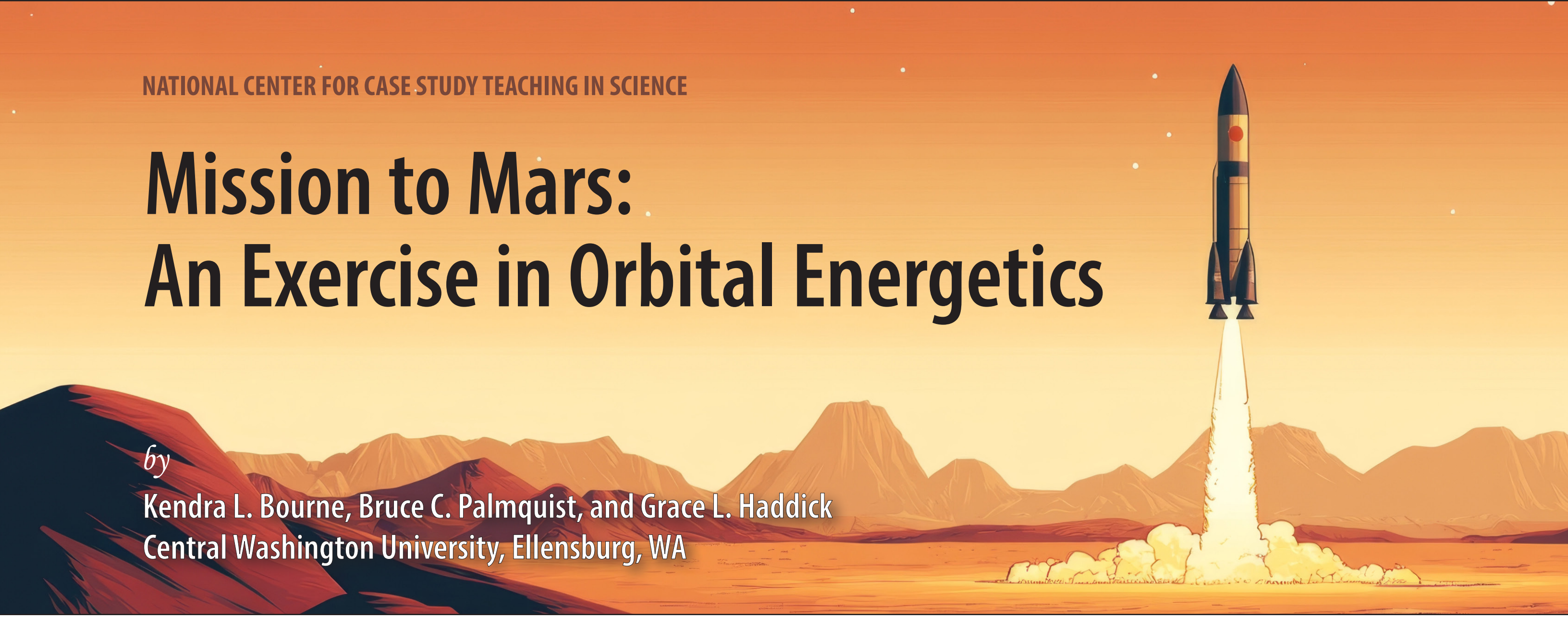

NATIONAL CENTER FOR CASE STUDY TEACHING IN SCIENCE

Mission to Mars: An Exercise in Orbital Energetics

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Introduction

Welcome, astronaut cadets! Congratulations are in order as you have all been selected to be the first group of humans to reach the surface of Mars. However, much needs to be done before that can be accomplished. As a part of your training at the academy, you will need to understand orbital energetics to ensure successful navigation of your 10,000 kg ship from Earth's orbit to Mars' orbit, and then from a ship's orbit around Mars to its surface. Key information on the prominent celestial bodies within the solar system (Table 1) as well as some relevant energy equations (Figure 1) are provided below for your convenience. As you work through the activity, be sure to review the two diagrams (Figures 2 and 3) on the following pages to demonstrate the special type of path, the Hohmann transfer orbit, that you will be taking for each of these tasks.

We are going to make a few assumptions as we work through this problem. First, because Earth's and Mars' orbits are very close to circular, we are going to treat them as circular. We will assume that Mars itself and the orbit that a spaceship travels around Mars are perfectly circular as well. Second, the size of the orbits are much larger than the planets we will be orbiting, so we can treat the planets as point masses.

Table 1. Data on celestial bodies.

Celestial Body	*Mass (kg)*	*Radius (m)*	*Orbital Radius (m)*
Sun	1.99×10^{30}	6.96×10^{8}	N/A
Mercury	3.30×10^{23}	2.44×10^{6}	5.79×10^{10}
Venus	4.87×10^{24}	6.05×10^{6}	1.08×10^{11}
Earth	5.97×10^{24}	6.37×10^{6}	1.50×10^{11}
Mars	6.42×10^{23}	3.39×10^{6}	2.28×10^{11}
Jupiter	1.90×10^{27}	6.99×10^{7}	7.78×10^{11}
Saturn	5.68×10^{26}	5.82×10^{7}	1.43×10^{12}
Uranus	8.68×10^{25}	2.54×10^{7}	2.87×10^{12}
Neptune	1.02×10^{26}	2.46×10^{7}	4.50×10^{12}
Pluto	1.31×10^{22}	1.15×10^{6}	5.91×10^{12}

Figure 1. Useful equations.

$$U_G = -\frac{GMm}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$K = \frac{1}{2}mv^2$$

$$K = -\frac{1}{2}U_G$$

$$E_{tot} = K + U_G$$

Part I – Switching from Earth's Orbit to Mars' Orbit

In order to transfer orbits, we need to know how much our energy must change between the two orbits (Figure 2). This change in energy is the amount of work our thrusters will need to do to change our orbit.

