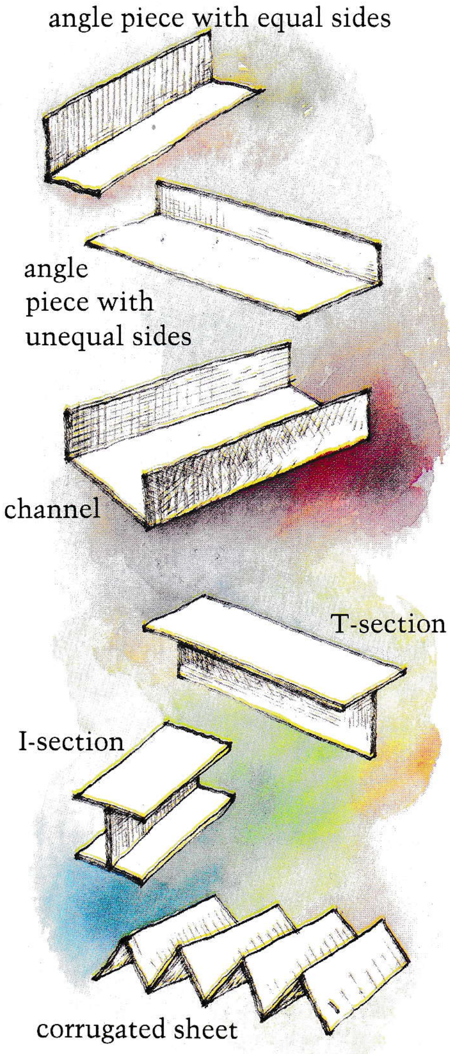


Figure 2

nal line in the middle whose length doesn't change. Obviously the entire layer of material behind this line experiences no deformation either. For this reason it's known as the "neutral layer" (or "neutral surface").

We may infer from the experiment that the farther a region of the bar is from the neutral layer, the greater the stress, or strain, it's subject to. But, according to Hooke's law, the force of elastic resistance becomes greater as the distance from the neutral layer increases, and the main contribution to the bar's rigidity is from the layer far

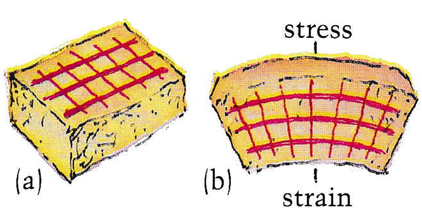


Figure 3

from the neutral one. Consequently, to enhance the structure's rigidity, we must place the main body of its material as far as possible from the neutral layer. This is why the sheet of paper in the Umov experiment can withstand only a very small load while the I-section made of the same material proves to be far more rigid.

But we should not be too fond of separating the main body of material from the neutral layer. For example, if the I-section bar is too thin in the middle, it becomes unstable and twists. On the other hand, if designed properly it is four times lighter than a solid bar with a square cross section that has the same rigidity.

In many situations the hollow cylinder turns out to be the best type of structure because of its axial symmetry—no matter how it is loaded, it behaves in the same way in every direction, and its material is far enough from the neutral layer. Compared to a solid cylindrical bar, the hollow one loses little in resistance to bending and gains a lot in material savings. For example, if inside a solid cylinder of diameter d we make a hole of diameter $d/2$, the bar's rigidity will decrease only by 6–7% and approximately 25% less material will be used in construction.

Now we see why nature uses tubular structures so extensively. When life existed mainly in the oceans, there was no particular need for the skeleton to have a small mass because buoyancy forces helped animals carry their own weight. Some sea species—sharks, for example—have inherited massive cartilage skeletons from their ancestors. But when animals began to crawl on land, the skeleton's strength had to be combined with the lowest possible mass. Over millions of years cartilage tissues evolved into tubular bones, resulting in a strong, lightweight structure that uses material very economically.

In our attempts to build a paper bridge between two piles of books, we learned from experience that the bridge must not be made flat and that shaped structures as in figure 2 are more desirable.

Another way to enhance the bridge's rigidity is to use a structure called a truss. You can easily make a very simple truss on your own (fig. 4). Glue a strip edgewise across the middle of a sheet of paper and connect it with taut threads to the ends of the sheet. (Think of some way to fasten the ends of the threads securely.) Now if you load the bridge, it begins to sag and the threads will be strained even more. Since it's difficult to break the threads, the structure is far more rigid than the flat sheet. Trusses are widely used in building bridges (fig. 5) because they transform bending tension into stress or strain on the bars and beams, whose role in our experiment is played by threads.

We might also mention another structure, the arched bridge (fig. 6), which has been known since ancient times—almost as far back as the 4th millennium B.C. It's based on the idea of transforming vertical loading into lateral compression of the arch, which is transmitted downward to the bridge's foundations.

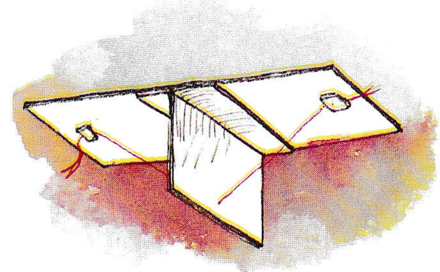


Figure 4

No doubt you've noticed that there are many different kinds of bridges. In addition to solving engineering and economic problems when constructing a bridge, the architect must design a beautiful shape that is in harmony with its surroundings. Perhaps it's a blessing in disguise that there is no general prescription for bridge-building. Indeed, a Russian will immediately recall Leningrad's bridges, which

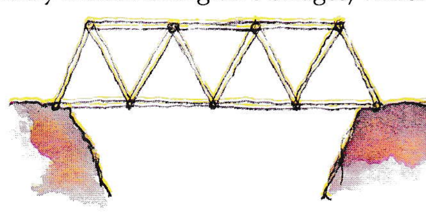


Figure 5

present a magnificent variety of possible solutions, while an American might think of the Golden Gate Bridge, the Mackinac Bridge, or the venerable Brooklyn Bridge.

Finally, it's worth pointing out that many famous inventors and engineers took their first career steps by making models of various machines and structures. "My interest in engineering arose during my early teens," wrote

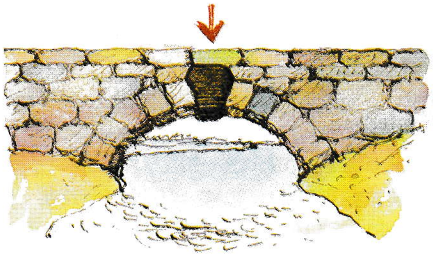
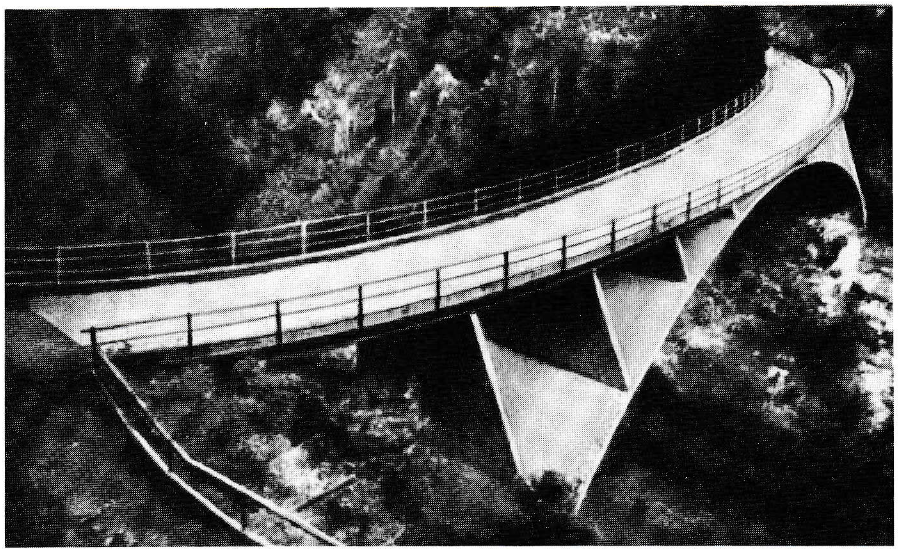


Figure 6

V. N. Obraztsov, an eminent Soviet scientist and transportation engineer. "I enjoyed making models of various structures—not just models but scaled miniature copies."



An arched bridge constructed early in this century by the Swiss engineer Robert Maillart. Photos and blueprints of it can be found in almost any treatise on architecture. The bridge overwhelms the viewer with its bold form and flawless construction.

We can see from our experiments that building a solid and beautiful paper bridge, using whatever "construction materials" come to hand, is not an easy task. If this activity fires your imagination and you make an interesting model, send us a photograph and a description of it—perhaps we can share it with your fellow readers of *Quantum*.

"Pigeonhole" from page 26

2. Given a square of size 1 x 1 and five points inside it, prove that among the given points there are two not more than $2^{1/2}/2$ apart.
3. A number of people (more than one) came to a party. Prove that at least two of them shook equal numbers of hands during the party.
4. Prove that among any 12 distinct two-digit numbers there are two numbers such that their difference D written in the decimal system looks like $\overline{aa}$, where a is a digit.
5. Prove that among any 15 distinct positive integers not exceeding 100 there are four numbers a, b, c, d such that $a + b = c + d$, or there are three numbers a, b, c forming an arithmetic progression.
6. Little grooves of the same width are dug across a long (very long!) straight road. The distance between the centers of any two consecutive grooves is $2^{1/2}$. Prove that no matter how narrow the grooves are, a man walking along the road with a step equal to 1 sooner or later will step into a groove. (We assume that the man's "feet" are so small that his footprints look like dots.) (Problem created by I. M. Gelfand)
7. Grandmaster Lev Alburt plays at least one game of chess a day to keep in shape and not more than 10 games a week to avoid tiring himself out. Prove that if he plays long enough there will be a series of consecutive days during which he will play exactly 23 games. (From the First Annual Southampton Mathematical Olympiad, 1986)
8. An organization consisting of n members ($n > 5$) has $n + 1$ three-member committees, no two of which have identical membership. Prove that there are two committees that have exactly one member in common. (First Annual Southampton Mathematical Olympiad)
9. Prove that for any 99 points located in a square of area 1 there is a circle of radius 1/9 that contains at least three points.
10. Prove that among any 6 points located in a 3 x 4 rectangle there are at least two points not more than $5^{1/2}$ apart.
11. Prove that there exists an integer divisible by 1989 whose last four digits in decimal representation are 1990.
12. Is there a positive integer n such that the last four digits of the decimal representation of 3^n are 0001?
13. Given 51 distinct two-digit numbers, prove that you can choose six of them such that any two of the six numbers have distinct digits in the "ones" place and distinct digits in the "tens" place. (Soifer and Slobodnik)
14. Given $r \times 10^{k-1} + 1$ distinct k-digit numbers, $0 < r < 9$, prove that you can choose $r + 1$ of them such that any two of the $r + 1$ numbers in any decimal location have distinct digits. (Soifer and Slobodnik)

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PROBLEM CORNER

Puzzlers in math and physics

Math

M1
Prove that for every odd number a there exists a natural number b such that $2^b - 1$ is divisible by a.

M2
Several circles are drawn inside a unit square. Prove that if the sum of their circumferences is equal to 10, there exists a straight line that intersects at least four circles.

M3
Four ones and five zeros are written on a circle in arbitrary order. The following operation is performed: a zero is written between any two equal numbers and a one between any two distinct numbers, then the previous numbers are removed. Prove that after any number of such operations you will never obtain nine zeros.

M4
Every side of an equilateral triangle is divided into n equal parts. Lines parallel to the sides of the triangle are drawn through these points, thus dividing the triangle into n^2 smaller triangles. Let's call any sequence of different triangles a chain if every two successive triangles have a common side. What is the greatest possible number of triangles in a chain? (M. Serov)

M5
(a) Let E, F, G lie on the sides AB, BC, CA of triangle ABC and $AE/EB = BF/FC = CG/GA = k$, where $0 < k < 1$. Let K, L, M be the intersection points of the lines AF and CE, BG and AF, CE and BG, respectively. Find the ratio of the areas of triangles KLM and ABC.
(b) Use six lines to cut a triangle into parts such that it is possible to compose seven congruent triangles from them. (A. Soifer)

Physics

P1
Figures 1 and 2 show the boundaries of disturbance regions created by a ship in two stretches of its route. The red arrows indicate the direction of the ship's travel. There is no current in the first stretch (fig. 1). The direction of the current in the second stretch (fig. 2) is indicated by the blue arrow. Determine the current's velocity if the ship's velocity relative to the banks is the same in both cases—18 km/h. (V. Belonuchkin)

P2
You've probably noticed that as soon as you set foot on wet sand, its color becomes lighter. This is due to the fact that the sand becomes dryer. But as soon as you remove your foot, water immediately occupies the footprint. Explain this phenomenon. (L. Aslamazov)

P3
When the humidity r_1 of air is 50%, water poured into a saucer evaporates in the open air in $t_1 = 40$ min. How long will it take for the water to evaporate if $r_2 = 80\%$? (A. Zilberman)

P4
Three uncharged capacitors of capacitance C_1, C_2, C_3 are connected to one another and to points A, B, D at potentials φ_A, φ_B, φ_D (fig. 3). Determine the potential φ_O at point O. (Moscow Physics Olympiad, 1984)

P5
Why does the presence of the ultraviolet component in the spectrum decrease the sharpness of pictures obtained on photographic film? (L. Ashkenazy)

