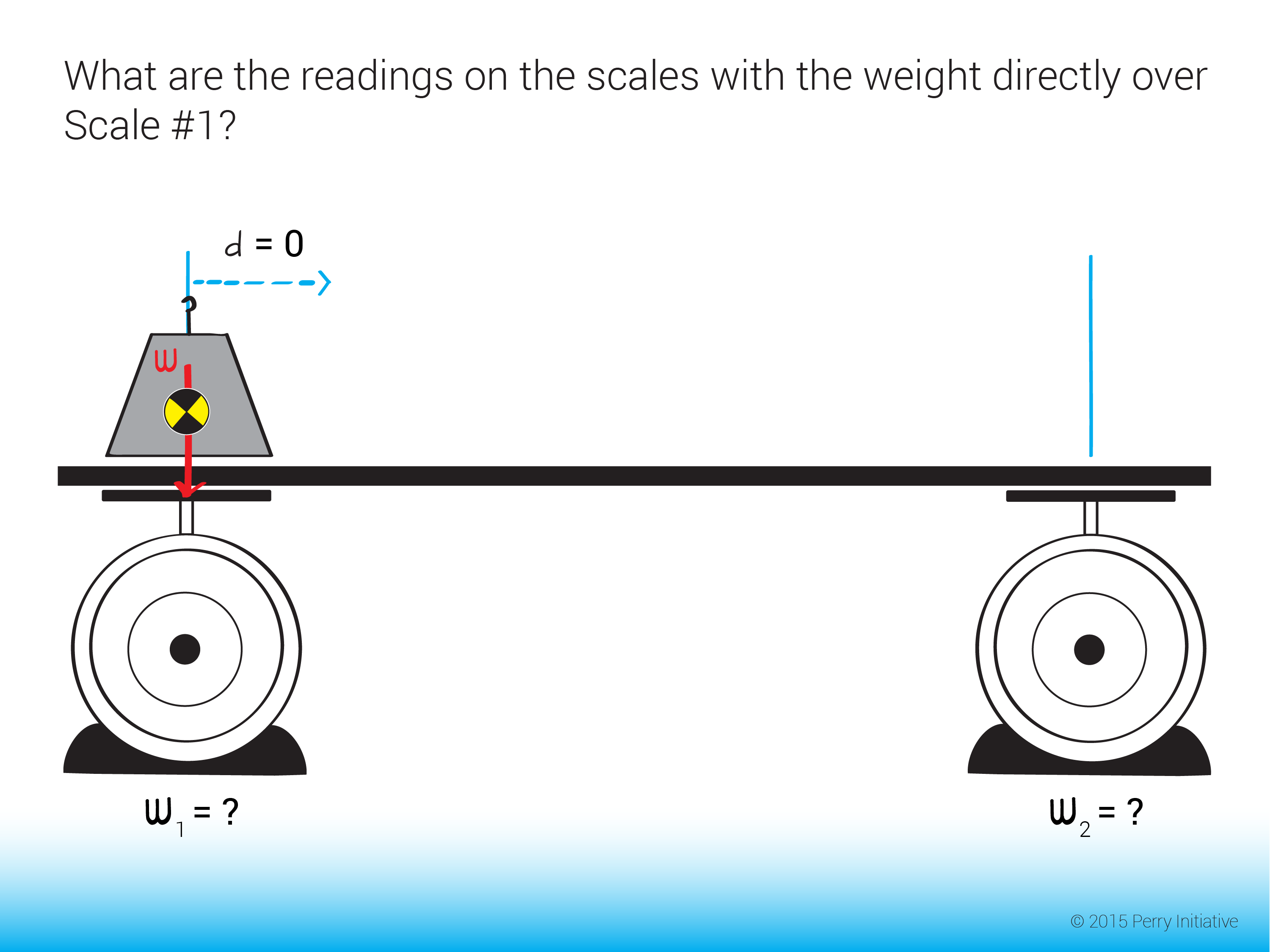
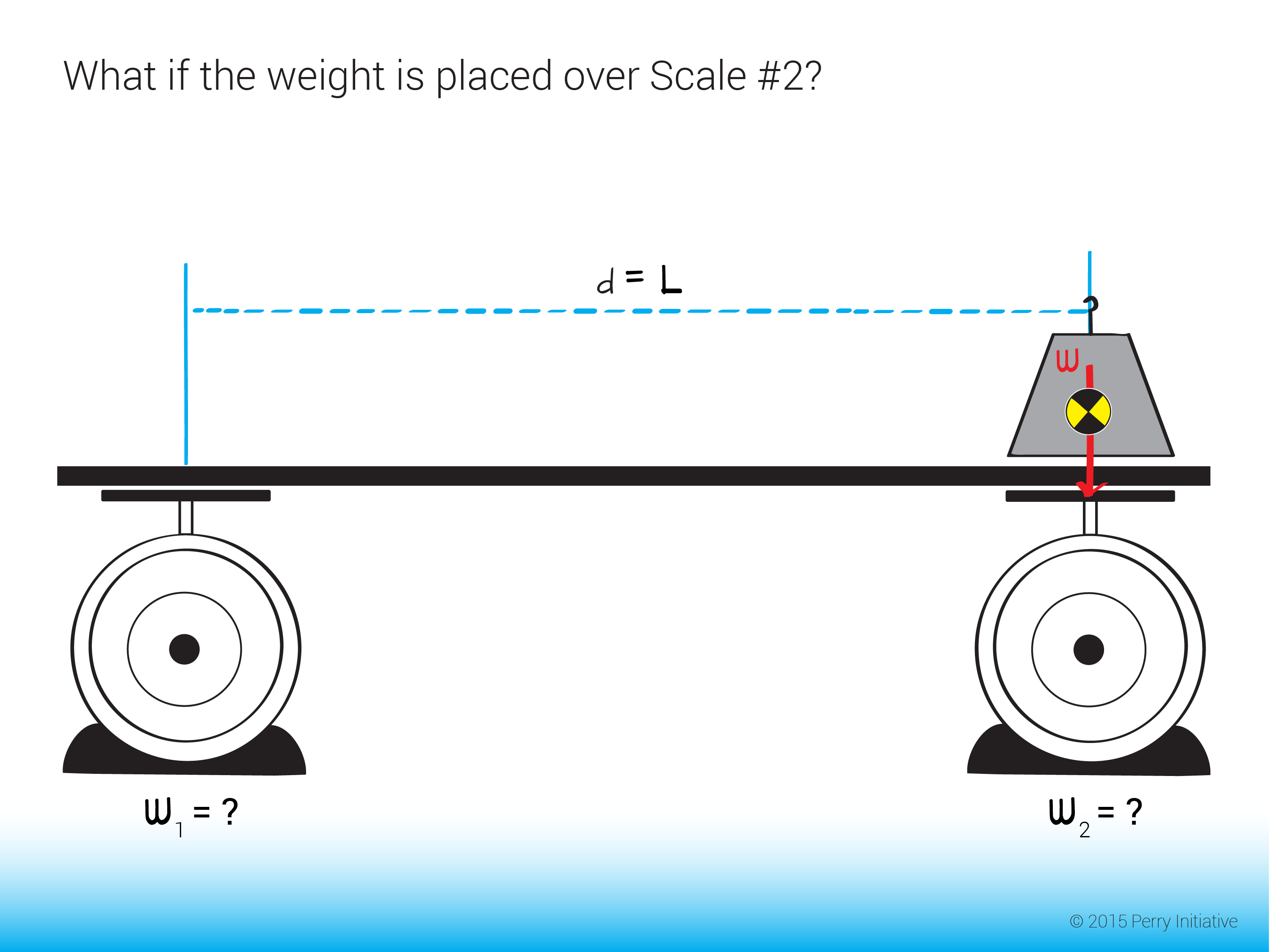
**Derivation of Balance Board Equation**

