Zombie Attack! An Introduction to Quantitative Modeling

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Outbreak!

It has happened, Zombies have appeared. They are beginning to take over large urban centers. In just a matter of weeks, several cities have been completely disabled. You are a disease researcher with the Centers for Disease Control & Prevention (CDC). Despite the CDC's Zombie Preparedness campaign, an effective strategy for dealing with the rapidly developing Zombie outbreak has still not been established. It is your responsibility to figure out how fast the number of Zombies will increase and whether or not human survival is possible. To do this, you develop a mathematical model to examine rates of spread and establish whether or not you can take action to slow or prevent the spread of Zombies.

The World Health Organization (WHO) believes the outbreak may be the result of a new parasitic fungal strain that has adapted to infect the brains of human hosts, similar to the fungus that attacks the brains of ants in the Brazilian rainforest (see below).

Zombie Ants

[The following is excerpted from the Wikipedia article "*Ophiocordyceps unilateralis*" accessed August 30, 2012 at http://en.wikipedia.org/wiki/Zombie_ants, CC BY-SA 3.0]

Ophiocordyceps unilateralis is a parasitoidal fungus that infects ants such as *Camponotus leonardi*, and alters their behavior in order to ensure the widespread distribution of its spores.

The species can be identified at the end of its lifecycle by its reproductive structure, consisting of a wiry yet pliant darkly pigmented stroma stalk extending from the back of the deceased ant's head.

The fungus's spores enter the body of the insect … where they begin to consume the non-vital soft tissues. Yeast stages of the fungus spread in the ant's body and presumably produce compounds that affect the ant's brain and change its behaviour by unknown mechanisms … .

The fungus then kills the ant … . When the fungus is ready to reproduce, its fruiting bodies grow from the ant's head and rupture, releasing the spores. This process takes 4 to 10 days.

The changes in the behavior of the infected ants are very specific, giving rise to the term zombie ants, and tuned for the benefit of the fungus. … According to David Hughes, "You can find whole graveyards with 20 or 30 ants in a square metre. Each time, they are on leaves that are a particular height off the ground and they have bitten into the main vein [of a leaf] before dying."

O. unilateralis has been known to destroy entire ant colonies. In response, ants have evolved the ability to sense that a member of the colony is infected; healthy ants will carry the dying one far away from the colony in order to avoid fungal spore exposure.

Disease researchers around the world have proposed several strategies for dealing with this outbreak. A few of the main strategies are outlined below:

- Assuming the parasitic-fungal-infection hypothesis is correct, release a broad-spectrum anti-fungal medication into the water supply, thereby decreasing the rate of new fungal infections occurring in humans.
- Develop a vaccine for the infection, so the currently healthy humans are no longer susceptible to the infection. It will take some time to develop an effective vaccine.
- Launch a wide-scale military attack with the goal of killing as many Zombies as possible. This may involve the complete decimation of populations in highly infected regions.
- Some combination of the above strategies.

Given the current rapid spread of the outbreak, you don't have the time needed to conduct the appropriate series of experiments required to determine the best approach, so a mathematical model seems to be the ideal way to evaluate the effectiveness of the different strategies outlined above. To develop the model, you need to be familiar with some basic Zombie natural history.

Zombie Natural History

- Zombies eat the living flesh of humans.
- They have minimal brain function but are able to hear and smell to locate food sources.
- They don't die unless killed by removing the head or destroying the brain.
- Humans can be "infected" by close contact with an infected Zombie (i.e., a bite or scratch). Once a human is infected, it can subsequently go through the process of zombification, passing from undead human into full-blown Zombie.

Your Challenge

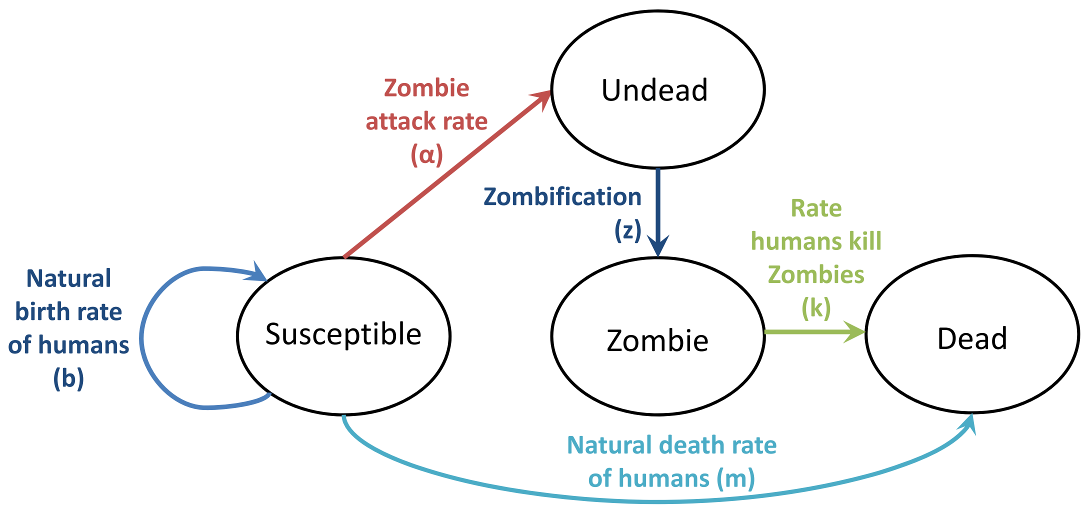
1. Develop a model of the human-zombie interaction.
2. Analyze the model predictions to determine the best course of action for human survival.
3. Critically evaluate the model.