Solutions on page 53

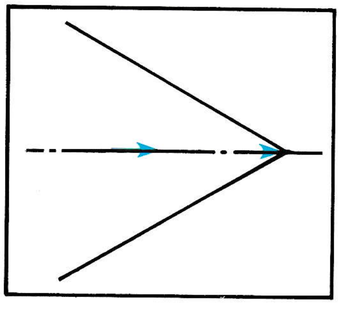


Figure 1

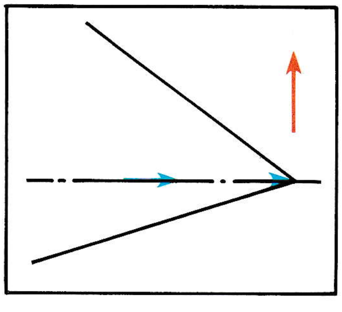


Figure 2

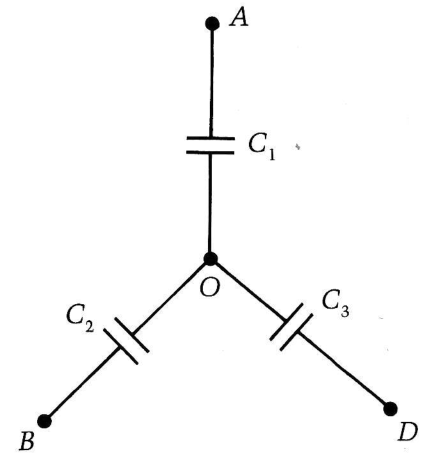
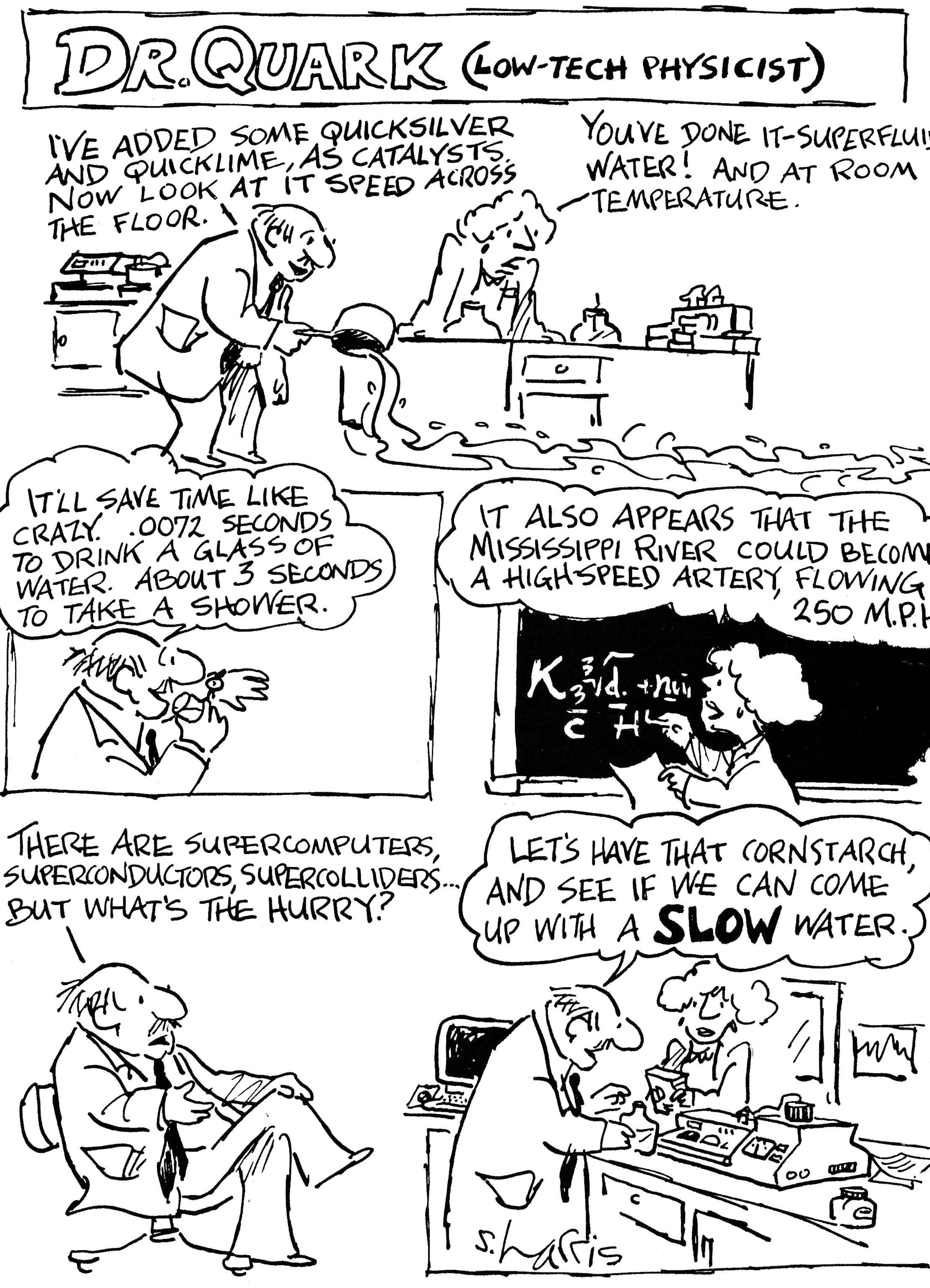


Figure 3


DR. QUARK (LOW-TECH PHYSICIST)
I'VE ADDED SOME QUICKSILVER AND QUICKLIME, AS CATALYSTS. NOW LOOK AT IT SPEED ACROSS THE FLOOR.
YOU'VE DONE IT—SUPERFLUID WATER! AND AT ROOM TEMPERATURE.
IT'LL SAVE TIME LIKE CRAZY. .0072 SECONDS TO DRINK A GLASS OF WATER. ABOUT 3 SECONDS TO TAKE A SHOWER.
IT ALSO APPEARS THAT THE MISSISSIPPI RIVER COULD BECOME A HIGH-SPEED ARTERY, FLOWING 250 M.P.H.
THERE ARE SUPERCOMPUTERS, SUPERCONDUCTORS, SUPERCOLLIDERS... BUT WHAT'S THE HURRY?
LET'S HAVE THAT CORNSTARCH, AND SEE IF WE CAN COME UP WITH A SLOW WATER.
S. Harris

The superfluidity of helium II

A slippery idea that stuck

by Alexander Andreyev

THE DISCOVERY OF LIQUID HELIUM'S special properties was one of the greatest achievements of modern physics. By now these startling and paradoxical properties have been unified within the concept of superfluidity, the phenomenon discovered in 1938 by the Soviet physicist P. L. Kapitsa. Superfluidity doesn't reduce to a simplistic qualitative statement—"liquid helium flows much better than any other liquid." As the Soviet physicist L. D. Landau showed in 1941 in his theory of superfluidity, the properties of superfluid helium are the most striking evidence of general laws that govern the behavior of any substance at very low temperatures. This is why studying liquid helium has had such a profound influence on many diverse fields in physics.

Properties of superfluid helium

Helium is rightly said to be an inert gas. Its atoms interact very weakly with other atoms, and especially among themselves. This is why helium turns into liquid from gas at a record low temperature (4.2°K at normal atmospheric pressure) without becoming solid at still lower temperatures (down to absolute zero). Helium exists in a solid state only at higher pressures (about 25 atmospheres at temperatures close to absolute zero) when the decrease in interatomic distances enhances interaction between its atoms, resulting in solidification.

At temperatures from 4.2°K to 2.2°K, liquid helium behaves in all respects as an ordinary, "normal" liquid. At 2.2°K, helium transforms from the normal liquid state (so-called helium I) into a special state (helium II) possessing the property of superfluidity. In the following pages only a few experiments with helium II—the most important ones—will be described. You'll see that their results utterly contradict our understanding of the concept of ordinary liquid and require new ideas to explain them.

Viscosity and superfluidity

In studying the viscosity, or internal friction, of helium II, we're confronted with the first puzzle of superfluidity. There are two methods of determining the coefficient of viscosity; with ordinary liquids they produce the same result.

The first consists of measuring the liquid's flow through a narrow capillary under the force of gravity. (See figure 1, in which the length of the arrows represents the velocity of the liquid.) Because of intrinsic friction, the flow velocity is different at different points of a cross section of the capillary—it's greatest in the middle and decreases toward the walls, which exert a backward drag on the fluid. Measuring the amount of liquid passing through the capillary per unit of time, we can find the viscosity coefficient.

The second method consists of studying the damping of torsional oscillations of a disk immersed in the liquid. Here we have a physical picture very similar to the previous one. The fluid is practically at rest in regions far from the disk, while the disk exerts a drag on the layer close to it. Different layers of fluid move at different velocities, and internal friction between them transforms the energy of torsional vibrations into heat. We can find the viscosity coefficient by measuring the damping time of the oscillations.

Measurements of viscosity by the first method showed that the viscosity of helium I is appreciable and measurable, but at transition to helium II it suddenly drops to a quantity too small to measure, which in all likelihood is zero. By all appearances there might be a chance of considering helium II a liquid subject to customary laws but with very low viscosity. Measuring the viscosity of helium II with the rotating disk method, however, gives a quantity of the same order of magnitude as that for helium I. Thus, in contrast to the behavior of ordinary liquids, helium II doesn't show any signs of viscosity under some conditions, while under others its viscosity is appreciable.

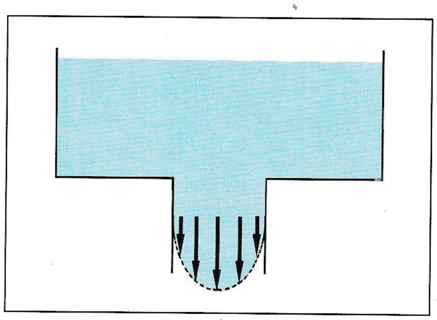


Figure 1

Transfer of Heat and Motion

In normal liquids there are two mechanisms for heat transfer: heat flow and convection.

"Heat flow" means that heat is transferred from one region to another exclusively because of a difference in temperature. To explain this more clearly, let's consider the following experiment. A stationary heater *H* emits energy in the direction given by the arrow (fig. 2), while the liquid remains at rest. To generate heat transfer in this direction by heat flow, the temperature T_1 recorded on the left must be higher than the temperature T_2 recorded on the right. Quantitatively, the rate of heat flow is determined by thermal conductivity, which is the ratio of the heat to the temperature difference. So for the same temperature difference there's greater heat flow from one region of the liquid to another when the thermal conductivity is greater. To put it the other way around, a smaller temperature difference is needed to produce the same heat flow.

"Convection" occurs when heat is transferred by the actual motion of the fluid. Therefore, in the situation discussed above, convective heat transfer may take place at the same thermometer readings T_1 and T_2 if the fluid starts to move from left to right for some reason. Convection is generally associated with large heat transfer. If heat transfer is small enough, we can usually neglect the portion due to convection and determine the thermal conductivity by measuring heat flow and temperature difference.

If we use helium II in this experiment, we'll find that extremely small differences in temperature are sufficient to produce large heat transfer. We reach a twofold conclusion. On the one hand, let's assume that in helium II, as in ordinary liquids, we may neglect convection when heat transfer is small and assume that heat flow plays the major role. We must then assign infinite thermal conductivity to helium II. On the other hand, we may suggest that heat transfer in helium II is always caused by convection and, consequently, temperature differences are absent.

Helium II is both viscous and nonviscous—it depends on how you measure viscosity.

To see which of the hypotheses is in agreement with real life, let's put a petal *P* that can rotate freely about a fixed axis *A* in a bath of helium II (fig. 2). We'll find that the petal always turns in the direction of heat transfer whenever it occurs in helium II. This is a clear indication that motion of the liquid accompanies heat transfer. Therefore, the second hypothesis is verified.

But the situation is far more complicated than it seems at first glance. Let's consider the experiment Kapitsa performed in 1941. A heater *H* is placed in a closed container partially filled with helium II (fig. 3); the container has an outlet to a surrounding bath of helium II. The same petal *P* with axis *A* is placed close to the outlet. If we turn on the heater, heat transfer caused by the liquid's motion should flow from inside the container to the outside. And in fact the petal turns to the right, indicating that helium flows from the container. But the crucial point here is that the liquid's level doesn't drop during the experiment. To all appearances, the liquid keeps flowing but the level stays the same.

This last result is convincing proof that the motion of helium II is subject to laws different from those for ordinary liquids. In particular, the nature of convection in helium II is quite unusual.

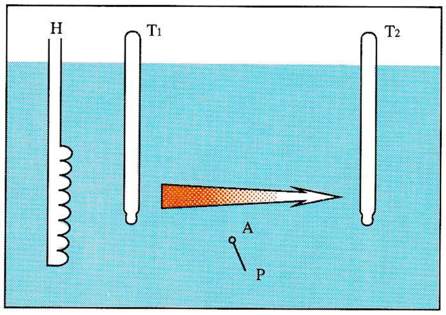


Figure 2

The theory of superfluidity

At room temperature certain solids, liquids, and gases may exist. If we increase the temperature, all liquids and solids turn into gases—that is, systems of single molecules moving freely. If we increase the temperature still further, the thermal motion of the constituent atoms becomes so violent that molecules begin to decompose into separate atoms. At still higher temperatures—of the order of a few tens of thousands of degrees centigrade—atoms decompose into electrons and nuclei. At these high temperatures any substance is a gas consisting of electrons and nuclei.

Superfluid helium II is a liquid that exists only at sufficiently low temperatures (2.2°K and lower). Consequently, to explain its properties we first need to know the general laws governing the changes of thermal motion in any substance when the temperature is lowered.

Elementary excitations

Let's assume the temperature is being lowered, beginning from tens of thousands of degrees centigrade. What happens when electrons and a nucleus unite to form an atom? Before unification, each of the electrons and the nucleus have three degrees of freedom (which means they can move freely in three-dimensional space), while after unification only the atom as a whole can move freely. Thus, the total number of degrees of freedom is diminished. We may say that lowering the temperature brings about a decrease in the possible types of thermal motion. In fact, other types of thermal

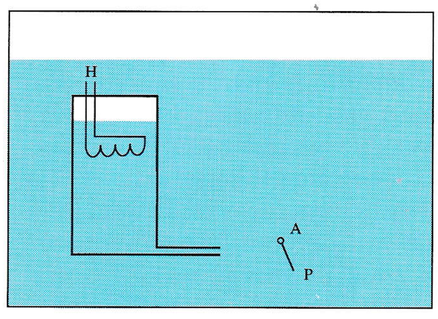


Figure 3

motion disappear, or freeze out, as the temperature drops still further.

Because the motion of electrons relative to nuclei and of atoms relative to the centers of mass of molecules has frozen out at room temperature (or lower temperatures), we may ignore the fact that molecules consist of atoms and assume that thermal motion is merely the motion of molecules considered to be separate particles. It should also be noted that when a gas or liquid becomes solid, its molecules have no means of moving freely in space to any appreciable distance and can only effect small oscillations at certain equilibrium sites.

The process of freezing out various kinds of thermal motion also takes place, of course, below room temperature. Because all kinds of thermal motion must disappear at absolute zero, we may assume that for any substance there is a region of sufficiently low temperatures in which a unique kind of thermal motion—the one most resistant to being frozen out—persists. This kind of thermal motion is called "elementary excitation." Different materials exhibit different kinds of elementary excitation, so the main point in studying a substance at very low temperatures is to determine the nature of its elementary excitations.

The elementary excitations of liquid helium II are not motions of single helium atoms, if only because the concept of thermal molecular motion is the very basis for the theory of normal liquids—which, as we have seen, the properties of helium II obviously contradict. Therefore, the elementary excitations we need are to be found among other kinds of thermal motion. Landau suggested that the elementary excitations of helium II are collective motions of the liquid's atoms—that is, sound vibrations; he demonstrated that a consistent theory of superfluidity can be built on this concept.

There are simple arguments indicating that the hypothesis is in accord with real life. The point is that for ordinary substances the process of freezing out the motion of single molecules is related to the solidification of liquid in that it is accompanied by a transition to a collective, soundlike motion of molecules as a whole.