Method 1. Thought experiment using algebra basics

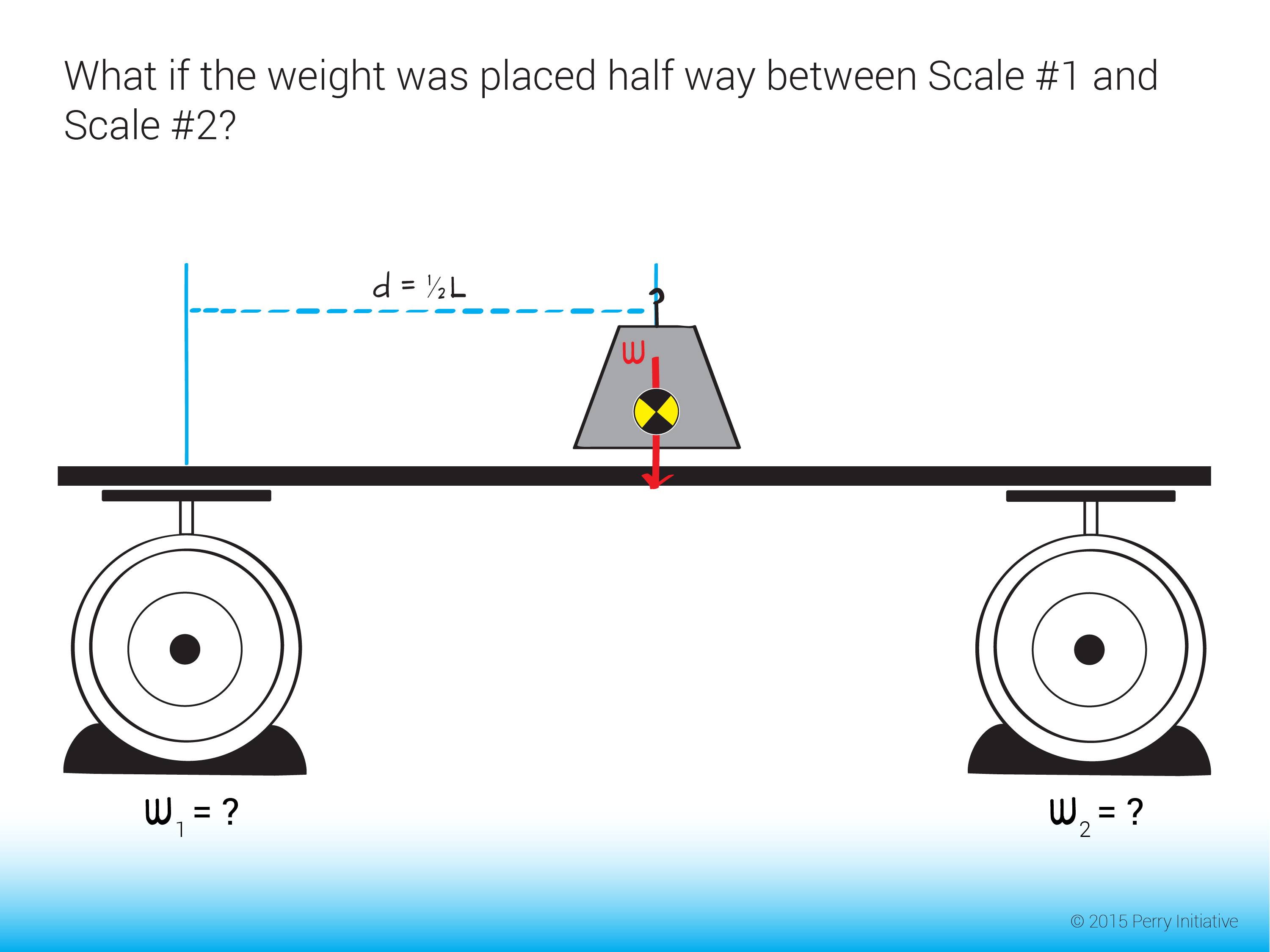
1. What proportion of the weight is directly over scale #1? Scale #2? (Ans.: Scale #1 = 100%, Scale #2 = 0) *Note: Teacher could mark measurement arm on each scale’s face*.



