Packing Pickles: Picking and Performing a Statistical Test

by
Christopher J. Cheng and Clinton Stevenson
Department of Food, Bioprocessing and Nutrition Sciences
North Carolina State University, Raleigh, NC

Part I – Pickle Problem

"If we don't fix this soon, this company won't last another year."

Jeff, a manager at the main pickle production plant at Perpetually Perfect Pickle Company, was meeting with his boss Pepper Pimpernell, the owner of the company.

"Based on the raw materials we are buying, we just aren't making enough pickles. A large amount of material is being wasted somewhere," continued Ms. Pimpernell

"I've audited the process several times, Ms. Pimpernell. Our waste streams aren't out of specification. In fact, our waste levels have actually been going down," said Jeff.

"I don't care what the problem is—just get it fixed. Now get out of here and get back to work," replied Ms. Pimpernell.

"Yes ma'am."

Jeff spent the next few hours auditing the pickle packing process with his intern Lauren. He was again unable to discover any variances in material usage that would cost the company this much money.

"Maybe this loss stemmed from discrepancies between what the line workers were recording as the fill weights of some jarred pickle products and what the actual weights were," chimed in Lauren as they were overseeing pickles punching process. It was here where workers would fill jars with pickles, pack more product in to meet weight specifications as needed, and record the weights of the jars that were being produced in the quality control records.

"What do you mean?" asked Jeff.

"Maybe the workers are filling their jars with more product than what their upper specification limit is and reporting them to be normal."

Jeff wondered whether their line workers responsible for sampling five jars of product every 15 minutes were not truthfully recording the weights of these jars if they were overweight. Maybe the workers were waiting to find a jar that fell within specifications before marking their weights on the quality control records. Or, maybe the line workers were flat-out lying when they recorded the weights of the jars of product. Whatever was happening, if Lauren's hunch was correct, it had implications for the net profit the company was making, as they would essentially be giving away free product.

"Lauren you're brilliant! Maybe our QC records are indeed wrong! I want you to go buy our pickle jars in the stores. Find out if they are truly heavier than our own recorded weights!" exclaimed Jeff.

Questions

1. Given the data below, are Lauren's records higher than the factory's records?

Average Sample Jar Weight in Ounces

	Quality Control Records	*Lauren's Records*
Number of Observations	24	24
Average	14.738	18.511
Standard Deviation	1.763	5.674

2. What hypothesis do you think Lauren should test?
3. How can she test this hypothesis? What kind of data does she need?

Part II – Calculations

Lauren decided to perform a hypothesis test to determine if the average weights of the jars from her data and the quality control records significantly differed at a 95% significance level.

Questions

1. What is the null hypothesis?

   U_0 = ____

2. What is the alternative hypothesis?

   U_a = ____

3. What is the sample mean and standard deviation?

   X_{bar} = __________________________

   S = __________________________

4. Calculate a test statistic. Which distribution should be used?

5. Is this a one tail test or a two tail test?
   a. one tail test
   b. two tail test

6. How many degrees of freedom exist in this problem?

   Df = __________________________

7. What is the critical *t* value? ($\alpha = 0.05$)

   t_{crit} = __________________________

Table of *t*-Distribution Critical Values

	Upper-tail probability *p*											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level *C*											

Part III – Determine Significance

Null Hypothesis: There is no significant difference between Lauren's records and the quality control records.

Alternative Hypothesis: There is significant difference between Lauren's records and the quality control records.

Using the hypotheses stated above, Lauren calculated the following test statistics:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(s_1^2/n_1 + s_2^2/n_2)}}$$

$t = -3.039$

$Df = n_1 - 1$ or $n_2 - 1$, whichever is less if $n_1 =/= n_2$

If $n_1 = n_2$ then $Df = n_1 + n_2 - 2$

$Df = 46$

tcrit:

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	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level *C*											

Question

1. What conclusion should Lauren draw?