To see this, let's recall that in solid bodies molecules can perform only small oscillations at certain equilibrium sites, and the oscillations interact among themselves. In fact, oscillations of a molecule are immediately passed on to its neighbors. As a result, vibrations of the whole set of molecules appear—that is, the whole solid body vibrates. These vibrations are sound waves.

Sound waves are the only kind of thermal motion in helium II—all others have been frozen out.

Of course, sound can propagate in liquids, too. In normal liquids, though, it fades out because its energy is transformed into the thermal motion of separate particles. Since we have a liquid (helium II) in which motion of single particles is frozen out, sound inevitably becomes the only kind of thermal motion.

In fact, this process of freezing out must occur at sufficiently low temperatures so that only soundlike, collective motion remains. But temperatures that cause all other substances to become solid don't lead to solidification of helium because there's virtually no interaction among atoms. Consequently, the transition from individual to collective motion in helium occurs in the liquid state.

This property of liquid helium singles it out from all other fluids and, as we'll see later, provides the basis for all the unusual phenomena we discussed earlier. The statement that elementary excitations in helium II are sound waves means the thermal motion in liquid helium at low temperatures is due to the presence of sound waves that propagate in all directions. Consequently, the energy of the sound waves must also increase as the internal energy of the liquid increases with temperature.

Some properties of sound waves in liquids

The first important property of sound waves in liquids is that the propagation of sound is accompanied by a transfer of mass in the same direction. In fact, if we look at the motion of a particle in the liquid, we'll see that a slow translational motion is superimposed on its oscillations and has the same direction of propagation as the sound wave. But the presence of mass transfer in a system of particles means that the system has a nonzero momentum. Hence, a sound wave in a liquid has a momentum in the direction of its propagation. This can be verified by a number of experiments. For example, when a sound wave hits a wall, momentum is an exchanged. The force acting on the wall is called "sound pressure."

Sound waves that determine thermal motion usually travel in various directions and don't generate any transfer of mass. In contrast, if sound waves under some conditions acquire a privileged direction of propagation, mass transfer does occur.

It should be noted here that in order to emit sound, a body moving in a liquid at subsonic speed must vibrate. If, however, the body is traveling at supersonic speed, it can generate sound waves (like a jet airplane breaking the sound barrier) even though it doesn't vibrate. We can also turn the picture around: assuming the liquid moves and the body is at rest, we must infer that there is no sound emission if the liquid's velocity is subsonic.

An explanation of helium II's properties

Helium II's superfluidity arises directly from the properties of sound waves in liquids. In fact, the friction from the flow of a normal liquid through a capillary causes the kinetic energy of the liquid to turn into heat because of the interaction of particles of the fluid with the roughness of the walls. Since the thermal motion in helium II is due to sound waves, energy transfer would cause the emission of sound. But, as we have seen, this is impossible at low speeds of flow.

continued on page 40

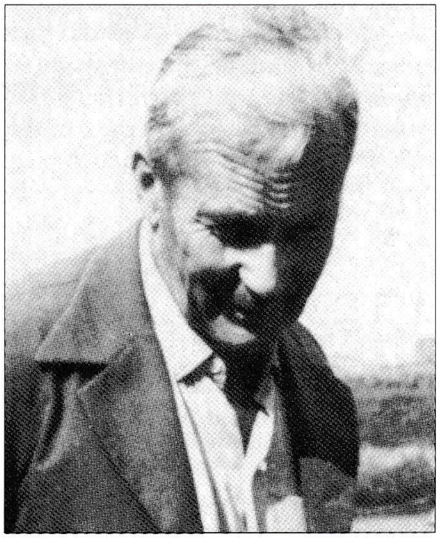
A.N. Kolmogorov

A man of many hats

Andrey Nikolayevich Kolmogorov was one of the greatest scientists in Russian history. His work in probability theory, turbulence, and dynamic systems was fundamental and is now considered classic. The range of his contributions was enormous—from poetics to stratigraphy, from genetics to celestial mechanics, from topology to mathematical logic and algorithmic complexity theory.

Kolmogorov was born on April 25, 1903, in the central Russian city of Tambov. At 17 he graduated from the secondary school there and entered the University of Moscow. Early on he showed a keen interest in Russian history. His first work was a scientific paper on the registration of real estate in the medieval Novgorod republic. But when he found out that history professors required at least five different proofs of every assertion, he switched to mathematics, where one proof suffices! At this time Kolmogorov found himself attracted to the ancient Russian arts as well, and he retained this interest for the rest of his long life.

At the age of 19 Kolmogorov constructed an integrable function with a Fourier series divergent almost everywhere. This unexpected result created a tremendous sensation and made Kolmogorov an internationally recognized mathematician overnight.



At that time, mathematics graduate students at Moscow University had to pass 14 examinations in various mathematical subjects, but it was possible to substitute an original article on a relevant topic in place of the exam. Kolmogorov never took any of the examinations, choosing instead to write the kind of papers he would make his life's work. Even at the outset of his career, his articles contained new results in function theory, set theory, topology, mathematical logic, probability theory, and other topics.

In May 1934, a little before James Alexander came up with the same idea, Kolmogorov introduced the cohomology ring, one of the most important topological invariants of a space. The idea came to him from physics. He generalized such notions as the distributions of charges and currents in space, on surfaces, and on lines, considering the similar "functions of sets" for a more abstract mathematical situation.

Though educated in abstract, set-theoretical mathematics, Kolmogorov was always interested in the natural sciences and other applications, in which he would put aside the shackles of mathematical rigor to obtain a concrete result. But after guessing a result, he invariably tried to formulate it rigorously as a mathematical theorem or conjecture whose proof might be deduced from the fundamental postulates of the theory.

Kolmogorov's work in 1941 on turbulent motions changed the face of the theory of turbulence. Here he introduced the ideas of self-similarity and scaling, leading to the famous Kolmogorov law of 2/3. These ideas, and the modern developments they spawned, are now crucial elements of statistical physics and field theory.

What did Kolmogorov consider his most difficult achievement? His work from 1955 to 1957 on the 13th Hilbert problem, which involved the representation of continuous functions of many variables as the superposition of continuous single-variable functions and on the summation operation.

Kolmogorov's last work before retiring from active research was dedicated to applying the ideas of information theory to the theory of algorithmic complexity and to the foundations of probability theory. He proved, for instance, that any "computer" containing N elements of fixed diameter related to no more than k other elements by "wires" of fixed thickness may be packed in a cube with an edge of approximately $\sqrt{N}$. He had guessed this result by starting from the observation that the gray substance of the brain (the neurons) forms its surface, while the white substance (the junctions) is inside.

In addition to his many mathematical theories, Kolmogorov expounded a theory of a more human sort: that it is impossible to do good mathematical research after the age of 60. And so, after half a century of original and often pathfinding work, he became a high school teacher. This was his main occupation for the last 20 years of his life. He was also appointed chairman of the Commission for Mathematical Education in the Academy of Sciences of the USSR and in that position instituted new programs to more fully develop the scientific interests of schoolchildren.

In 1970, together with I. K. Kikoyin, Kolmogorov created a new magazine for Soviet youth—*Kvant*. He wrote articles for it and remained active in managing it right up until his death in 1987.

Kolmogorov once told a student of his that he thought of humanity as individual flames wandering in a fog, each only vaguely aware of the light given off by the others. A lifetime spent enlightening his fellow human beings belied this somber worldview.

A. N. Kolmogorov stood out among the great mathematicians of the 20th century in that he revolutionized both mathematics and physics, much as Newton had done two centuries earlier. His mind roamed freely in many fields and tirelessly sought connections. A brilliant guesser and a hard worker, Kolmogorov was a mentor to students and younger colleagues. And even in his retirement Kolmogorov nurtured yet another generation—your fellow readers of *Quantum* in the Soviet Union.

—*V. I. Arnold,* Physics Today *(abridged and adapted)*

Sally Ride

Flying high on solid ground

When the space shuttle Atlantis roared into the clear Florida sky in October, the center of attention was the Jupiter-bound Galileo space probe. Environmentalists had expressed concern about the probe's nuclear reactor and the risk of serious atmospheric contamination if an accident occurred during takeoff or when the probe zipped past the Earth after its boomerang trip around the sun. Many scientists, on the other hand, were eager to see the long-delayed probe sent on its way toward the giant planet, about which Voyager had transmitted such tantalizing information.

Once Galileo was released, the five-member crew could turn to its many other tasks—measuring the amount and height distribution of ozone in the Earth's atmosphere, aiming a 70-millimeter camera at various terrestrial targets, studying the causes of space motion sickness (which afflicts nearly half of all shuttle passengers), and so on.

Dr. Ellen S. Baker, a physician, conducted the motion sickness tests. Her trip into space was accompanied by no fanfare. She was accompanied instead by another woman—Dr. Shannon W. Lucid, a biochemist. Three men and two women in a space vehicle—no big deal.

But when Sally Ride became the first American woman in space six years earlier, it *was* a big deal. True, the Soviet Union had been sending women into space for years. Maybe that's why the anticipation had been so great—the US space program was finally "catching up," as the vocabulary of competition has it.

Because of the fame she instantly achieved on June 18, 1983, simply by being in orbit, Sally Ride risked being pigeonholed as "America's first woman in space," as if "woman in space" is a career choice. The fact is, Ride was a physicist before she was an astronaut and while she was an astronaut. She is a physicist today.

Sally K. Ride was born on May 26, 1951, in Encino, California. She attended Stanford University, earning a B.S. in Physics and a B.A. in English in 1973; an M.S. in 1975; and a Ph.D. in 1978. Her research in physics has focused on free-electron lasers. Seeing that English degree tucked away in that list, one is reminded of Werner Heisenberg's remark that "even for the physicist the description in plain language will be a criterion of the degree of understanding that has been reached."

Ride began astronaut training in 1978. At this time she was also a part-time adjunct professor of space science at Rice University in Houston. As part of her preparation, she served as the capsule communicator at Mission Control for the first two shuttle missions.

In addition to her flight in 1983, Ride took part in a shuttle mission in October 1984. On both flights she was in charge of the scientific experiments aboard. During her first flight, the crew deployed and retrieved a satellite with the shuttle's robot arm for the first time and conducted materials and pharmaceutical research. Her second flight lasted eight days, during which the crew deployed a satellite, conducted scientific observations of the earth, and demonstrated the potential for satellite refuelling by astronauts.

In January 1986 Ride's training for a third shuttle flight was interrupted by the Challenger explosion. For the next six months, Ride served on the Presidential Commission investigating the accident. She was then assigned to NASA headquarters in Washington, DC, where she created the Office of Exploration and produced a report on the future of the US space program.

As if taking Heisenberg's injunction to heart, Dr. Ride wrote a book for children, *To Space and Back*, describing her experiences as a shuttle astronaut in simple yet evocative language.

In 1987 Ride left NASA to become

a Science Fellow at the Stanford University Center for International Security and Arms Control, where she worked as a physicist. In July 1989 she was appointed director of the California Space Institute and professor of physics at the University of California, San Diego. As director of Cal Space, Ride will oversee a $3.3 million research institute headquartered at the university's Scripps Institution of Oceanography.

Now that women routinely ride the space shuttle to work, will we see more women running physics laboratories and teaching mathematics on *terra firma*? That depends on those who open doors—and the women who would walk through them. Not so long ago, Albert Stasenko's article in this issue of *Quantum* would have ended with the suggestion that works of art be used "when discussing the laws of physics with your little brother"—period. The achievements of Sally Ride demonstrate that "little sisters," too, are capable of reaching great heights in science and math.

—*Timothy Weber*

"Helium II" from page 37

Let's take a closer look at helium II flowing through a capillary. At any temperature other than zero, there are sound waves in the liquid because of thermal motion. To study their interaction with the walls, let's imagine a coordinate system moving with the liquid so that we can consider the liquid to be at rest while the walls move in the opposite direction. Superfluidity means that the walls don't exert a drag on the fluid. Nonetheless, they interact with the sound waves. As a result, there is an exchange of momentum between them and a privileged direction for the propagation of the sound. Consequently, there is transfer of mass in the liquid because of the drag on the liquid by the walls of the capillary, although the mass of the liquid involved in the motion is far less than the total mass of the liquid because at low temperatures the energy of sound waves is very small.

We may visualize helium II as though it consists of two components that can move independently of each other. The motion of one of them isn't accompanied by friction; therefore, it's called the superfluid component. The other, called the normal component, exerts a drag on the walls and has internal friction like that of normal liquids. The sum of the components' masses is equal to the total mass of the liquid.

Of course, separating helium II into two components is only a manner of speaking. As we have seen, there are two kinds of *motion* in helium II, each accompanied by its own mass transfer; the sum of the masses is equal to the total mass of the liquid. One motion is related to the propagation of sound waves and is accompanied by friction; the other is superfluid motion. *All* helium atoms participate in *both* motions—they aren't divided into "superfluid" and "normal" atoms.

The mass of the normal component increases as the energy of sound waves increases at higher temperatures, so that at a certain point it equals the total mass of the liquid. Then superfluid motion disappears because there's no more mass to be transferred, and helium II turns into helium I, which behaves as a normal liquid capable only of normal motion.

If a body moves at subsonic speed in helium II, the resulting superfluid motion doesn't resist it. In fact, if the force of resistance isn't equal to zero, we need to use a force that performs some work to move the body. The work can only turn into heat—that is, can only result in emitting sound. This, as we know, is impossible. Therefore, we may say there is no pressure exerted on the body by the superfluid component flowing around it. On the contrary, sound waves falling on the body's surface exchange momentum with it (as explained above in the example of capillary flow). So there is pressure on the body from the normal component.

Now it's not so hard to explain the helium II experients described earlier.

If we measure viscosity by the first method (capillary flow), we don't find any viscosity in helium II because the superfluid component flows out very rapidly through the leak. It doesn't matter at all that its density is somewhat less than the overall density because any normal, viscous fluid will flow through a narrow enough leak far more slowly. Measuring the viscosity by the rotating disk method gives a value different from zero because the disk moves in a fluid that has two components; the oscillations are damped through interaction with the normal component.

We may say that the superfluid component shows itself in the experiment with the capillary flow and the normal component in the experiment with the rotating disk.

In helium II, the process of emitting sound waves releases heat, which is transferred in a certain direction because the energy of the sound waves in that direction is greater than in the opposite direction. Therefore, the direction of heat transfer is at the same time the direction of privileged propagation of sound waves and the related mass transfer of the normal component. Thus, heat transfer in helium II is always accompanied by a convective motion of the normal component. This is why in the experiment shown in figure 2 the thermometer readings are always the same yet the petal always turns in the direction of heat transfer—the normal component is flowing around it.