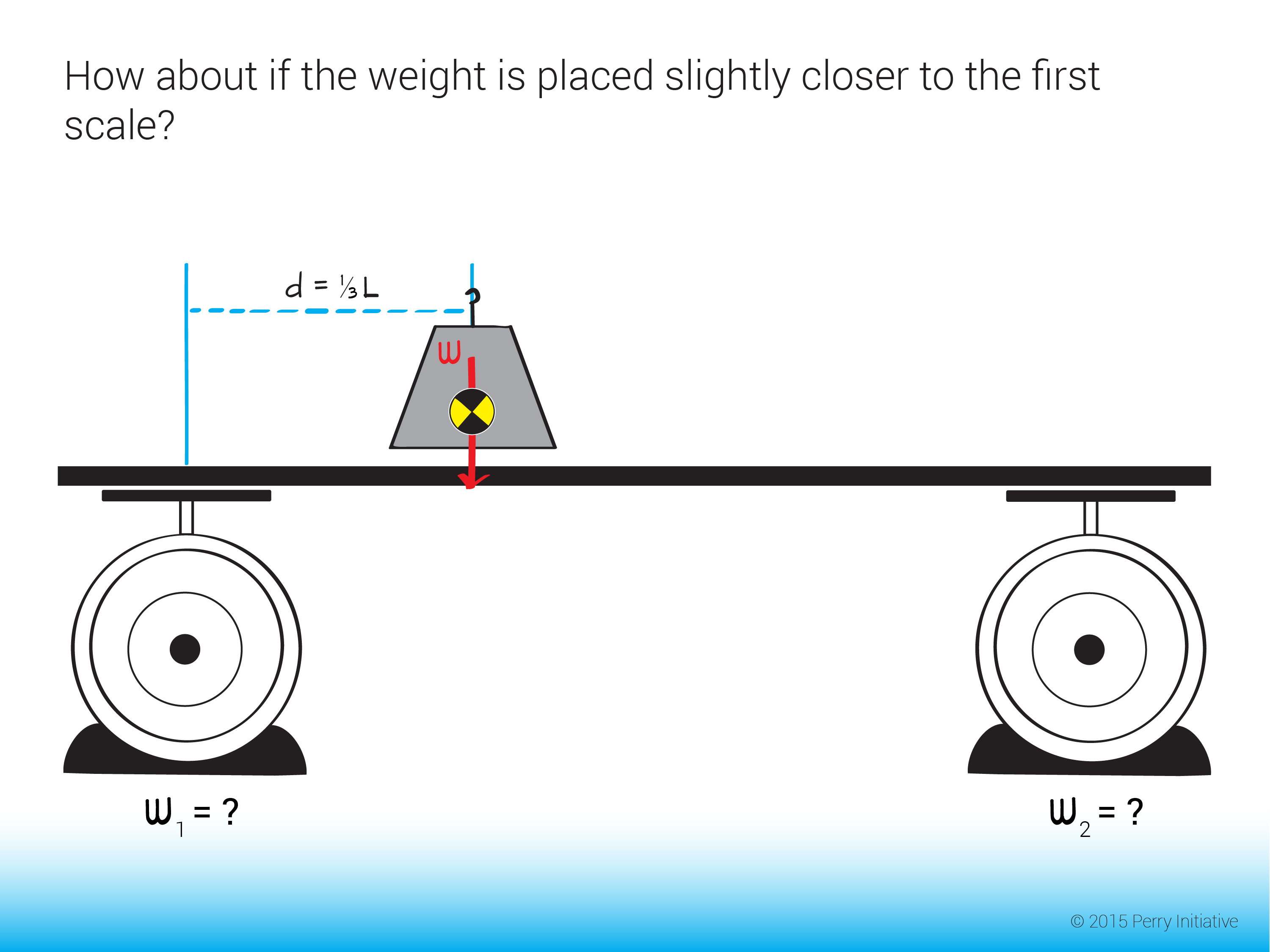
1. What if the weight is placed over scale #2? (Ans.: Scale #1 = 0, Scale #2 = 100%)\



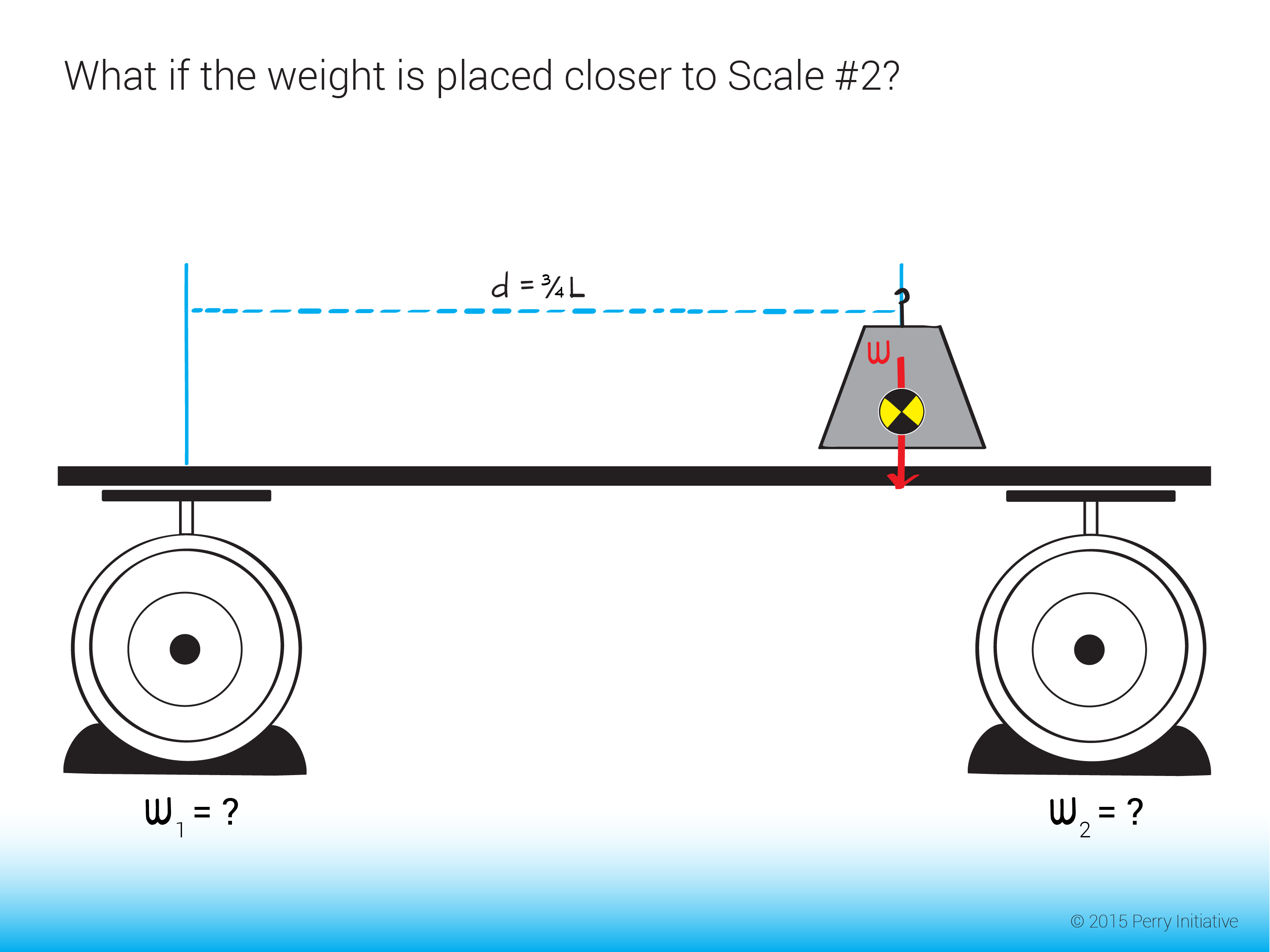
1. What if the weight is placed half way between scale #1 and scale #2? (Ans.: 50% on each scale)



