A Tale of Two Houses: A Case Study in Heat Transfer

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Part I—Thanksgiving Dinner

You are in your first semester as an engineering student at the University of North Dakota and Thanksgiving is fast approaching. Despite having serious reservations, you decide to appease your mother and attend Thanksgiving dinner at the house of your Uncle Bill. The deciding factor in your decision is the fact that your other uncle, Bubba, from Atlanta will also be there. You haven't seen him in a few years and know that things are never dull when he is around.

As you arrive, you immediately find Bill and Bubba in a heated discussion. As it turns out, Bill has been complaining about how horrible his heating bills will be during the upcoming winter. Not to be outdone in the pity-generating department, Bubba claims that things are far worse in the South, where keeping his house cool in the summer requires much more energy.

Since you have always been known as the "smart one" in the family, Bill and Bubba naturally turn to you to settle their argument. Put your vast engineering knowledge to good use and help them settle their debate.

Which house uses more energy: Bill's house in January or Bubba's house in August? Why do you think so?

Uncle Bubba's House
Atlanta, GA
2,100 square feet, 3 bedroom, 2 bath
2 story, aluminum siding, poorly insulated
Built 1997, cedar deck, green asphalt shingles

Uncle Bill's House
Walhalla, ND
4,000 square feet, 4 bedroom, 3 bath
2 story with half-basement, wood siding
Built 1925, renovated 2005, well-insulated, 3 fireplaces

As Uncle Bubba and Uncle Bill continue to argue, you slip away to another (quieter) room. Pulling out a pad of paper, you begin to think about what factors will have an influence on how much energy is needed to heat or cool each house.

List as many factors as you can think of that will affect a house's heating/cooling needs.

Part II—Acting Like an Engineer

After a little thought, you are able to narrow down your list to four important factors that influence the rate of heat transfer: wall thickness, wall surface area, thermal conductivity of the wall, and the difference in temperature between the inside and outside of the house. As an aspiring engineer, you like to think in mathematical terms and decide to give variable names to each of these factors. Pulling out your pad of paper again, you write out the following variable definitions:

- Q = rate of heat transfer
- x = wall thickness
- A = wall surface area
- ΔT = inside-outside temperature difference
- k = thermal conductivity

You know these factors are important, but aren't completely sure what effect each of them will have or how they are interrelated.

What will happen to the rate of heat transfer?

If…	***Heat transfer will…***		
Difference between inside and outside temperatures is increased?	increase	decrease	no change
Wall thickness is increased?	increase	decrease	no change
Wall area is increased?	increase	decrease	no change
Thermal conductivity is increased?	increase	decrease	no change

Which mathematical equation describes the relationship between…?

1. Q and x	$Q = mx+b$	$Q = mx$	$Q = mx^2$	$Q = m/x$
2. Q and A	$Q = mA+b$	$Q = mA$	$Q = mA^2$	$Q = m/A$
3. Q and ΔT	$Q = m\Delta T+b$	$Q = m\Delta T$	$Q = m\Delta T^2$	$Q = m/\Delta T$
4. Q and k	$Q = mk+b$	$Q = mk$	$Q = mk^2$	$Q = m/k$

Part III—I Remember Algebra (Don't I?)

As you look at all of your equations, you think back to algebra class and remember how one equation with many variables could be reduced to an equation with only two variables by holding everything else constant.

For example, the equation: $Y = X Z^2$ can become

$Y = a X$ (when Z is held constant)

or

$Y = b Z^2$ (when X is held constant).

You figure you can do the same thing in reverse and combine the four equations into one equation that describes how all those variables are interrelated.

Write a single equation that defines how Q varies with x, A, ΔT, and k.

Part IV—Finding an Answer

After much thought and considerable scribbling, you come up with a single equation that describes how these factors are interrelated:

$$Q = \frac{kA\Delta T}{x}$$

As you look at the equation, it seems vaguely familiar. Suddenly remembering where you've seen it before, you jump up to go find it.

"Hey! Have you figured out…?" yells Uncle Bubba.

"I've almost got an answer," you reply, cutting him off as you run upstairs.

Quickly closing the bedroom door, before Bubba and Bill can follow, you begin looking for the backpack with your school books. You pull out a copy of McCabe, Smith, and Harriot and flip through it until you find the section on heat transfer and discover that, not only is your equation in the book, but it is famous enough to have a name: Fourier's Law.

Confident that you are on the right track, you decide you are ready to do some calculations. You look up values for thermal conductivities in the textbook, use the house dimensions to estimate the surface area of each house, and make some reasonable estimates for wall thickness and temperatures. Soon you have an answer and return downstairs to talk to your uncles.

Estimate the missing values and calculate the heat lost/gained from each house.

Uncle Bubba's House	***Uncle Bill's House***
Area (A): ______ ft^2	Area (A): ______ ft^2
Effective Conductivity (k): 50 BTU in/(ft^2 °F day)	Effective Conductivity (k): 15 BTU in/(ft^2 °F day)
Temperature Difference (ΔT): ______ °F	Temperature Difference (ΔT): ______ °F
Thickness (x): ______ inches	Thickness (x): ______ inches
Q : ______________ BTU/day	Q : ______________ BTU/day