In the case of heat transfer in a container filled with helium II (fig. 3), there is no mass transfer in one direction because the pressure difference due to heat transfer generates a flow of the superfluid component in the opposite direction. The velocity of superfluid motion is such that total mass transfer is absent and the liquid's level doesn't change. The deviation of the petal indicates that the normal component flows out of the inner region, but the superfluid flow remains concealed because it doesn't result in pressure on bodies. ◘

Alexander Andreyev *is an academician and deputy director of the Vavilov Institute of Physics Problems. He is a theoretical physicist who works primarily in the field of solid-body and low-temperature physics.*

Just for the fun of it

These problems were proposed by Alexander Korshkov (10th grade), A. P. Savin, A. M. Domashenko, A. A. Panov, and B. B. Proizvolov.

B1
It's easy to show that the sum of the five acute angles of a regular star (like the ones in the American flag or the one in the Soviet flag) is 180°. Prove that the sum of the five angles of an irregular star (fig. 1) is also 180°.

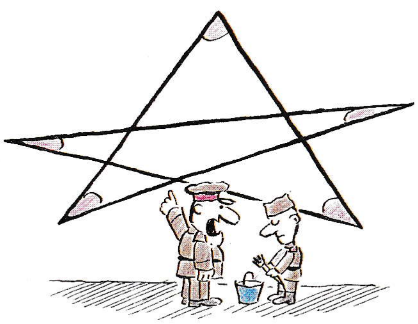


Figure 1

B2
Using each of the numbers 1, 2, 3, and 4 twice, I succeeded in writing out an eight-digit number in which there is one digit between the ones, two digits between the twos, three digits between the threes, and four digits between the fours. What was the number?

B3
Write the numbers 1 to 8 in the eight circles shown in figure 2 so that any two numbers inside circles joined by a line differ by no less than 2.

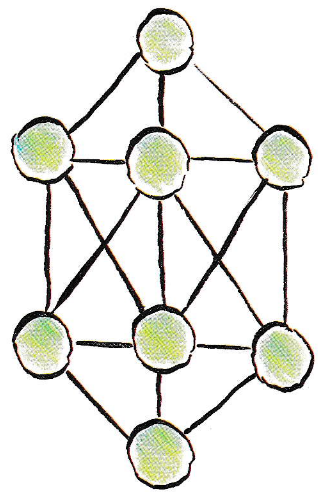


Figure 2

B4
My grandfather's clock behaves in a strange way. During the first half hour of every hour it's 2 minutes fast, but during the second half hour it's 2 minutes slow. How can that be explained?

B5
An arbitrary point inside an equilateral triangle is joined to the three vertices and perpendicular lines are dropped down to the three sides (figure 3). Show that the sum of the areas of the three red triangles equals that of the three blue ones.

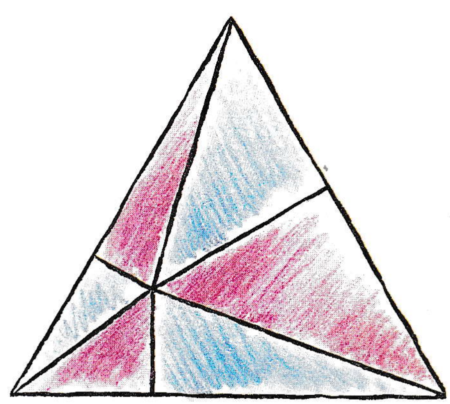


Figure 3

SOLUTIONS ON PAGE 53

Van Rooman's challenge

and Viète's triumph

by Yury Solovyov

IN THE FIRST DAYS OF October 1594, the king of France strolled along the beautiful lanes of Fontainebleau Park with the envoy from the Republic of the United Netherlands (better known as Holland—the name of its largest province). Having come into being as the result of a long and persistent struggle against Spanish dominion, the republic was very young—it was only in its twentieth year. The war with Spain was still continuing, and the Dutch government was determined to find new allies.

In France the flames of the long religious civil war had recently died out, and Henry of Navarre, overcoming furious resistance, had just become king of France. Henry the Fourth did not hide his interest in Holland as an ally in the struggle with Spain, but above all he was interested in the rapid growth of Dutch trade and seafaring. For this reason, walking in Fontainebleau Park, he listened very attentively to the envoy's story—about new silk manufactures in Rotterdam, paper mills in Utrecht, and shipyards in Zandam.

"There are a lot of talented engineers and scientists in Holland," the envoy recounted. "The mathematician and engineer Simon Stevin is working out a new system of locks and dams, new fortresses are being constructed according to the projects of the mathematician Ludolph Van Ceulen, while our mathematician Adriaen Van Rooman has become famous for his puzzling calculations. By the way," the envoy continued, "not so long ago, Van Rooman challenged all the mathematicians of the world. He sent a letter to many countries defying anyone to solve a problem of his own invention. But so far no one has succeeded."

"The winner will certainly be a Frenchman," the king laughed.

"Your Majesty," remarked the envoy, "I have this letter with me, but apparently France doesn't have any outstanding mathematicians, because Van Rooman didn't mention a single Frenchman among those to whom he addresses his challenge."

"But nevertheless, I've got such a mathematician, and a rather extraordinary one," Henry the Fourth answered. "Call Viète!"

That was how, on this fine autumn day in 1594, the destinies of two very dissimilar men crossed.

Adriaen Van Rooman was born in 1561 in the city of Leuven in the Spanish Netherlands (now Belgium). He studied medicine and mathematics at Leuven University, where he obtained his doctor's degree. He was a lecturer in mathematics at Leiden and Würzburg universities.

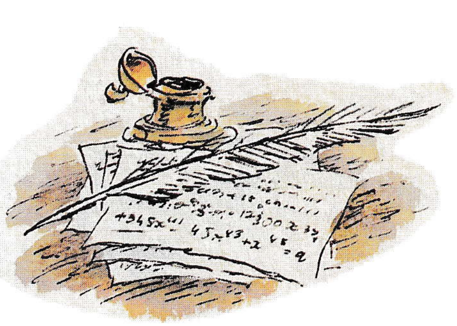
Van Rooman studied geometry and trigonometry and also dealt with practical astronomy and navigation problems. He worked on the problem of expanding the functions $\sin nx$ and $\cos nx$ in powers of $\sin x$ and $\cos x$, and met with some success. He determined the numerical value of π to seventeen decimal places, which was the greatest precision achieved at the time in Europe. During his lifetime, Van Rooman was very famous in Holland

and Germany, but as time went by his works lost their significance. Nowadays his name can only be found in the largest encyclopedias.

François Viète (or Franciscus Vieta in Latin) was born in the French town of Fontaine in 1540. He began studying law in 1559 but was drawn to mathematics and astronomy. In 1571 he moved to Paris, where he continued his career as a lawyer and became acquainted with Parisian mathematicians. In 1573 Viète started working as a counsellor of Brittany's parliament and later became Royal Privy Counsellor to Henry III. In 1580 he obtained the post of Royal Reporter on Requests.

During the last years of Henry's rule, Viète was a cipher clerk, studying the correspondence between the Spaniards and the king's enemies. He discovered the key to the difficult Spanish cipher used by Philip II, king of Spain. When Philip found out from intercepted French letters that his secret information was being read by the French, he complained to the pope, pointing out angrily that the French were using sorcery and black magic against him. After Henry III was murdered in August 1589, Viète offered his services to Henry of Navarre.

The author of numerous papers on algebra, geometry, trigonometry, and astronomy, Viète discovered the relationship between positive roots and coefficients of algebraic equations, still known as the Viète formulas. He found the formula for expanding the functions sin *nx* and cos *nx* in powers of sin *x* and cos *x*, and he is the author of a contemporary system of algebraic notation. His works, written in complicated language, were incomprehensible to his contemporaries, and only half a century after his death did they begin to influence the development of algebra and geometry.



But let's return to Fontainebleau. When Viète appeared, the envoy took out Van Rooman's letter. The letter proposed solving the following equation:

$$45x - 3795x^3 + 95634x^5 - 1138500x^7 + 7811375x^9 - 34512075x^{11} + 105306075x^{13} - 282676280x^{15} + 384942375x^{17} - 488494125x^{19} + 483841800x^{21} - 378658800x^{23} + 236030652x^{25} - 117679100x^{27} + 46955700x^{29} - 14945040x^{31} + 3764565x^{33} - 740259x^{35} + 111150x^{37} - 12300x^{39} + 945x^{41} - 45x^{43} + x^{45} = a,$$

in particular when

$$a = \sqrt{1\frac{3}{4} - \sqrt{\frac{5}{16} - \sqrt{1\frac{7}{8} - \sqrt{\frac{45}{64}}}}}.$$

To simplify the problem, Van Rooman gave the answers for two other values of *a*, expressed in a rather cumbersome way.

Viète read the letter and immediately wrote down the answer. The envoy said there was a sealed envelope with Van Rooman's answer at his residence and that he would open it in the presence of a notary to see if Viète was right. The next day the Dutchman confirmed the correctness of Viète's answer, and Viète, in turn, presented 22 other answers, unknown to Van Rooman. In addition, Viète pointed out a mistake in the statement of the problem, made by the copyist or by Van Rooman himself.

Let's try to reconstruct how Viète found the solution for such a monstrous (at first sight) equation.

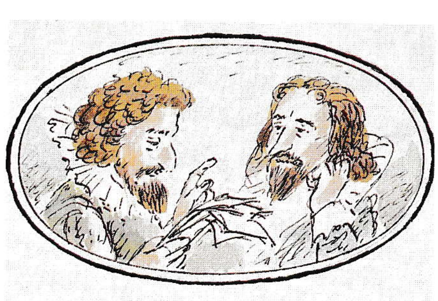
To this end, we must analyze some of his mathematical papers. His main trigonometrical results were formulas for the sines and cosines of multiple arcs. Viète obtained them in the form of a rule for mechanically computing them. It resembles the standard rule except that, instead of Pascal's triangle, Viète used the following table:

1	1	1	1	1	1	...	
1	2	3	4	5	6	...	
1	3	6	10	15	21	...	
1	4	10	20	35	56	...	
1	5	15	35	70	126	...	
1	6	21	56	126	252	...	
1	7	28	84	210	462	...	
.	.	.	.	.	.	...	
.	.	.	.	.	.	...	
.	.	.	.	.	.	...	

Every number here is the sum of the number to its left and the number above it. It's worth mentioning that Viète didn't express sin *nx* and cos *nx* in terms of sin *x* and cos *x*, as we do, but expressed 2 sin *nx* and 2 cos *nx* in terms of 2 sin *x* and 2 cos *x*. If we assume that the values of 2 sin *nx* and 2 cos *nx*, expressed in that way, are known, we get an equation of the *n*th degree for the unknown quantities 2 sin *x* and 2 cos *x*.

Viète's original aim was to find the formulas for expressing the sines of multiple arcs in terms of the sines of the small arcs—that is, by constructing tables of sines. Later these formulas were used in algebra and geometry. In particular, to solve the geometrical problem of trisecting angle α, Viète used the equation $3x - x^3 = a$, which is satisfied by the values $a = 2\sin\alpha$; $x = 2\sin(\alpha/3)$. Viète interpreted the positive solutions as the chords corresponding to the arcs $2\alpha/3$ and $(360° - 2\alpha)/3$. He didn't take the negative roots into consideration at all, which was accepted practice at the time. Likewise, to divide an angle into 5 equal parts, Viète considered the equation $5x - 5x^3 + x^5 = a$, which is satisfied by the values $a = 2\sin\alpha$, $x = 2\sin(\alpha/5)$.

Now it's clear how Viète succeeded in solving Van Rooman's problem so quickly. He saw at once that the proposed value of *a* is the length of the side of a regular polygon of fifteen



sides inscribed in the unit circle (check that!) or, which is the same, the chord corresponding to the arc 24°. Coefficients for the first and subsequent terms of the left side of Van Rooman's equation suggested that the left side is nothing more than the expression of 2 sin 45α in terms of 2 sinα. Since a = 2 sin 12°, then α = 12°/45 = 4°/15, which means that x = 2 sin (4°/15). It is this specific solution that Viète gave to the Dutch envoy.

After the royal audience, Viète checked his supposition. Unfortunately, after the necessary calculations, he discovered that the left side of the given equation did *not* coincide with the expansion of 2 sin 45 α in powers of 2 sin α, contrary to what he'd expected after glancing at Van Rooman's challenge!

At that moment he probably didn't feel too good. Most likely, he felt just awful. What had happened? Maybe there was a mistake in his exhaustive calculations? Apparently at that moment Viète found a totally different—geometric—approach to the expression of 2 sin 45 α in terms of 2 sin α: in order to divide the arc into 45 parts, it's first necessary to divide it into five parts, then every part into three, and each of those into three parts again. In short, the left side of Van Rooman's equation can be obtained from the system

$$3z - z^3 = a,$$
$$3y - y^3 = z,$$
$$5x - 5x^3 + x^5 = y.$$

Only after analyzing Van Rooman's answers for the two other values of a was Viète sure that it was a question of dividing the arc into 45 parts, and he corrected the mistake in the statement of the problem without any doubts. But Viète didn't limit himself to finding one solution. The 22 other solutions, which he announced the next day, took the following form:

$$2\sin\frac{360°k + 12°}{45} = 2\sin\frac{120°k + 4°}{15},$$

$k = 1, 2, \ldots, 22.$

So Viète succeeded in finding all the positive roots (remember, only positive roots were regarded as solutions in his time).

We could end here, but perhaps we should mention that the mathematical competition between Viète and Van Rooman continued. After a while, Viète proposed the following problem to Van Rooman: With compass and ruler, construct a circle tangent to three given circles (Apollonius's problem). Viète himself soon came up with a beautiful geometric solution.

It's said that, after his second defeat, Van Rooman became a zealous admirer of Viète and even came to Paris to study under him. ◙

Of amoebas and men

A true tale of topology gone awry

by Alexey Sosinsky

This is a true story. It happened a long time ago in Moscow at the International Congress of Mathematicians. August 1966—I was still a postgraduate student back then.

After one of the ICM workdays, a group of mathematicians, mostly topologists, gathered for a very informal discussion. Besides the host and other well-known Soviet mathematicians, the group included the outstanding British algebraic topologist J. F. Adams; his fellow countryman E. C. Zeeman, an equally famous geometric topologist; and Colin Rourke, a postgraduate student of Zeeman's.

The conversation turned to one of the ageless topics in mathematical discussions—the comparative merits of geometry and algebra. The enthusiastic geometer Zeeman attacked the rather aloof and phlegmatic Adams, accusing him (and thereby all algebraists) of a complete lack of imagination and practical ineptitude.

"With all your sophisticated algebraic invariants, you're totally incapable of solving the simplest topological problems," Zeeman was saying. "For instance, this one."

Zeeman joined the index fingers and thumbs of his left and right hands, forming interlocking rings (fig. 1).