1. How about if the weight is placed slightly closer to the first scale? (Ans.: 66% on scale #1 and 33% on scale #2)



1. How about if the weight is placer closer to scale #2? (Ans: 25% on scale #1, 75% on scale #2)



1. Using this information, we can derive a mathematical relationship among the variables in this system.

|  |  |  |
| --- | --- | --- |
| ***d*** | **W1** | **W2** |
| 0 | W | 0 |
| L | 0 | W |
| ½ L | ½ W | ½ W |
| 1/3 L | 2/3 W | 1/3 W |
| ¾ L | ¼ W | ¾ W |

Sum of the readings on the scale is equal to the total weight: W = W1 + W2

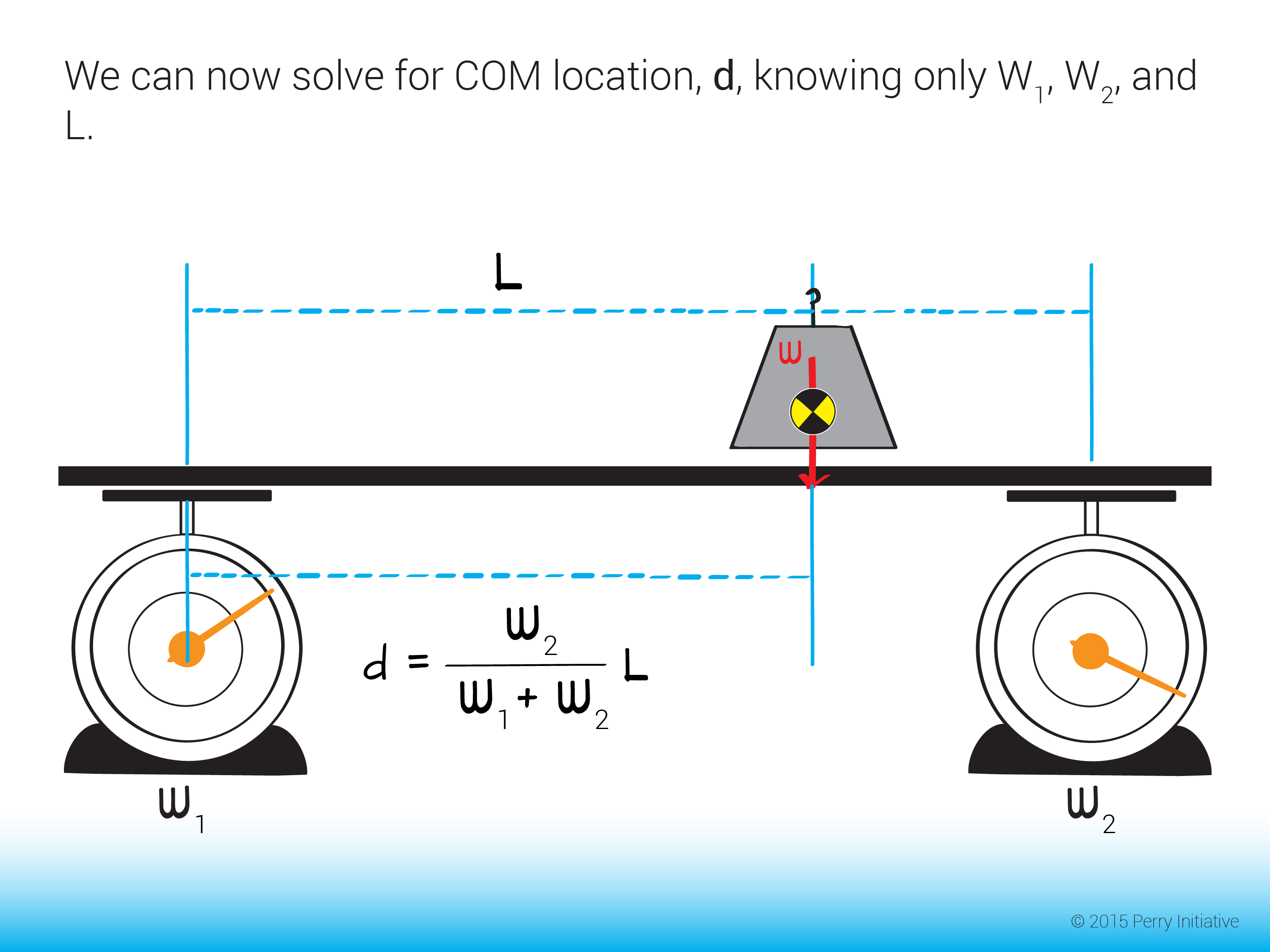
1. The reading on scale #2 is directly proportional to COM location: W2 = a**.**d