Part I – State Variables and Flow Diagram

When you are developing the Zombie model, you start by thinking about what "state" a human can be in, given this new Zombie-infested world. You decide that a "human" can be in one of four states:

1. *Susceptible*—currently un-infected, but susceptible to infection by a zombie.
2. *Undead*—infected by a scratch or bite of a zombie, but not yet infectious (i.e., capable of infecting) nor presenting full-blown zombie symptoms. This is a state of latency.
3. *Zombie*—Infected, infectious, with full symptoms; acute state.
4. *Dead*—this state can result either by the usual causes for humans, or, in the case of zombies, by being killed by an uninfected human.

Once you've established the different states that a "human" can be in, you examine the rates with which individuals move from one state to another. Often it is easiest to consider the transition rates from one state to another by drawing a flow diagram and connecting the different states with arrows from one state to another. A flow diagram is simply an illustration of the interconnections between state variables that provides a visualization of how each state variable influences each other. The flow diagram you have developed for the Zombie outbreak is shown below.



Here you've defined what each of the parameters means:

Parameter	Interpretation
b	The number of babies born to each individual per year
m	The probability of death for each individual per year
α	The probability that a human is infected in an encounter with a zombie
z	The probability that an undead human goes through the zombification process to become a zombie
k	The probability that a zombie is killed in an encounter with a human

Question 1: In your own words, summarize how individuals can move from one state to another and at what rate (or with what probability).

Part II – Developing the Mathematical Equations

Now that you have thought about the state variables (Susceptible, Undead, Zombie, and Dead) and the rates connecting the different state variables, you can quantitatively describe the system by writing down equations detailing the information in the flow diagram. You are interested in examining how each of these state variables will change in time. You can think of this as the change in your state variables (Δ(State Variable)) over a period of time. As this period of time gets smaller and smaller, this will produce an *instantaneous rate of change* represented as:

$$d(\text{State Variable})/dt$$

which is just a fancy way of writing the change in the state variable over an infinitesimally small period of time. When we think about the change in the state variable this way, we can model the change in the state variable as a function of the flow rates entering a circle in the flow diagram minus the flow rates leaving the circle (Δ(State Variable) = input rates – output rates). For example, the change in the number of Susceptibles (S) over time is just simply the birth rate of each susceptible multiplied by the number of susceptibles (bS), minus the death rate of each susceptible multiplied by the number of susceptibles (mS), minus the attack rate of the Zombies multiplied by the number of both Susceptibles and Zombies (αSZ):

$$dS/dt = bS - mS - \alpha SZ$$

So to summarize this equation in words, you could say that the change in the number of susceptibles is the rate new susceptibles are born minus the rate susceptibles die due to natural causes, minus the rate that susceptibles become Undead due to attack by Zombies.

The equations for the remaining states are shown below:

$$dZ/dt = zU - kSZ$$

$$dU/dt = \alpha SZ - zU$$

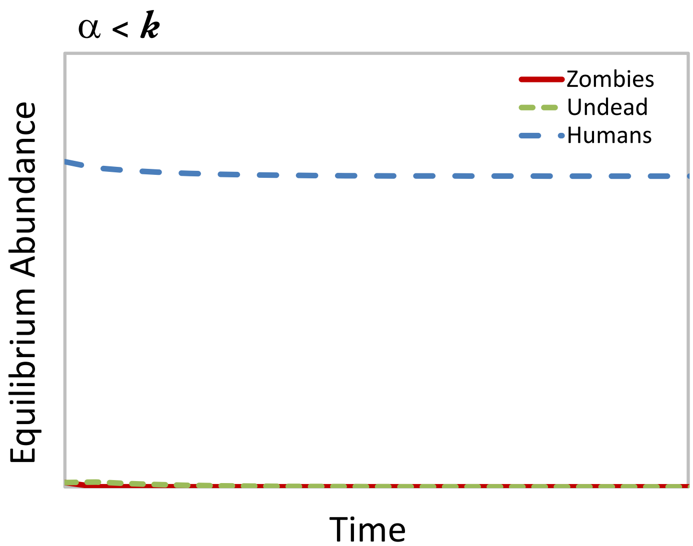
$$dD/dt = kSZ + mS$$

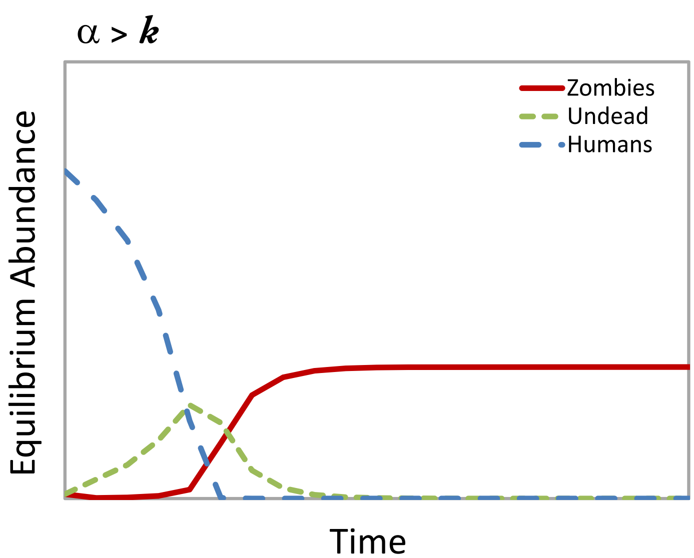
Question 2: Using the equation for the change in the numbers of susceptibles (S) over time (above) as a guide, describe in your own words what each of the equations represents.

Part III – Model Predictions

You have produced a model describing the Zombie outbreak! Now you are interested in understanding what the model predicts.

One interesting result from the model is shown below for two different values of α and k.





Question 3: It appears based on this model that these two rates in particular are important for determining whether humans can survive a Zombie outbreak. Based on the graphs above, describe the consequences of the relative values of k and α on the fate of humanity. What does this mean regarding a strategy for the survival of humans? Does this outcome make intuitive sense?

Part IV – Evaluating the Model

The terms in the model describing the attack of Zombies on humans (αSZ) and the rate at which humans kill Zombies (kSZ) are referred to as mass action terms (very common in predator-prey models and disease models). This means that the rate of interaction between two variables is directly proportional to the numbers of each variable. Consequently this also means that both the susceptibles and Zombie individual movements are random, both groups are homogeneously mixed, and that attacks and kills happen instantaneously upon contact (like particles bouncing around in a beaker).

Question 4: Given what we know about Zombies' ability to locate food sources, how might this way of modeling the Zombie attack rate be inaccurate? How might human behavior also influence this term? Hypothesize how incorporating the time it takes for a Zombie to feed on a human and the time it takes for a human to kill a Zombie would alter this term.

Question 5: What are some advantages of using a model to examine this problem? How might the model be overly simple? Can you suggest modifications to the model that might improve its ability to predict the outcome of a Zombie invasion?

Useful Websites/Further Reading

1. http://www.youtube.com/watch?v=XuKjBIBBAL8
   Planet Earth video of Zombie Ants.

2. http://www.livescience.com/13046-zombie-fungus-carpenter-ant-brain-altering.html
   More Zombie Ant video.

3. http://mysite.science.uottawa.ca/rsmith43/Zombies.pdf
   A published Zombie model.

4. http://www.cdc.gov/phpr/zombies.htm
   CDC's Zombie Preparedness Guidelines.

5. http://news.nationalgeographic.com/news/2011/03/pictures/110303-zombie-ants-fungus-new-species-fungi-bugs-science-brazil/
   National Geographic news release, detailing the zombie ant biology.

6. http://www.plosone.org/article/info%3Adoi%2F10.1371%2Fjournal.pone.0017024
   Hidden Diversity Behind the Zombie-Ant Fungus Ophiocordyceps unilateralis: Four New Species Described from Carpenter Ants in Minas Gerais, Brazil, PloS ONE

7. http://www.nature.com/scitable/knowledge/library/disease-ecology-15947677
   "Disease Ecology," by A. Marm Kilpatrick and Sonia Altizer, 2010, Nature Education Knowledge *2(12): 13.*

Otto, S.P. and T. Day. *A biologist's guide to mathematical modeling in ecology and evolution.* Princeton University Press. New Jersey, USA. (http://press.princeton.edu/chapters/s8458.pdf)

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Jackson, L., A.S.Trebetz, and K.L. Cottingham. 2000. An introduction to the practise of ecological modelling. *BioScience* 50: 694–706.

Credit: Licensed image in title block © Robert Kneschke | Fotolia.com #23728100. Photo of ant from Pontoppidan M-B, Himaman W, Hywel-Jones NL, Boomsma JJ, Hughes DP (2009), Graveyards on the Move: The Spatio-Temporal Distribution of Dead Ophiocordyceps-Infected Ants, *PLoS ONE* 4(3): e4835. doi:10.1371/journal.pone.0004835, CC BY 2.5.