"Is it possible to unlock these rings without moving a thumb or index finger apart?" he asked. "Can I change from that position to this one"—he unlinked his two hands and dramatically moved his arms sideways and upwards, joining the fingers of each hand again but with his hands at a distance from each other—"if my thumbs and index fingers are glued to each other?

"Of course," he continued, "this is a topological problem, which means that my body can change its shape at will, without any tearing, cutting, or gluing—like an amoeba."

Adams didn't hurry to answer. The

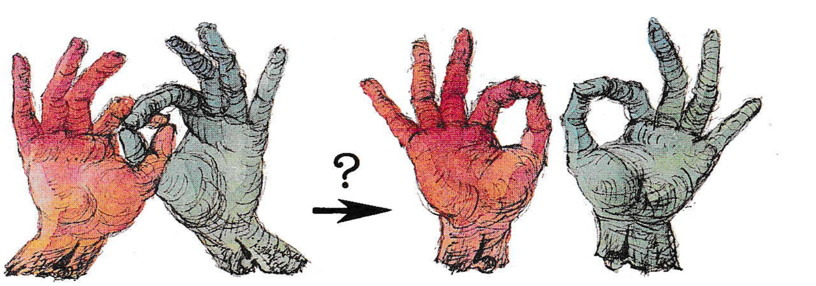


Figure 1

Figure 2

detached and somewhat bored expression on his face hid what must have been an intense attempt to solve the problem in his mind. Quite unexpectedly Rourke, who until then had remained modestly silent, as a graduate student should, joined the conversation.

"Can't be done, sir," he said.

This really surprised me. I knew the correct answer to the problem—it's fairly well known among topologists—and I couldn't understand how Rourke could make such an obvious blunder. Zeeman's surprise, however, was much greater.

"You . . . but . . . I mean," he spluttered, "I can understand that one of these algebraists . . . But *you*! You're supposed to be a pupil of mine. This is a perfectly *triv*ial problem for anyone who claims to be a geometer!"

But Rourke held his ground and, as often happens when two Englishmen don't agree on something, the argument resulted in a bet. The terms having been agreed upon, Zeeman immediately took a paper napkin and, right then and there, sketched the required transformations of the amoeba (fig.2).

"Yes, of course," said Rourke, "but your jacket, sir.

Zeeman's expression changed from triumph to bewilderment, and he began to reason out loud.

"Certainly all the amoeba's transformations can be carried out without taking off the jacket—all you have to do is twist and stretch it in places. But then after the hands are unlinked, the jacket will be wrapped around the amoeba in a very incongruous way. And it won't be fair to say that I will look the way I claimed I would. In fact, one sleeve of the jacket is enough to get me all swaddled up in fabric after all those transformations, so I won't be able to take the sleeve off without cutting it . . . Colin, I concede I've lost."

While a pound note was changing hands, Adams added to Zeeman's misery by addressing him in a bored monotone:

"Oh by the way, Chris, how do you prove that you can't take off your jacket without unlocking your fingers if you don't use invariants?"

As AN EXERCISE in three-dimensional topology, try to imagine what the jacket looks like after the amoeba has unlocked its fingers. Figure 3 shows where the right sleeve of the dinner jacket ends up. As for invariants—perhaps *Quantum* will turn to that topic in upcoming issues. ◘

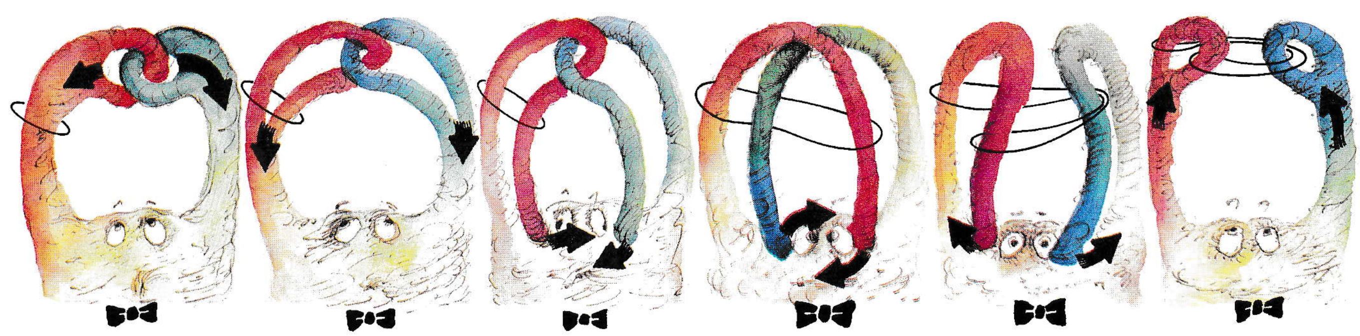


Figure 3

AT THE BLACKBOARD

Don't unplug your brain just yet, but when it comes to weeding out false assumptions . . .

Equations think for you

by V. Nakhshin

SOLVING A COMPUTATIONAL problem in physics usually involves two stages: first, we think over the conditions of the problem, analyze the physical laws describing the given phenomenon, and find the appropriate system of equations; second, we try to find an efficient method for obtaining the solution in general form. In a sense, we're physicists during the first part of our work, mathematicians during the second. But, having obtained the answer, we turn to physics again to see whether our result is reasonable—for example, whether it stands the test of physical dimension analysis.

Sometimes a review of the result shows that it's absurd. Problems where this happens generally have a feature in common: they require a careful preliminary examination of the physical process involved and suggest several versions, or scenarios, that are all reasonable enough at first sight. Ultimate success depends on a wise choice of the scenario, and it should be noted that a little bit of mathematics helps us make that choice. Indeed, suppose we choose a scenario, come up with a system of equations, solve it, and then convince ourselves that the answer is absurd. The conclusion is that we should have chosen another scenario to analyze with our mathematical tools. Eventually (and we can be thankful that physical laws severely restrict the number of possible scenarios), mathematics will indicate the right one.

Let's consider a few examples.

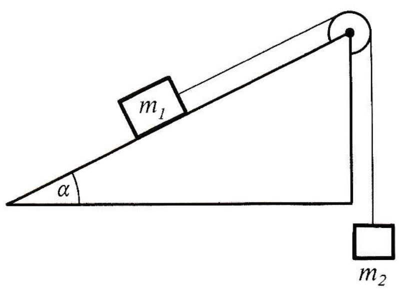


Figure 1

Problem 1. *There is a pulley at the top of a rough inclined plane (fig. 1). Two bodies of masses* $m_1 = 3$ *kg and* $m_2 = 2$ *kg are attached to the ends of a rope slung over the rim of the pulley. Find the acceleration of the system and the friction between the first body and the plane if the coefficient of friction* $\mu = 0.5$ *and the plane's angle of inclination* $\alpha = 30°$ *from the horizontal.*

Three different scenarios are clearly possible: (1) the second body moves downward and the pulley rotates clockwise; (2) this body moves upward and the pulley rotates counterclockwise; (3) the system does not move at all.

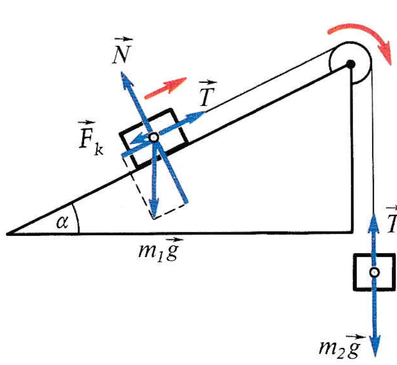


Figure 2

The first two cases differ from the third in that they are concerned with kinetic (sliding) friction between the first body and the inclined plane, whereas the third is concerned with static friction. These two kinds of friction are quite different. The length of the vector for kinetic friction is given by the equation $F_k = \mu N$, where N is the length of the vector of normal reaction. For static friction, the only requirement is that its force be less than that of kinetic friction.

Let's choose the first scenario. By writing Newton's second law, we may resolve the forces acting on the first body as those parallel to the inclined plane and those perpendicular to it; for the second body, we need to consider only the vertical direction. The equations read

$$T - m_1 g \sin\alpha - \mu N = m_1 a,$$
$$N - m_1 g \cos\alpha = 0,$$
$$m_2 g - T = m_2 a.$$

It should be noted that the acceleration a is positive because of the choice of directions (fig. 2). But solving the system, we get a negative a:

$$a = g\frac{m_2 - \mu m_1 \cos\alpha - m_1 \sin\alpha}{m_1 + m_2} \cong -1.6 \text{ m/sec}^2.$$

Therefore, we must reject the first scenario.

Let's take a look at the second one. At this point, a common mistake is to set the acceleration $a \cong 1.6$ m/sec^2 on the grounds that we have reversed all directions. But the force of friction remains negative, and the equations take the form

$$m_1 g \sin\alpha - T - \mu N = m_1 a,$$
$$N - m_1 g \cos\alpha = 0,$$
$$T - m_2 g = m_2 a;$$

so that

$$a = g\frac{m_1 \sin\alpha - \mu m_1 \cos\alpha - m_2}{m_1 + m_2} \cong -3.6\ \text{m/sec}^2.$$

We see that the acceleration is again negative, and we must reject the second scenario. The only option left is the third one—that is, $a = 0$. Then friction is static, and the formula $F_k = \mu N$ doesn't hold any longer. We have found the solution, at least partially. To find the force of friction, notice that the forces acting on the first body parallel to the inclined plane are the projection of the force of gravity ($m_1 g \sin\alpha = 15$ newtons) and the rope's tension, equal to the weight of the second body ($T = m_2 g = 20$ newtons). Consequently, the force of static static friction is directed parallel to the plane downward:

$$F_s = T - m_1 g \sin\alpha = 5 \text{ newtons}.$$

This is, in fact, less than that of kinetic friction:

$$\mu N = \mu m_1 g \cos\alpha \cong 13 \text{ newtons}.$$

Problem 2. *Two kilograms of water at temperature +80°C are poured into a calorimeter containing three kilograms of ice at –10°C. What temperature establishes itself inside the calorimeter as the result of heat transfer? (The heat capacity of the calorimeter should not be taken into account.)*

Obviously, the final temperature T_f is greater than –10°C and less than +80°C. The main point is that the specific heat of water and that of ice are different, and the process of melting requires some additional heat. Therefore, the process of heat transfer essentially depends on whether some ice melts or some amount of water freezes. To find out which possibility actually occurs, we'll consider the equations of thermal balance for three different cases: (1) $T_f > 0$°C; (2) $T_f < 0$°C; (3) $T_f = 0$°C.

First, let's suppose $T_f > 0$°C—that is, the calorimeter will end up containing only water. We can visualize the heat transfer as follows: the mass m_1 (3 kg) of ice is heated from T_1 = –10°C to 0°C, the ice melts, the water obtained from the ice is heated from 0°C to T_f, and the mass m_2 (2 kg) of water initially contained in the calorimeter is cooled from T_2 = 80°C to T_f. The thermal balance equation reads

$$c_1 m_1 (0 - T_1) + \lambda m_1 + c_2 m_1 (T_f - 0) + c_2 m_2 (T_f - T_2) = 0,$$

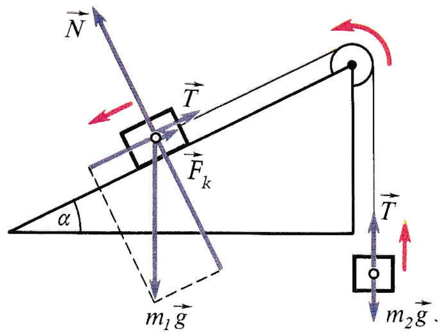


Figure 3

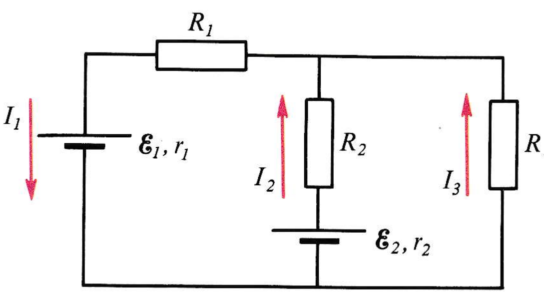


Figure 4

where c_1, c_2 represent the specific heat of ice and of water and λ is the specific heat of melting. Hence,

$$T_f = \frac{c_1 m_1 T_1 + c_2 m_2 T_2 - \lambda m_1}{c_2 (m_1 + m_2)}.$$

However, for our numerical data T_f turns out to be less than 0°C, so there is a contradiction with the hypothesis we had assumed.

Suppose $T_f < 0$°C—that is, the calorimeter will contain only ice. Heat transfer should proceed as follows: the ice is heated from –10°C to T_f and the water is cooled down to 0°C, freezes, and cools down, in the form of ice, to T_f. The thermal balance equation then reads

$$c_1 m_1 (T_f - T_1) + c_2 m_2 (0 - T_2) + c_1 m_2 (T_f - 0) - \lambda m_2 = 0,$$

which for our numerical data gives $T_f > 0$°C. Consequently, the second hypothesis is also wrong, so we come to the conclusion $T_f = 0$°C.

At this point you may be irritated with me and asking yourself: "Why does he purposely select examples in which all initial hypotheses turn out to be wrong and such cumbersome solutions lead to the comparatively simple results $a = 0$, $T_f = 0$°C? Why doesn't he take these simple cases right off the bat?"

The answer is that such an approach is possible, but generally it doesn't result in serious simplifications. For example, in the first problem we could assume that the system is at rest, write out equations, solve them, and find the force of friction. But then we have to find the force of kinetic friction to convince ourselves that it's greater than the static friction. If, on the other hand, for some numerical values kinetic friction had turned out to be the lesser quantity, we would have to start from scratch.

In the second problem, if we had merely assumed $T_f = 0$°C, we wouldn't have known how to write the equations of heat balance because, without making any assumptions about the process taking place in the calorimeter, we don't know whether some ice melted or some water froze. Of course, we could suggest some way out of these difficulties, but the number of possible options would be even greater than in the solutions discussed above.

Problem 3. *Find the currents in all parts of the circuit given in figure 4. Here* $E_1 = E_2 = E = 1$ *volt,* $r_1 = r_2 = r = 1$ *ohm, and* $R_1 = R_2 = R_3 = R = 10$ *ohms.*

The key to solving this problem is to find the right directions of the currents, which determine the equations of charge balance at the nodes, or branching points, of the electrical circuit. First, let's suppose that the currents I_1, I_2, I_3 are directed as shown in figure 4. Electrical charges may not accumulate at the branching points of a circuit because incoming and outgoing charges counterbalance each other; consequently, we have the equation for the currents $I_1 = I_2 + I_3$ in accordance with the hypothesis concerning their directions.