For example, ½ W = a . ½ L and ¾ W = a . ¾ L

So if a = W/ L, then W2 = (W/L) . *d*

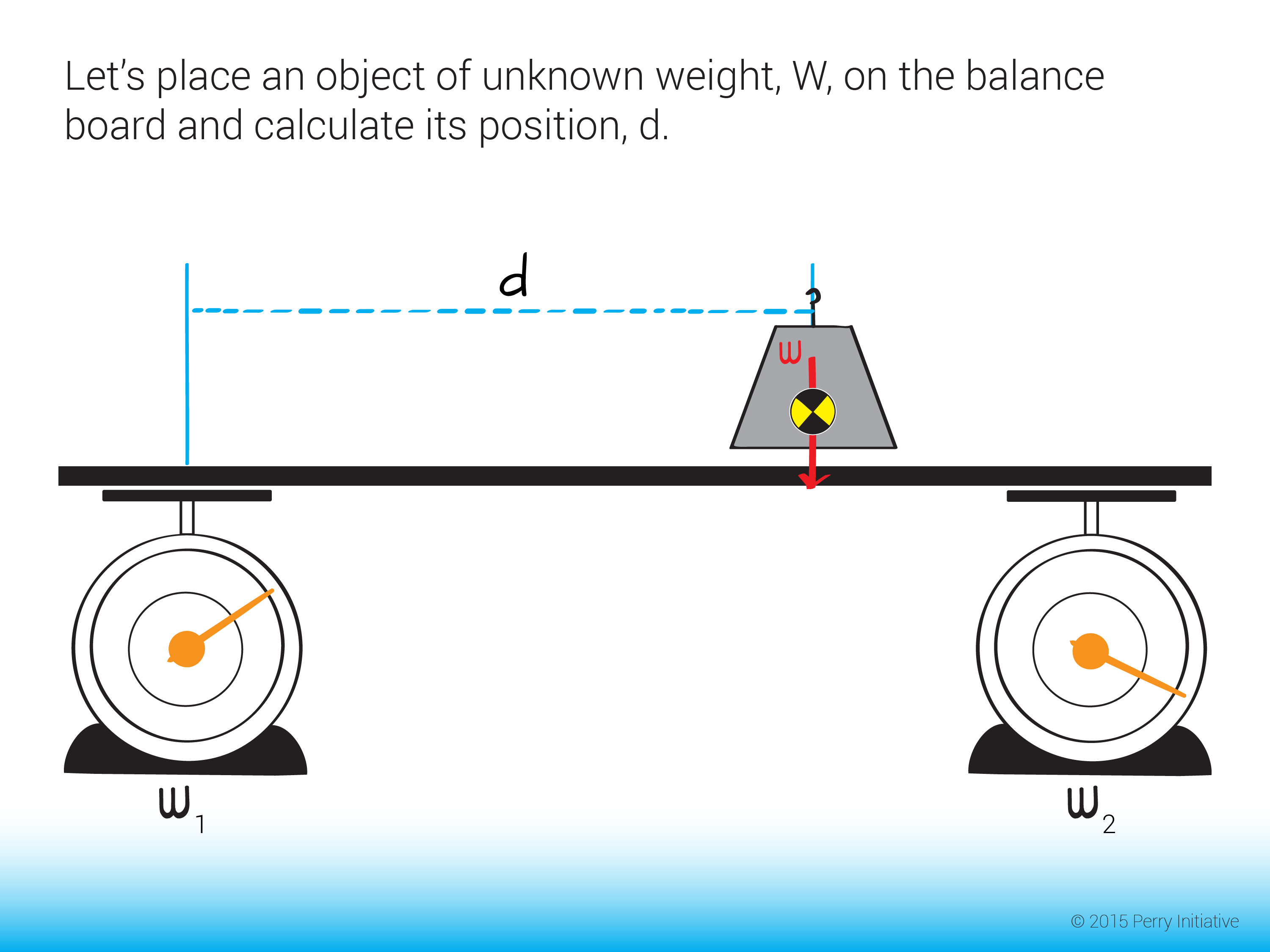
|  |  |  |
| --- | --- | --- |
| ***d*** | **W1** | **W2** |
| 0 | W | 0 |
| L | 0 | W |
| ½ L | ½ W | ½ W |
| 1/3 L | 2/3 W | 1/3 W |
| ¾ L | ¼ W | ¾ W |

1. Using the two equations, W = W1 + W2 and W2 = (W/L) . *d*, we can substitute the first into the second to get: W2 = (W1 + W2) . *d*/L and using algebraic reasoning to solve for d, we get:
2. Using this equation, we can solve for COM location, *d*, knowing only W1, W2, and L.