Below, calculate the amount of total mechanical energy your spaceship will have in Earth's orbit and the amount of total mechanical energy your spaceship will have in Mars' orbit, then answer the questions below.

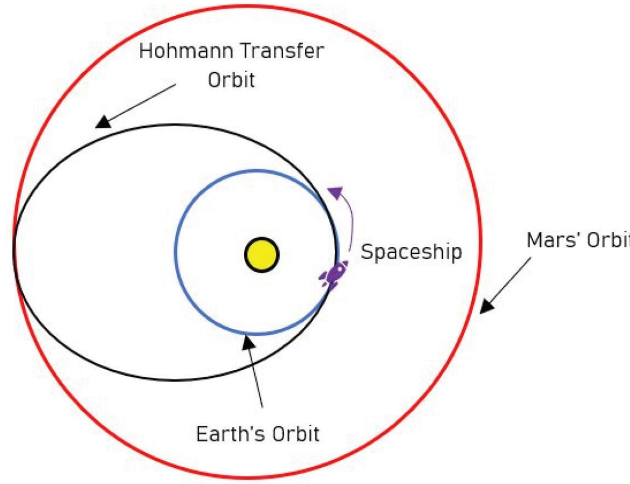

Figure 2. Hohmann transfer orbit.

Questions

1. How much work do our thrusters need to do to get us into Mars' orbit? Include at least three significant figures.

2. Is this an increase (positive work) or decrease (negative work) in total energy? How do you know?

In the URL given below, replace the "XX" with the first two digits from your answer to Question 1. Then type this URL into your browser and watch the video it links to: https://tinyurl.com/firsttwodigitsXX.

For example, if the first two digits from your answer to Question 1 are 8 and 6, you would type "https://tinyurl.com/firsttwodigits86." If your two digits are correct, you will be taken to a video about Mars. If your two digits are not correct, you will get an error message or be taken to a random video. Check your calculations, compare your work with a classmate, or ask your teacher for help so you get the correct digits.

Part II – Landing on the Surface of Mars

Congrats! You have made it to Mars' orbit, and now you are even orbiting around Mars itself (Figure 3). The final step is to make your way onto Mars' surface. At your current location, you are in geostationary orbit with Mars, so you are roughly 20,500 km above the Martian surface. For the purposes of this exercise, we will treat the surface of Mars as another "orbit."

Just as before, calculate the total mechanical energy your spaceship will have in your orbit around Mars and in the "orbit" of Mars' surface, then answer the questions below.

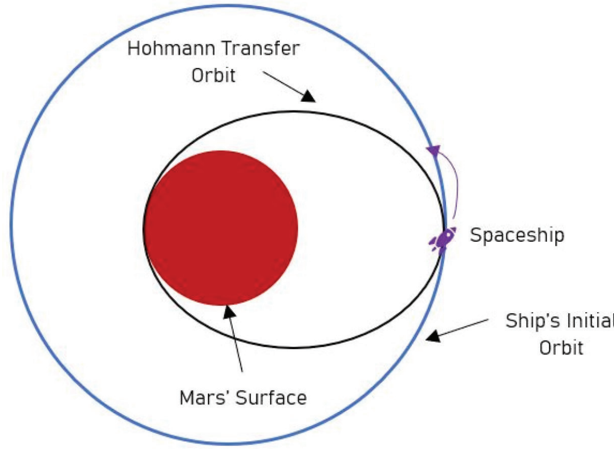

Figure 3. Orbiting Mars.

Questions

1. How much work do our thrusters need to do to get us to Mars' surface? Include at least three significant figures.

2. Is this an increase (positive work) or decrease (negative work) in total energy? How do you know?

Replace the XX at the end of the URL with the first two digits from your answer to question three. Type this URL into your browser and watch the video it links to: https://tinyurl.com/firsttwodigitsXX.

For example, if the first two digits from your answer to question three are 8 and 6, you'd type https://tinyurl.com/firsttwodigits86. If your two digits are correct, you will be taken to a video about Mars. If your two digits are not correct, you will get an error message or be taken to a random video. Check your calculations, compare your work with a classmate, or ask your teacher for help so you get the correct digits.

Part III – Reflection

Congratulations! You have successfully calculated the changes in energy required to transfer between orbits, therefore completing this phase of your training. In order to demonstrate your understanding of the Hohmann transfer orbit, please answer the following questions.

Questions

1. In which situation did we need to increase our energy to change orbits? In which situation did we need to decrease energy our energy to change orbits?

2. What conclusion can you draw about change in energy and the resulting movement between orbits?

3. How was energy increased or decreased in our system?

4. What did it mean if our thrusters did negative work on our spaceship? Explain this in terms of energy.

5. What conclusion can you draw about the relationship between total energy and work done on a system? When work is done on a system, energy is … ?