Let's choose two loops inside the circuit—for example, the left and the right one—and choose a certain direction for tracing a path—for example, counterclockwise. The energy conservation principle requires the algebraic sum of electromotive forces to be equal to the algebraic sum of voltages:

$$E - E = I_1R + I_1r + I_2r + I_2R,$$
$$-E = -I_2R - I_2r + I_3R.$$

For the numerical data of the problem these equations read

$$I_1 = I_2 + I_3,$$
$$10I_1 + I_1 + I_2 + 10I_2 = 0,$$
$$10I_2 + I_2 - 10I_3 = 1,$$

and have the solution

$$I_1 = -1/31 \text{ A}, \; I_2 = 1/31 \text{ A}, \; I_3 = -2/31 \text{ A}.$$

Note that the first and third currents are negative, whereas the assumption we made about the directions of the currents requires them to be positive. If we reverse the directions of the first and third currents, we'll obtain the right answer—three positive values of I_1, I_2, I_3, whose absolute values equal those obtained above. It should be noted that the equation for current conservation in this case reads $I_3 = I_1 + I_2$.

Problem 4. *A body is thrown straight up at an initial speed* $v_0 = 25$ *m/sec. In what time will it reach the height* $h = 40$ *m?*

Let's write the equation for the height of the body:

$$h = v_0t - \frac{gt^2}{2},$$

where v_0 is the initial velocity and g is the free-fall acceleration. From the equation we infer

$$t = \frac{v_0 \pm \sqrt{v_0^2 - 2gh}}{g}.$$

Substituting $v_0 = 25$ m/sec, $h = 40$ m, and $g = 9.81$ m/sec^2, we obtain a negative number under the square root. Obviously, the result indicates that the body will not reach this height. In fact, the maximum height is given by $h_{max} = v_0^2/2g \cong 31.5$ m (that is, less than 40 m).

So, in order to obtain reasonable answers, we must ask reasonable questions.

The English naturalist T. H. Huxley used to compare mathematics to millstones, which grind only the seeds poured between them—nothing more. The problems discussed above are examples of this general principle, but they also teach us that, in helping us reject incorrect assumptions, mathematics gives us a clue about the direction that will lead to a solution.

Exercises

1. A lever with a mass of 10 kg leans on a prop 25 cm from its left end. The lever is 1 m long. A weight with a mass of 2 kg is suspended from the left end of the lever. What force must be applied at the right end at an angle of 30° above the horizontal so that the lever is in equilibrium?

2. A 500-gram mass of steam at 100°C is put inside a calorimeter containing 1 kilogram of ice at 0°C. What temperature results after heat transfer has taken place? (The heat capacity of the calorimeter should be ignored.)

3. A body is thrown straight up at an initial speed of 15 m/sec. What height does it attain in 2 sec?

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The Tournament of Towns

A more relaxed approach to math competition

by Nikolay Konstantinov

THE TOURNAMENT OF TOWNS resembles ordinary mathematics olympiads in that high school students get together to solve math problems. It differs in many ways, however, from math olympiads as they are traditionally organized in the USSR. For instance, olympiads are multistage competitions at the school, city, region, republic, national, and finally international levels. In contrast, the Tournament of Towns is a one-day affair in which any high school student can participate without leaving his or her home town.

The tournament takes place each year in the spring and autumn simultaneously in many cities of the Soviet Union and—in the last couple of years—in a few cities in Poland, Bulgaria, and Australia as well. All these cities have their own organizing committees consisting of local educators and research mathematicians who advertise, organize, and supervise the competition and correct the papers.

The central organizing committee (based in Moscow) provides the problems and sums up the overall results. In particular, it double checks the grades given to the best papers, which are sent to Moscow by the local organizers for that purpose.

If there is no organizing committee in your city, it's possible to have one set up. All that's needed is a group of math problem enthusiasts—teachers, research mathematicians, or college students—who should write to *Kvant* magazine (Moscow 103006, ul. Gorkogo 32/1, *Kvant,* Tournament of Towns) for more details, competition problems, and instructions (which need not be dogmatically complied with— our traditions allow local organizers to modify the rules to suit local conditions).

Some of the new tournament problems will undoubtedly become part of the classical folklore of math competitions.

I think the problems used in the competition are the most interesting feature of the Tournament of Towns. During the 10 years the tournament has been in existence, 214 problems have been proposed to the competitors. These include a certain number of "practice problems," usually chosen from past olympiads, but almost all the contest problems are original. Some of the new problems are real masterpieces—beautiful discoveries in miniature that will be remembered for a long time by the contestants (whether they solved them or not) and will undoubtedly become part of the classical folklore of math competitions.

The authors of many of the best problems are past winners of various contests—Moscow, Leningrad, Riga, and national math olympiads as well as the Tournament of Towns itself. Most of the authors are connected with *Kvant* in one way or another—as subscribers, readers, authors, or editorial board members. It's traditional in the Soviet Union to send new mathematics problems to *Kvant*, where a highly competent team of problem specialists, headed by N.B. Vasilyev, picks out the best ones to use in the Tournament of Towns or publish directly in *Kvant*'s problem section.

An important feature of the choice of problems for the tournament is that it involves two levels of competition—one for beginners and one for experienced problem solvers. Here experience is not synonymous with age. There are easier and harder problems for the two age groups participating in the tournament.

It should also be pointed out that the selection of the winners is not, by any means, the main goal of the Tournament of Towns. Participation in the competition often helps confirm a beginner's interest in mathematics. In order to deemphasize the purely competitive stimulus of problem solving (doing as much as you can within a given time limit under stressful conditions) and to avoid transforming the tournament into something that helps produce young "professional math problem solvers," the central organizing committee has recently been sending participants "research topics" on which they can work at their leisure, without the nervous strain involved in a timed contest.

Problems from the 10th Tournament of Towns

To give you a more specific idea of the competition, here are the problems from the spring round, March 1989.

Grades 7 and 8 (ages 13 to 15), O-level (beginners)

1. The positive numbers a, b, c satisfy $a \geq b \geq c$ and $a + b + c \leq 1$. Prove that $a^2 + 3b^2 + 5c^2 \leq 1$. (3 points)

2. Let AM be the median of triangle ABC. Can the radius of the incircle inside triangle ABM be exactly twice that of ACM? (3 points)

3. What digit must be written instead of the question mark in the number 888...88?999...99 (there are 50 eights and as many nines in it) in order to make the number divisible by 7? (3 points)

4. Is it possible to draw a closed curve on the surface of Rubik's cube so that it goes through each colored square exactly once without passing through any vertex? (3 points)

Grades 7 and 8, A-level ("professionals")

1. A stairway has 100 steps. Nick intends to go down the stairway in a special way. He starts at the top and jumps down, then jumps up, then down, then up again, and so on. He may jump 6 steps up or down (say, from the ninth step to the fifteenth or to the third one) or 7 steps or 8 steps, and he may not jump onto the same step twice. Will he make the descent? (3 points)

2. A pawn is placed in one of the squares of a chessboard (8 x 8 squares). Two players move the pawn in succession to any other square but each move (beginning with the second) must be longer than the previous one. The player who cannot make such a move loses. Who wins in an errorless game? (The pawn is always placed in the exact center of a square.) (3 points)

3. Convex quadrilaterals $ABCD$ and $PQRS$ are cut out of cardboard and paper, respectively. Let's say that they fit each other if the following conditions hold: (1) The cardboard quadrilateral can be placed on the paper one so that its vertices A, B, C, D lie on the "paper sides," one vertex on each side; (2) if the four visible paper triangles are then folded over the cardboard quadrilateral, they cover it entirely without overlapping. Prove that (a) if two quadrilaterals fit each other, the paper one either has two parallel sides or has perpendicular diagonals (2 points); (b) if $PQRS$ is a paper parallelogram, it's possible to cut out a cardboard quadrilateral $ABCD$ that fits it (3 points).

4. Prove that if m is even, then the integers from 1 to $m - 1$ can be written in sequence so that no sum of successive numbers in the sequence is divisible by m. (5 points)

5. The end points of n unit vectors with origin at the center of a unit circle divide its circumference into n equal parts. Some of the vectors are blue, the others are red. Calculate the sum of all the angles from a red vector to a blue one (measured counterclockwise) and divide it by the total number of all red–blue pairs of vectors. Prove that the "mean value" of the angle obtained in this way is always 180°. (7 points)

6. Prove that (a) if $3n$ cells of a $2n$ x $2n$ square table are marked by stars, then n rows and n columns of the table can be crossed out so that all the stars will be crossed out (4 points); (b) $3n + 1$ stars may be placed in the table ($2n$ x $2n$) so that crossing out any n rows and any n columns leaves at least one star not crossed out. (4 points)

Grades 9 and 10 (15 and older), O-level

1. The positive numbers a, b, c, d satisfy $a \leq b \leq c \leq d$ and $a + b + c + d \geq 1$. Prove that $a^2 + 3b^2 + 5c^2 + 7d^2 \geq 1$ (3 points)

2. A circle can be inscribed in trapezium $ABCD$. Prove that the circles whose diameters are the nonparallel sides of the trapezium are tangent. (3 points)

3. Find six different positive integers such that the product of any two is divisible by their sum. (3 points)

4. Is it possible to draw a diagonal on each of the colored squares of Rubik's cube so as to obtain a connected line without self-intersections? (3 points)

Grades 9 and 10, A-level

1. Find two six-digit numbers whose concatenation (the number obtained by writing them one after another) is divisible by their product. (3 points)

2. The point M is chosen inside triangle ABC so that $BMC = 90° - BAC/2$ and the line AM contains the circumcenter of triangle BMC. Prove that M is the incenter of triangle ABC. (4 points)

3. One thousand linear functions are given:

$f_k(x) = p_k x + q_k$, $k = 1, 2, ..., 1000$.

Prove that to find the value of their composition

$f(x_0) = f_{1000}(f_{999}(...f_2(f_1(x_0))...))$

at the point x_0, no more than 30 stages of "parallel computations" are required. At each stage any arithmetical operations are allowed on any pairs of numbers obtained at the previous stages. (At the first stage the given numbers p_k, q_k, x_0 are to be used.) (5 points)

4. An exclusive tennis club with 11 members is ruled by a directorate consisting of three or more club members. The club's charter forbids having the same directorate twice. At each meeting of the club the directorate must be changed—either a new member is added to it or an old member is excluded from it. Can it happen one day that all imaginable directorates (of three or more members) will have ruled? (6 points)

5. Given n lines ($n > 1$) in general position (no three of them intersect at one point, no two of them are parallel) in a plane, prove that each of the parts into which the lines divide the plane can be labeled by a nonzero integer whose absolute value is not greater than n so that the sum of the integers in each of the two sides of each of the lines is equal to 0. (7 points)

6. Given 101 rectangles whose sides are integers not greater than 100, prove that there are three (A, B, C) that can be placed one inside another. (8 points) ■

Bulletin Board

We invite you to submit reports of interesting events, especially if you have taken part in them yourself and can provide firsthand impressions. And we ask you to send us announcements of exciting things to come—math and physics events that our readers around the country (and the world) can attend. We want to be your bulletin board and diary—let *Quantum* know what's happening!

A PROMYS from Boston University

Ambitious high school students from all over the United States are invited to take part in Boston University's summer Program in Mathematics for Young Scientists (PROMYS). Students entering this six-week residential program take a challenging course in number theory and may also study algebra. Returning students choose from algebra, the theory of equations, and experimental dynamical systems. PROMYS emphasizes active problem solving, including the formulation, criticism, and modification of conjectures. Special lectures by outside speakers help give a broad view of mathematics and its role in the sciences.

For more information, write to PROMYS, Department of Mathematics, Boston University, 111 Cummington Street, Boston, MA 02215, or call 617 353-2560.

Moscow conference to focus on energy-efficiency

Scientific, political, and spiritual leaders from around the world will gather in Moscow in January for a week of interdisciplinary workshops and forums on critical environmental and development issues. The Global Forum on Environment and Development for Survival will feature such luminaries as astronomer Carl Sagan, oceanographer Jacques Cousteau, and Soviet scientist Yevgeny Velikhov. Several U.S. senators are expected to attend, and Soviet President Mikhail Gorbachev will address the 600 participants.

An exhibition of energy-efficient technologies will serve as a centerpiece of the forum. In the context of global warming, acid rain, deforestation, and nuclear waste, the exhibition will emphasize the potential for massive energy savings in the large-scale use of new technologies in transportation, building, lighting, appliances, heating, and cooling. Compact fluorescent lights, for example, use only one-fourth as much electricity as their incandescent cousins.

Further information about the forum can be obtained from the Center for Global Change, University of Maryland at College Park, Executive Building, Suite 401, 7100 Baltimore Avenue, College Park, MD 20740.

New source of competition info

Early in 1990 the National Science Teachers Association is publishing a booklet on cooperating and competing in science and math. Along with a detailed and annotated list of science and math competitions at national and international levels, the book provides advice on organizing interest groups and starting fairs. In addition, it publishes the results of surveys about contest experiences provided by competition sponsors and judges, by student contestants and their teacher/mentors, and by Nobel Prize and Medal of Science winners.

Problem corner

M1

Consider $a + 1$ numbers $2^0 - 1, 2^1 - 1, \ldots, 2^a - 1$. Since there are only a different remainders modulo a, we can find two of the above numbers giving the same remainders modulo a (the Dirichlet principle). If these numbers are $2^k - 1$ and $2^m - 1$ with $k < m$, then $(2^m - 1) - (2^k - 1) = 2^k(2^{m-k} - 1)$ is divisible by a. Since a is odd, $2^{m-k} - 1$ is divisible by a.

One can also prove a more general fact: if a and c are coprimes, there exists a natural number b such that $c^b - 1$ is divisible by a.

M2

Project all the circles perpendicularly onto some side AB of the square (fig. 1) and assume that every line intersects no more than three circles. Any point of the segment AB is thus covered by projections of circles no more than three times. It follows that the length of all projections is less than or equal to 3. But this length is also equal to the sum of the diameters of all the circles. Thus, the sum of the circumferences of the circles is less than or equal to 3π. Since $3\pi < 10$, this contradicts the original assumption that this sum is equal to 10.

M3

We'll prove an even more general statement: If there is an odd number N of units and zeros (and both units and zeros are actually present), one will never obtain N zeros. Assume the converse. If N zeros are obtained after t operations for the first time, then after $t - 1$ operations you have N equal numbers different from zero—that is, N ones. It follows that after $t - 2$ operations every two neighboring numbers are different, which contradicts N being odd.