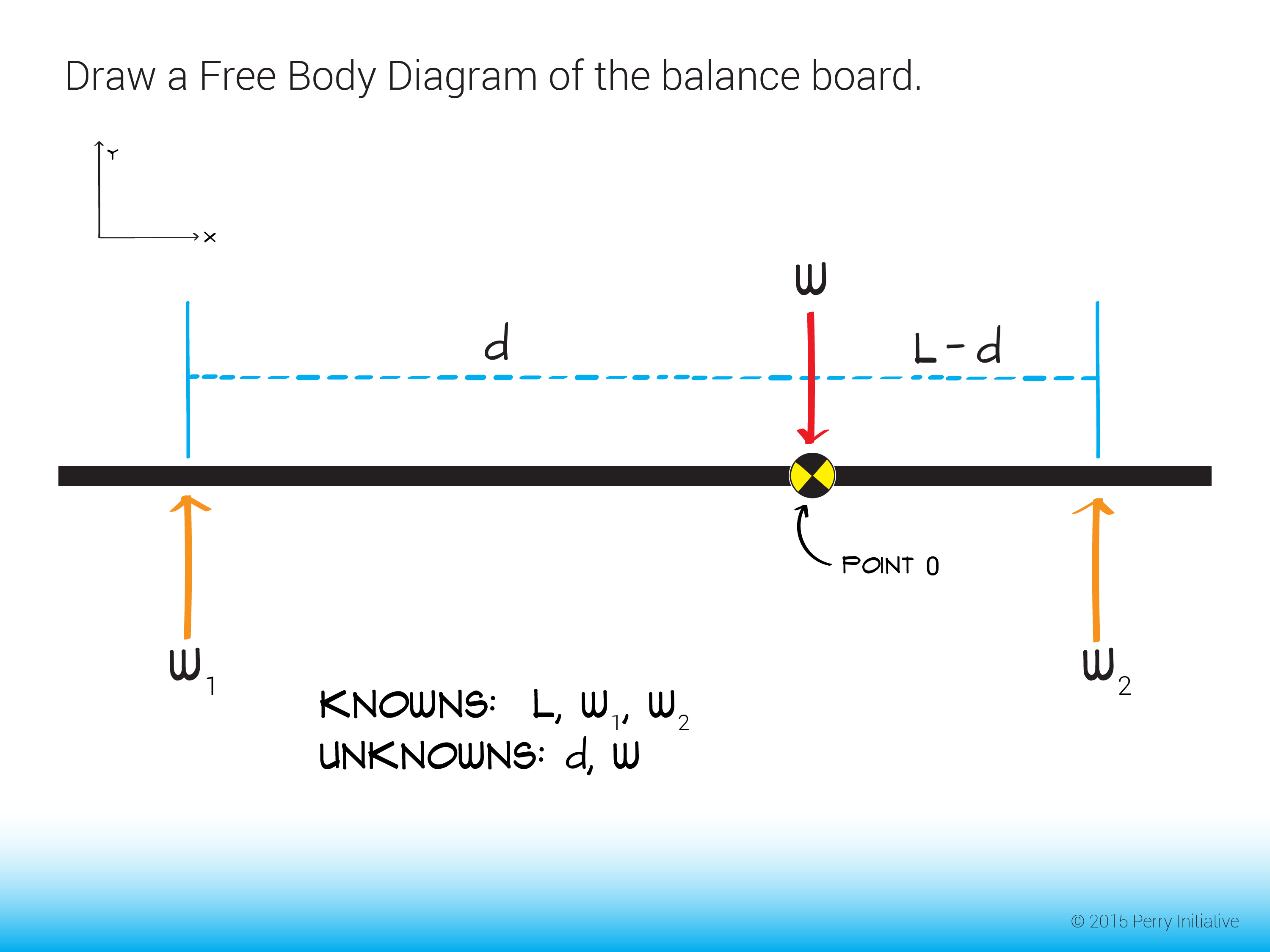


Method 2. Equation derivation

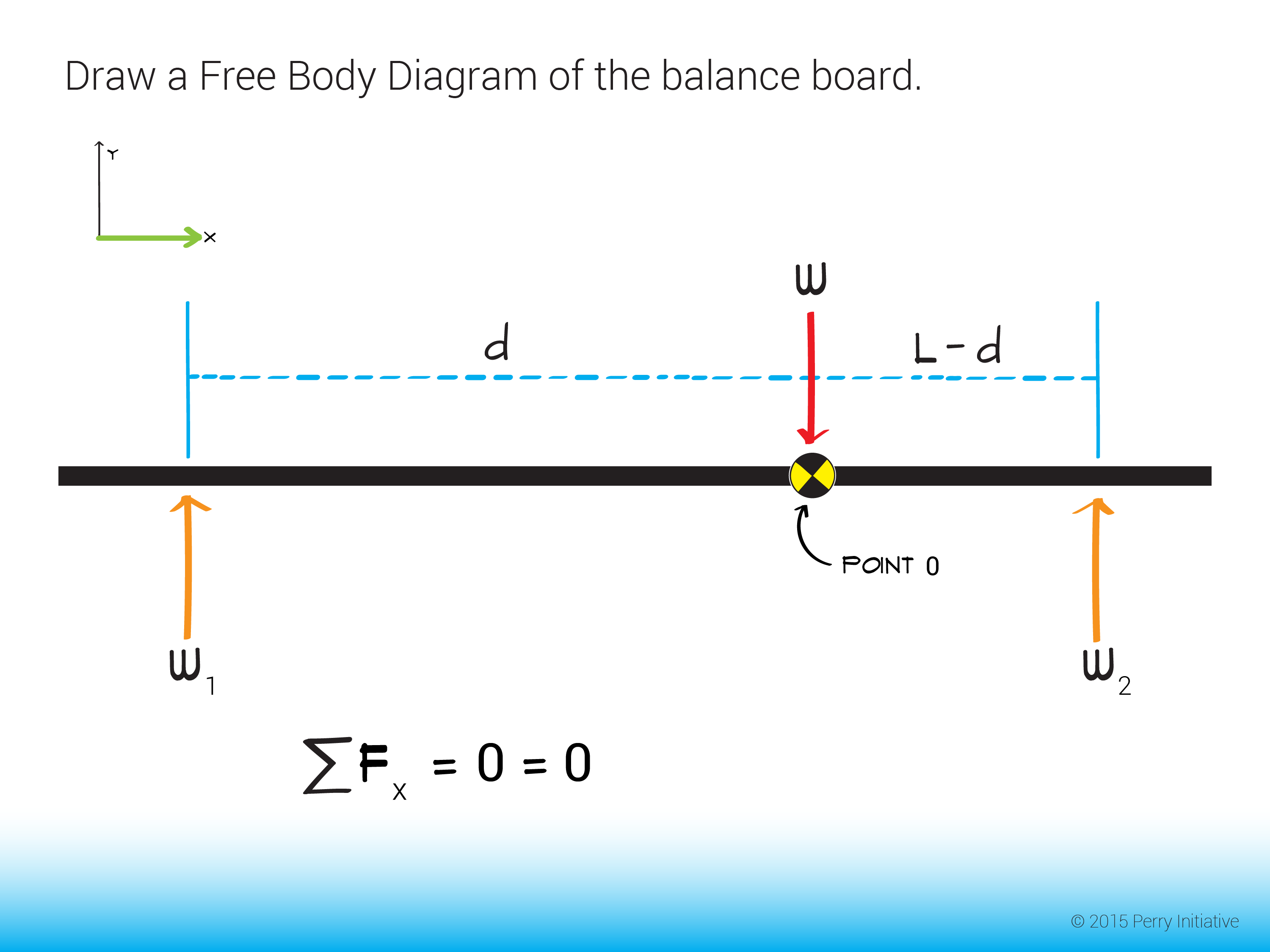
1. When an object of unknown weight, W, is placed on the balance board, we can measure its position, *d*.

