Chuck A. Luck Wagers a Buck: Probabilistic Reasoning and the Gambler's Ruin

by
Christopher M. Rump
Department of Industrial Engineering, University at Buffalo, State University of New York

Part I – What Are the Odds?

Consider the following question posed by Daniel Reisman of Niverville, New York, to Marilyn vos Savant in her column *Ask Marilyn* (vos Savant 1999):

> *At a monthly "casino night," there is a game called Chuck-a-Luck: Three dice are rolled in a wire cage. You place a bet on any number from 1 to 6. If any one of the three dice comes up with your number, you win the amount of your bet. (You also get your original stake back.) If more than one die comes up with your number, you win the amount of your bet for each match. For example, if you had a $1 bet on number 5, and each of the three dice came up with 5, you would win $3.*
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> *It appears that the odds of winning are 1 in 6 for each of the three dice, for a total of 3 out of 6—or 50%. Adding the possibility of having more than one die come up with your number, the odds would seem to be slightly in the gambler's favor. What are the odds of winning at this game? I can't believe that a casino game would favor the gambler.*

Although Marilyn didn't answer the question directly, she provided the following response:

> *Chuck-a-Luck has a subtle trick that many people don't recognize unless they analyze it. If the three dice always showed different numbers, the game would favor no one. To illustrate, say that each of six people bets $1 on a different number. If the three dice showed different numbers, the operator would take in $3 from the losers and pay out that same $3 to the winners. But often, the three dice will show a doublet or triplet: two or three of the dice will show the same number. That's when the operator makes his money.*
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> *For example, say the dice show 3, 3 and 5. The operator collects $4 ($1 each from the people who bet on 1, 2, 4, and 6) but pays out only $3 ($2 to the person who bet on 3, and $1 to the person who bet on 5). Or say the dice show 4, 4, and 4. The operator collects $5 ($1 each from the people who bet on 1, 2, 3, 5, and 6) but still pays out only $3 (to the person who bet on 4).*
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> *The collections and payoffs change according to the bets (sometimes the house wins, sometimes the gamblers win), but with this game, you can still expect to lose about 8 cents with every $1 you bet over the long run.*

Questions

1. Can you answer Daniel's question: "What are the odds of a bettor winning at this game?"
2. What is wrong with Daniel's analysis? Can you verify Marilyn's claim that the bettor will lose, on average, about 8 cents for every $1 bet?
3. Following Marilyn's reasoning, suppose six people each bet $1 on a different number. What are the operator's expected winnings from these players? Can you use this to verify Marilyn's claim?
4. Imagine a rules change where doubles pay x:1 rather than 2:1 and triples pay y:1 rather than 3:1. Derive an equation in terms of the unknowns x and y that makes this modified Chuck-a-Luck a fair game. Give some specific values for x and y that satisfy the equation and are easy to remember and implement.
5. Grab three dice and play repeated rounds (at least 30) of Chuck-A-Luck with a friend, "wagering" $1 on each play. Have your friend record your data as you play, and then switch roles. Develop a statistical point estimate for your average winnings. What would you expect to happen if you played more rounds?

Part II - Going for Broke

Chuck A. Luck has brought $2 to the casino. He will play Chuck-a-Luck (wagering $1 on each play) until he has $4 or until he goes broke, whichever comes first.

Questions

1. Formulate a discrete-time Markov chain for this gambler's ruin problem by defining an appropriate state space. Write the transition probability matrix for this chain. Does the Markov assumption hold in this case? Explain.
2. What is the probability that Chuck goes home broke?
3. On average, how many chucks does Chuck chuck, i.e., how long does the game last?
4. Suppose Chuck considers a strategy of wagering $2 instead of $1 on each play. How would these answers change? Which strategy should Chuck employ if he wants to minimize his probability of going broke? If he wants to maximize the number of plays? Explain.
5. Play repeated rounds of this Chuck-a-Luck game against a friend. Choose one player to act as the "chucker" and one person to play as the "house." Both the "chucker" and the "house" should start with $2 each. Here, each game consists of either you or your friend going broke. Be sure to keep track of how long each game lasts and who won. Do your "simulated" results match the expected theoretical results?