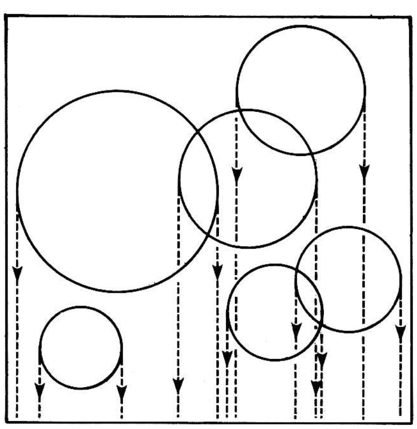


Figure 1

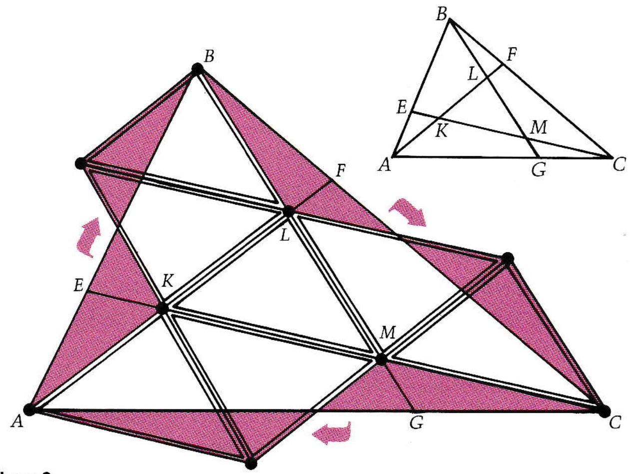


Figure 3

M4

The answer is $n^2 - n + 1$. A chain of this length is shown in figure 2. To prove that chains can't be any longer, paint the triangles alternately in two colors (as shown in the figure). It is easily seen that there are n white triangles more than colored ones. On the other hand, the colors of triangles in a chain must alternate. So there can be at most one white triangle more than colored ones. Thus, at least $n - 1$ white triangles do not belong to any chain.

M5

(a) Denote the area of any triangle XYZ by $S(XYZ)$. Since the ratio of the area of two triangles with the same altitudes (with respect to their bases) is equal to the ratio of their bases (with respect to their altitudes), it's easy to see that

$$S(ACK) = \frac{1}{k} S(ABK) = \frac{k+1}{k^2} S(AEK)$$

$$= \frac{k+1}{k^2+k+1} S(AEC) = \frac{k}{k^2+k+1} S(ABC).$$

Similarly,

$$S(BLA) = S(CMB) = \frac{k}{k^2+k+1} S(ABC).$$

Thus,

$$\frac{S(KLM)}{S(ABC)} = 1 - \frac{3k}{k^2+k+1} = \frac{(1-k)^2}{k^2+k+1} = \frac{(1-k)^3}{1-k^3}.$$

(b) Note that if $k = 1/2$, the above ratio is equal to $1/7$ and each of the segments AF, BG, CE is split by the two others in the ratio 3:3:1. If you draw lines through points K, L, M parallel to the segments AF, BG, CE, every side of triangle ABC will be split in the ratio 2:1:1:2.

Now it isn't difficult to compose 7 equal triangles from the 13 parts shown (fig. 3). (One of these 7 triangles is merely triangle KLM).

P1

Let's combine the figures from the problem into a new figure (fig. 4 below). From an arbitrary point A on the ship's route we drop a perpendicular onto the boundary of the disturbance region in the stretch where there is no curent. The length of the perpendicular (AB or AB') determines the distance traversed by the wave during the time the ship traveled the distance AO. The distance BC (or $B'C'$) deter-

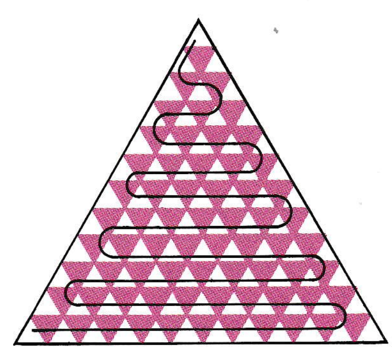


Figure 2

mines the drift of the disturbance regions's boundary due to the current during the same interval of time. Hence, the ratio BC/AO (or $B'C'/AO$) determines the ratio of the current's velocity v to the ship's velocity v_O. From the figure we find $v/v_O : 1/5 :: v : 3.6$ km/h.

P2

In order to explain what occurs with sand on a riverbank, we'll begin with some background on "close packing." Identical balls can be placed on a plane so that each of them touches six other balls. We can create another layer by placing balls in the spaces between balls in the first layer. Each of them will touch three balls in the lower layer and six balls in its own layer, and so on. The arrangement obtained in this way is called close packing of the balls. If we disturb the close packing by taking balls in one layer out of the

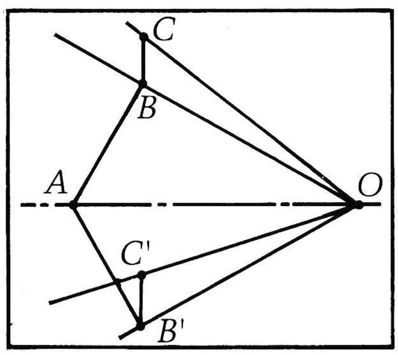


Figure 4

spaces between balls in the lower layer, the gaps between the balls will grow. The volume of the whole system will also increase. This means that if a system of closely packed balls is acted upon by forces leading to a disturbance of the close packing, the volume of the system increases because the gaps between the balls increase.

Any granular medium behaves like this. Take, for example, millet grains (or coffee beans), fill a glass with them, and shake it slightly so that the grains form the closest possible packing. Then press the grains. The pressure will cause an increase in the volume occupied by the grains—that is, it will disturb the close packing. Some of the grains will pour out. If we now tap the glass so that the grains become closely packed again, the glass will not be filled to the brim.

Now let's return to the sand on the riverbank, which is also closely packed. When pressure is exerted on the sand, the close packing is disturbed and the volume of the sand grows because of an increase in the gap between the grains of sand. Water from the upper layers of sand moves deeper down, filling the increasing gaps. The sand seems to dry out. When you remove your foot the close packing is restored, and the water forced out of the reduced gaps fills the footprint.

P3

Along with evaporation, condensation also occurs. The evaporation rate in both cases ($r_1 = 50\%$ and $r_2 = 80\%$) is the same—it depends only on the temperature of the liquid. But the condensation rate is proportional to the concentration of vapor molecules in air—that is, it's proportional to the relative humidity; so it's higher in the second case than in the first.

Obviously, the rate of decrease of water in the saucer is $v_r = v_v - v_c$ (v_v, v_c are the rates of evaporation and condensation). At 100% humidity $v_v = v_c$.

Taking into account all that has been said, we may write:

$$\text{at } r_1 = 50\%,\ v_{r1} = v_v - v_{c1} = 1/2\, v_v;$$
$$\text{at } r_2 = 80\%,\ v_{r2} = v_v - v_{c2} = 1/5\, v_v.$$

Since $t_2 : t_1 :: v_{r1} : v_{r2}$, the time t_2 during which water would evaporate at $r_2 =$ 80% is equal to

$$t_2 = v_{r1} t_1 / v_{r2} = 5/2\, t_1 = 100 \text{ min}.$$

Note: If the air convection over the surface of the liquid is insufficient, a layer of saturated vapor forms near the surface and the rate of water decrease will be lower. In our solution we have ignored this effect.

P4

Taking into account the relation between the capacitance, voltage, and charge of a capacitor, we can write the following equations for the three capacitors:

$$\varphi_A - \varphi_O = q_1/C_1,$$
$$\varphi_B - \varphi_O = q_2/C_2,$$
$$\varphi_D - \varphi_O = q_3/C_3,$$

where C_1, C_2, C_3 are the capacitances of the corresponding capacitors and q_1, q_2, q_3 are the charges on their plates. According to the law of charge conservation, $q_1 + q_2 + q_3 = 0$; hence, the potential of the common point O is

$$\varphi_o = \frac{\varphi_A C_1 + \varphi_B C_2 + \varphi_D C_3}{C_1 + C_2 + C_3}.$$

P5

The refraction index of glass and, consequently, the focal distance of a lens depend on the wavelength of the radiation. By selecting the shape and material of the lens and the optimum distance to the film plane, we practically exclude blurring when we photograph using visible light. But when ultraviolet rays fall on film, blurring occurs because the refraction index for ultraviolet light differs from the index used in designing the lens. To overcome this deficiency, photographers use filters that eliminate the ultraviolet component. For the same reason some lenses have a separate scale marked on them for photographing in the infrared range.

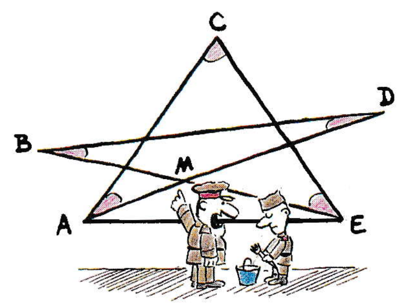


Figure 5

Brainteasers

B1

Join two of the star's adjacent vertices—say, A and E (fig.5). Since the angles at M of triangles AME and BMD are congruent, the sum of the angles B and D of triangle BMD equals the sum of the angles A and E of triangle AME; but then the total sum of angles at the vertices of the star is equal to the sum of the angles of triangle ACE—that is, 180°.

B2

There are two such numbers: 41312432 and 23421314.

B3

The solution is shown in figure 6. You first determine that the numbers 1 and 8 must be put in the middle (since they are each joined to 6 other circles); then the position of the numbers 2 and 7 can be uniquely determined; then the number 3 and the others can be placed. (Obviously, the solution can be flipped along its vertical axis, in which case the outer pairs 3-5 and 4-6 would swap positions.)

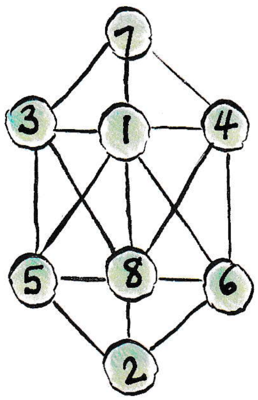


Figure 6

B4

The clock keeps time correctly, but the minute hand is slightly loose on its spindle—it can move freely 2 minutes from its correct position. Under the action of the force of gravity, the minute hand always stays below its correct position. In the left half of the clock dial, this makes the clock two minutes slow; in the right half, two minutes fast.

B5

If lines parallel to the sides of the triangle are drawn through the chosen point, pairs of congruent triangles of different colors are formed (figure 7).

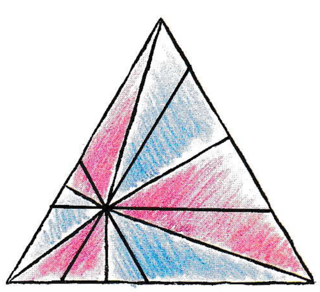


Figure 7

SOLUTION TO COVER PROBLEM

Figure 8 shows the diagonal section of the cube with inscribed spheres. Triangles $O_1O_2M_1$ and AO_1D_1 are similar. So are triangles $O_3O_2M_2$ and AO_1D_1, and so on. Note also that $AO_1 = 3^{1/2}$; $O_1D_1 = r_1 = 1$; and $O_1O_2 = r_1 + r_2$, $O_2O_3 = r_2 + r_3$, and so on. Therefore,

$$\frac{O_1O_2}{O_1M_1} = \frac{AO_1}{O_1D_1} = \sqrt{3}; \quad \frac{O_3O_2}{O_2M_2} = \frac{AO_1}{O_1D_1} = \sqrt{3}; \ldots \qquad (1)$$

or

$$\frac{r_1 + r_2}{r_1 - r_2} = \sqrt{3}; \quad \frac{r_3 + r_2}{r_3 - r_2} = \sqrt{3}; \ldots \qquad (2)$$

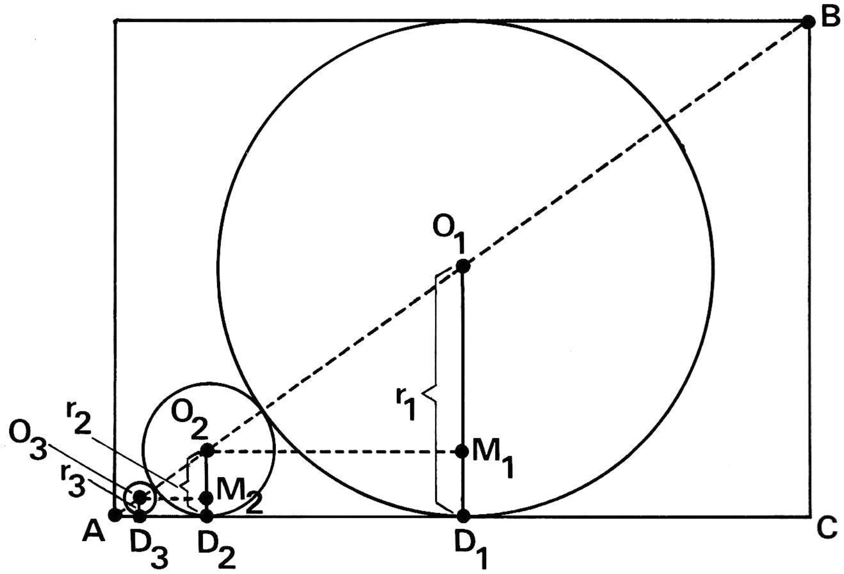


Figure 8

From (2) we get

$$r_2 = \frac{\sqrt{3}-1}{\sqrt{3}+1} r_1; \; r_3 = \frac{\sqrt{3}-1}{\sqrt{3}+1} r_2 = \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2 r_1; \ldots$$

We can continue this process indefinitely. Therefore,

$$r_{1990} = \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^{1990}.$$

The sum of all diameters can be found by using the formula for the sum of an infinite geometric progression with the factor $|q| < 1$; that is,

$$\sum_{\infty} = \frac{d_1}{1-q},$$

where

$$d_1 = 2, \; q = \frac{\sqrt{3}-1}{\sqrt{3}+1}.$$

Of course, the same result can be obtained without any calculations by simply realizing that the infinite series of diameters will converge to half of the cube's diagonal plus the radius of the first sphere.

Invincible Mephisto

A devilishly good computer chess program

by Y. Gik

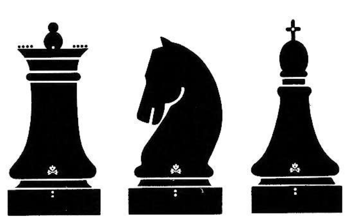
THE 6TH WORLD CHAMPIONSHIP for microcomputer chess was held in Rome, where for the first time the computer crown was decided in tournament play. Three copies of Richard Lang's renowned Mephisto, a three-time world champion, constituted one team; three copies of D. Levy's less well known Sphinx made up the other.

The competition raged through six rounds—the three Mephistos (A, B, and C) played the three Sphinxes (also A, B, and C) in two games each: white and black. The results were disappointing for the Sphinxes—the Mephistos won all six rounds and garnered the crown for the fourth straight time.

Along with the basic competition in Rome another battle was taking place, a sort of "junior world championship." Seven programs for personal computers participated in a round-robin tournament. The winner—Psion Chess; its creator—Richard Lang again.