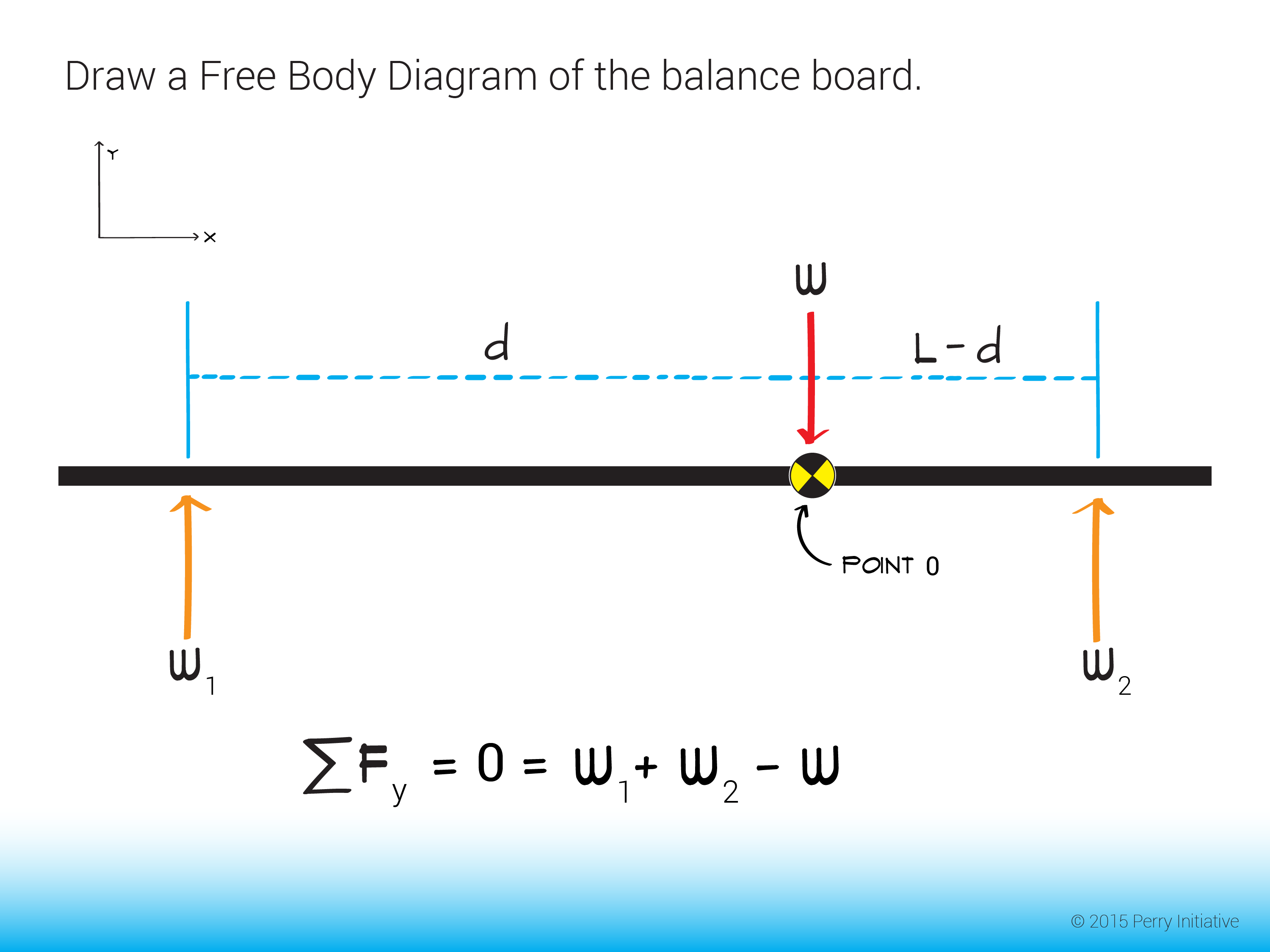


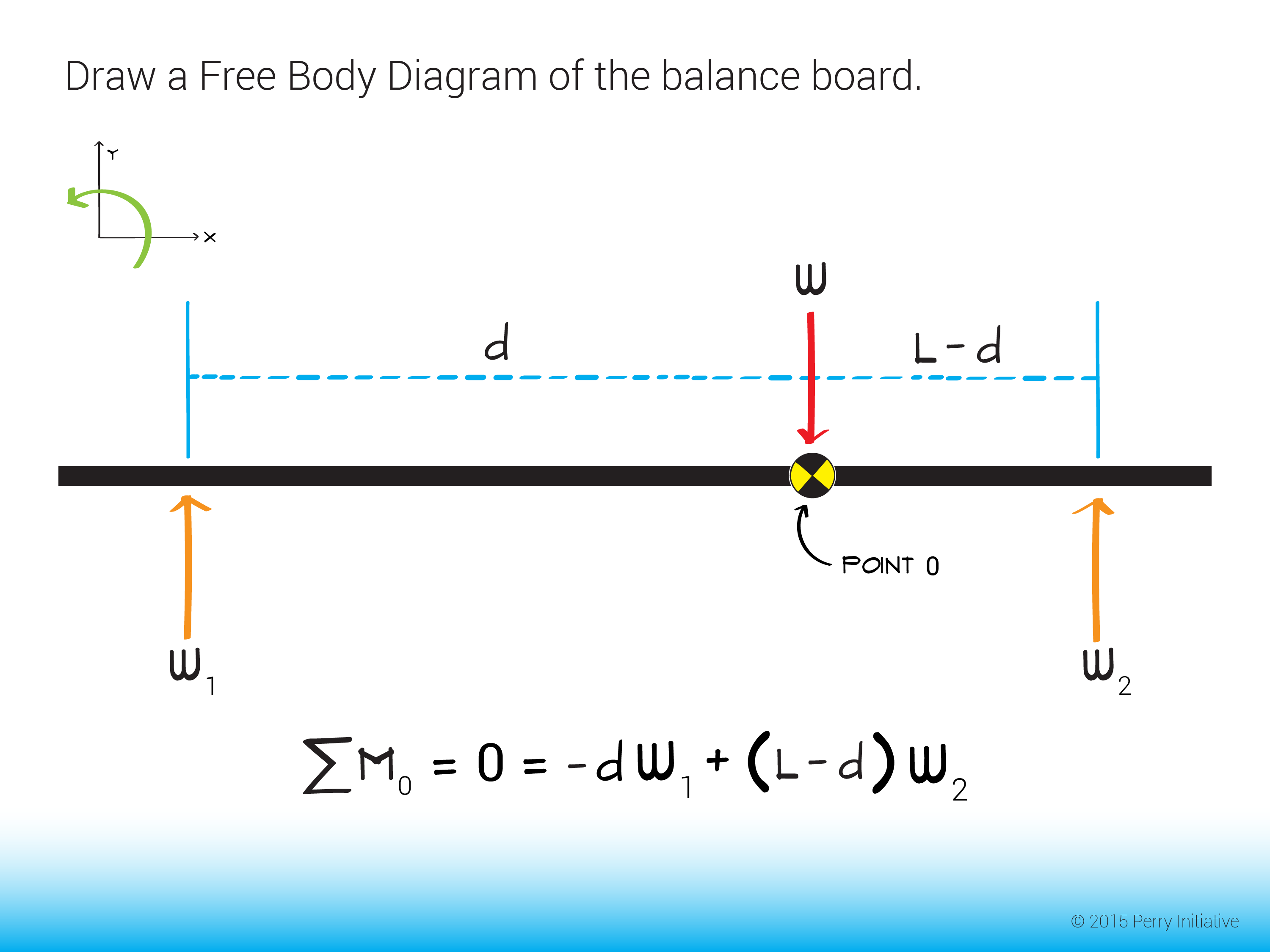
1. Draw a free body diagram of the balance board.



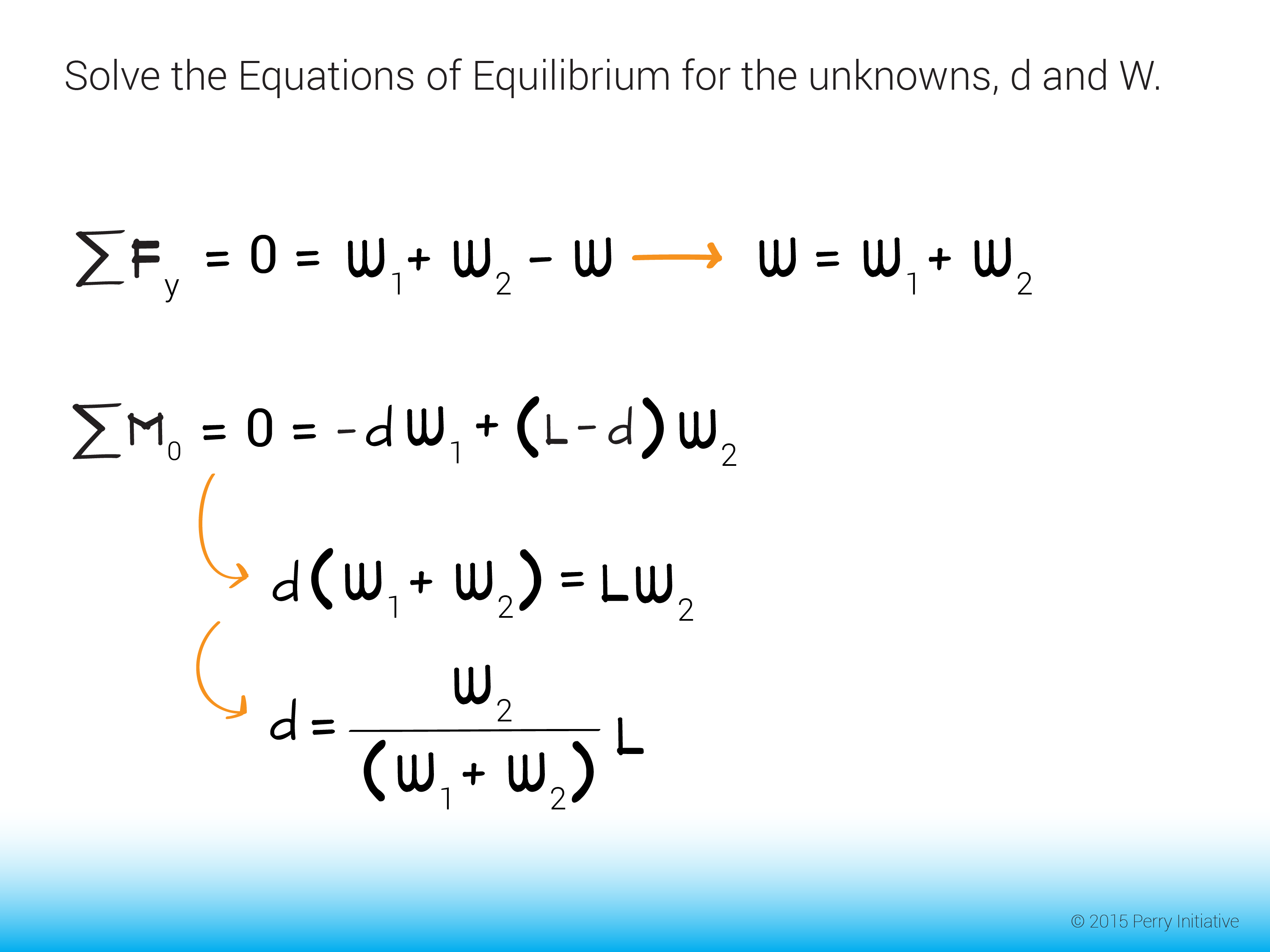
1. Write out equations of equilibrium.







1. Solve the equations of equilibrium for the unknowns, *d* and W.



1. Using this equation, we can solve for COM location, *d*, knowing only W1, W2, and L.