It's interesting that, despite his great expertise in computer science, Lang is anything but an "absent-minded professor." Between computer competitions he runs in marathons, and does quite well at it.

Here are some samples of computer play at the Rome tournament with commentary on the the more interesting episodes in the games.



Mephisto A—Sphinx A

English Opening

1.	c2-c4	e7-e5
2.	Nb1-c3	Ng8-f6
3.	Ng1-f3	Nb8-c6
4.	e2-e3	Bf8-e7
5.	d2-d4	e5xd4
6.	Nf3xd4	0-0
7.	Bf1-d3	Nc6-e5
8.	e3-e4	Be7-c5
9.	Bd3-e2	Bc5-b4
10.	Bc1-g5	h7-h6
11.	Bg5xf6	Bb4xc3+
12.	b2xc3	Qd8xf6
13.	Nd4-b5	Qf6-g6
14.	0-0	Qg6xe4
15.	Nb5xc7	Ra8-b8
16.	Nc7-b5	b7-b6
17.	Nb5-d6	Qe4-c6
18.	Qd1-d5	Ne5-g6
19.	Be2-f3	Qc6-c5
20.	Ra1-d1	Bc8-a6
21.	Rf1-e1	Ng6-f4
22.	Qd5xc5	b6xc5
23.	Re1-e7	Rb8-b2
24.	a2-a4	Nf4-g6
25.	Re7xd7	Ng6-e5
26.	Rd7xa7	Ba6xc4
27.	Nd6xc4	Ne5xc4
28.	Ra7-c7	Rb2-a2
29.	Bf3-d5	Ra2xa4
30.	Rc7xc5	Nc4-b6
31.	Bd5-b3	Ra4-a7
32.	Rc5-c6	Nb6-d7
33.	f2-f3	Rf8-b8
34.	Bb3-c2	Ra7-a2
35.	Bc2-f5	Nd7-f8
36.	Rd1-e1	Rb8-d8
37.	Rc6-c8	Rd8-d2
38.	Bf5-h3	g7-g6
39.	c3-c4	Rd2-b2
40.	c4-c5	Kg8-g7
41.	Rc8-c7	Nf8-h7

The correct moves would have been 41. ... Nf8-e6! 42. Bh3xe6 Rb2xg2 stalemate.

42.	f3-f4	Nh7-f6
43.	c5-c6	Nf6-d5

The activity of the black pieces compensates for the lack of a pawn. Mephisto, though, is carrying out an exchange combination, preserving its chances for success.

44.	Rc7xf7+!	Kg7xf7
45.	Bh3-e6+	Kf7-f6
46.	Be6xd5	Ra2-a7
47.	g2-g3	Rb2-d2
48.	Bd5-f3	Ra7-a2
49.	Bf3-e4	Ra2-a4
50.	Re1-c1	Ra4xe4
51.	c6-c7	Re4-e8
52.	c7-c8Q	Re8xc8
53.	Rc1xc8	Rd2-d1+
54.	Kg1-g2	Rd1-d2+
55.	Kg2-h3	h6-h5
56.	Rc8-c1	Kf6-f5
57.	Rc1-h1	Rd2-d3

Simplest of all would be to force a draw: 57. ... g6-g5. Now white's efforts are crowned with success.

58.	Kh3-h4	Rd3-d2
59.	h2-h3	Kf5-f6
60.	g3-g4	Rd2-f2
61.	Kh4-g3	Rf2-c2
62.	Rh1-b1	Rc2-c3+
63.	Kg3-h4	Rc3-c5
64.	Rb1-b6+	Kf6-f7
65.	f4-f5	g6xf5

66. g4-g5 Kf7-g7
67. Kh4xh5 Rc5-c7
68. h3-h4 Rc7-a7
Black resigns.

Plimat—Cirrus

Sicilian Defense

1. e2-e4 c7-c5
2. Ng1-f3 Nb8-c6
3. d2-d4 c5xd4
4. Nf3xd4 Ng8-f6
5. Nb1-c3 d7-d6
6. Bc1-g5 e7-e6
7. Qd1-d2 a7-a6
8. 0-0-0 Bc8-d7
9. f2-f4 Bf8-e7
10. Nd4-f3 b7-b5
11. e4-e5 b5-b4
12. e5xf6 b4xc3
13. Qd2xc3 g7xf6
14. Bg5-h4 d6-d5
15. Kc1-b1 0-0
16. Nf3-d4 Nc6xd4
17. Qc3xd4 Kg8-h8
18. g2-g4 a6-a5
19. g4-g5 Qd8-c7
20. Rd1-d3 Rf8-b8
21. Rd3-b3

Black's position contains some dangers, but it suddenly finds a way of forcing a draw.

21. ... e6-e5!
22. Qd4xd5 Bd7-c6
23. Qd5-c4 Qc7-d8
24. Rb3-d3 Qd8-b6
25. Rd3-b3 Qb6-d8
26. Rb3-d3 Qd8-b6
27. Rd3-b3 Qb6-d8

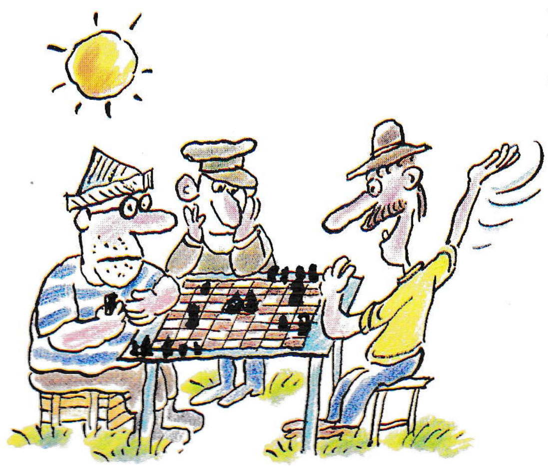
Draw.

Psion Chess—Plimat

Queen's Gambit

1. d2-d4 Ng8-f6
2. c2-c4 e7-e6
3. Nb1-c3 Bf8-b4
4. Qd1-c2 c7-c5
5. d4xc5 0-0
6. Ng1-f3 Nb8-c6
7. Bc1-f4 Bb4xc5
8. e2-e3 d7-d5
9. Ra1-d1 Qd8-a5
10. a2-a3 Rf8-d8

This exchange of moves produced a well-known position, one that occurs in world championship matches. (Bravo, computers!) At Bagio in 1978 (twenty-first game) against Korchnoi, Karpov chose 10. ... Rf8-e8; at Merano in 1981 (eleventh game), he selected 10. ... Bc5-e7. And the rook move to d8 has frequently been seen in actual games. The standard reaction by white is 11. Nf3-d2, while advancement of the pawn b2-b4 has never been examined in theory. Nevertheless, the future "junior world champion" chooses this very move, and its debut brings success.



"Ha! My *micro*computer could've made a better move than *that*!"

11. b2-b4!? Nc6xb4
12. a3xb4 Bc5xb4
13. Rd1-c1 Nf6-e4
14. Bf4-e5 f7-f6
15. Be5-d4 e6-e5

Everything is being forced. Black is winning back a piece and will be left with an extra pawn. Yet, incredibly, the ensuing endgame eventually works out in white's favor!

16. Nf3xe5 Ne4xc3
17. Bd4xc3 f6xe5
18. c4xd5 Bc8-f5
19. Bc3xb4 Qa5xb4+
20. Qc2-c3 Qb4xc3+
21. Rc1xc3 Bf5-e4
22. f2-f3 Be4xd5
23. e3-e4 Bd5-f7?

An unforgivable error. Black had been confidently engaged in tactical skirmishing; 23. ... Bd5-c6 would have solidified its clear advantage.

24. Rc3-c7 b7-b6?

Yet another blunder (24. ... Ra8-b8 would have been better), allowing white to solidly take control of the c-file. White flawlessly plays through to the end.

25. Bf1-a6! Bf7-e6
26. 0-0 Rd8-d7
27. Rc7xd7 Be6xd7
28. Rf1-c1 Bd7-e8
29. Rc1-c7 Kg8-f8
30. g2-g3 h7-h5
31. h2-h4 g7-g6
32. Kg1-f2 Be8-f7
33. Kf2-e3 Bf7-b3
34. f3-f4 e5xf4
35. Ke3xf4 Bb3-e6
36. Kf4-g5 Be6-f7
37. Kg5-f6 Bf7-b3
38. Kf6xg6 Ra8-e8

Black finally wakes up and brings the rook into play, but it's too late.

39. Rc7xa7 Re8xe4
40. Kg6xh5 Re4-a4
41. Kh5-g5 Bb3-c4
42. Ra7-a8+ Kf8-e7
43. Ba6-b7 Ra4xa8
44. Bb7xa8 Bc4-d3
45. h4-h5 Ke7-e6
46. h5-h6 Ke6-e5
47. Ba8-c6 Bd3-h7
48. Bc6-e8 b6-b5
49. Be8xb5 Ke5-e6
50. Bb5-e8 Ke6-e7
51. Be8-g6 Bh7-g8
52. h6-h7

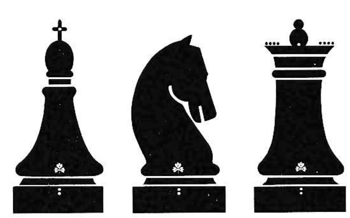
Black resigns.

Chet—Kempelen

Incorrect Opening

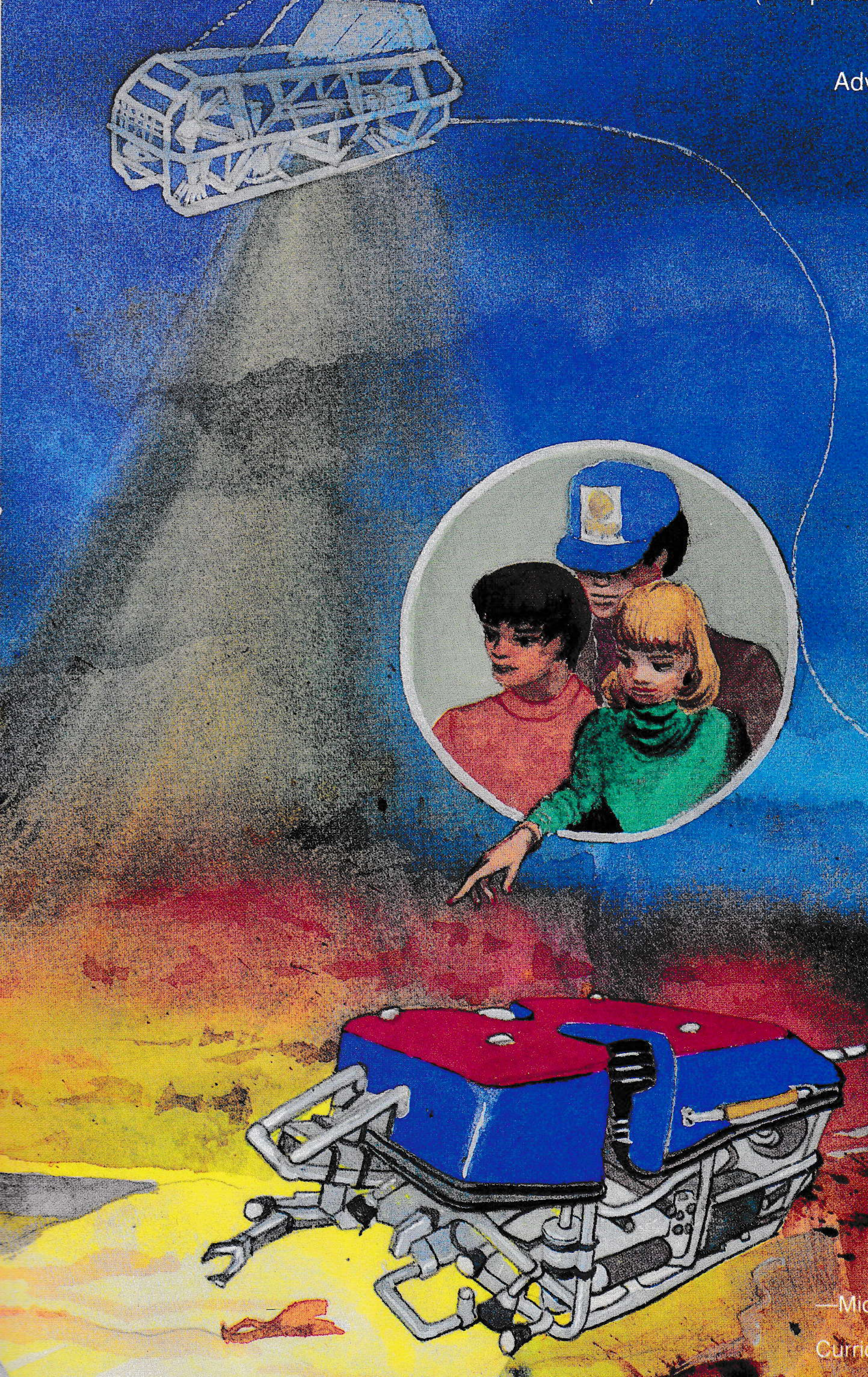
To close, here's an amusing example of how a person can still count on an electronic opponent to make a mistake.

1. d2-d4 f7-f5
2. h2-h3 Ng8-f6
3. g2-g4 f5xg4
4. h3xg4 Nf6xg4
5. Qd1-d3 Ng4-f6
6. Rh1xh7 Nf6xh7
7. Qd3-g6#



Dive Back in Time...
...with Project JASON

During the War of 1812, the warships *Hamilton* and the *Scourge* sank in a sudden squall on Lake Ontario. This spring Woods Hole scientist Dr. Robert Ballard will lead a group of fellow scientists in exploring and documenting these remarkably well-preserved ships. Their underwater eyes will be the remotely operated vehicle (ROV) JASON (an updated model of the vehicle used to find the *Titanic*).



Advances in technology have resulted in a new type of communication link between JASON and its surface vessel—fiber optic cable and a modulated laser. The fibers of ultra-pure glass can transmit 100 million bits of digitized data per second as opposed to just over a hundred thousand bits per second that can be transmitted through copper cable. Improvements in laser design have resulted in compact laser units whose amplified light beams can be switched on and off at incredible speeds. The far end of the fiber optic link contains a light-sensitive circuit. Light bursts transmit an on/off (binary) signal for the intricate video and control messages.

The surface vessel will use satellite communication technology to transmit live signals from the lake bottom to a network of participating museums and institutions. Students visiting the museums will be able to see and hear the live exploration of the underwater wrecks. The NSTA is developing a special curriculum to prepare participating students for the visit.

—Michael DiSpezio, Technology Writer, NSTA JASON Curriculum

For further information about the project write to: JASON Curriculum, National Science Teachers Association, 1742 Connecticut Avenue NW, Washington, DC 20009.