NATIONAL CENTER FOR CASE STUDY TEACHING IN SCIENCE

One Bad Apple:
Designing Sampling Plans for Better Food Quality

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ValleyTree Orchards is a brand name that a large beverage company uses for their fruit juices. They have been working on improving the taste of their apple cider. Customers keep complaining that the taste of the cider is variable: one bottle tastes great, the next bottle they buy tastes sour or too sweet or stale. The research and development team can produce a bench-scale product that has the same flavor from batch to batch. So can the pilot-scale production team. The problem seems to be with the full-scale production line.

The quality assurance (QA) team has reviewed the quality control data of products tested before the product is shipped and did not see anything unusual. Brix (amount of sugar), viscosity, and microbial counts are all within acceptable ranges for all samples that have been shipped out. One quality control technician mentions that some of the samples they test tasted a little different, but she admits that she's been tasting cider for years and might be more sensitive than the average consumer. She didn't notice any sort of pattern to the samples that taste different; they don't show up on particular days and don't seem to be correlated to any of the other quality control (QC) data collected.

The QA team has also gone over the process with several engineers and can't find anything that would cause variations in flavor. Cleaning and maintenance are performed as scheduled. Having failed to find any root causes in the process or the QC data, a frustrated member of the QA team, looking for any possible explanation to what has been happening, takes a look at the raw ingredients coming in. He watches a shipment of apples being received. The apples are shipped in large crates, about a meter high, and a meter and a half wide and long. As each crate comes in, they are checked for quality. The QA team member watches as a worker collects about two dozen apples from the side of a crate that is closest to the worker. She takes them over to a nearby table, checks each one quickly, then puts the apples back in the crate.

Curious, the QA team member walks over to the worker and asks her about the receiving process. She explains that each crate needs to be checked before being accepted. Depending on the size of the apple, each crate will contain 4500–7500 apples. She's been told by her boss that too many bruised, punctured, broken, or wormy apples means that the crate is bad. Her boss says that if more than half of a couple dozen apples are flawed in this manner, the crate shouldn't be accepted. So she's been looking at two dozen apples per crate and rejecting crates with more than a dozen bad apples in that set of two dozen. She also says that she doesn't really look at the apples as she picks them up, since she'd be tempted to pick up only the good apples.

The QA team member asks how long she's been working. She's been at the company for several months. However, the customer complaints have only been showing up for the past 6 weeks. The QA team member may have found another dead end. Or maybe not.

Questions

1. Apples are normally inspected before processing, and damaged or wormy apples are removed. What might happen if defective apples enter the process? What should be done with the defective apples? Why is it a good idea to reject shipments of apples that have a high number of defective apples?

2. Do you see anything wrong with the way the worker is checking the incoming crates of apples? Explain your answer.

3. The QA team would like to try a new sampling plan. Under this plan, 150 apples per crate will be checked. If more than 50 of these have bruises, punctures, or breaks, the crate is rejected. Create an operating characteristic (OC) curve based on this plan.* Use at least seven points in your curve, so that the general shape of the whole curve can be seen.

4. Several members of the QA team are worried that the sampling plan is too strict. They want to try a double acceptance sampling plan. This plan uses the same criteria as the plan in Question 2, except that the sample will be accepted if no more than 50 apples have bruises, punctures, or breaks and the sample will be rejected if the sample has more than 55 apples with bruises, punctures, or breaks. If the sample needs to be tested again, 20 more apples will be tested. If the total number of apples with defects from both the first and second samples is greater than 55, the lot is rejected. Otherwise, it is accepted. Create an OC curve based on this plan.†

5. If constructed correctly, the probability of acceptance on both the single and double acceptance sampling plans is equal to 1.00 until the percent nonconforming in the lot is well over 20%. Why do the curves look like this? Why does the probability of acceptance not decrease sooner? Is this good for the manufacturer? Explain your answer.

6. Which sampling plan do you think is better to use, the single acceptance or the double acceptance? Or would you choose a different sampling plan? Explain your answer.

7. Joe is using the new double acceptance sampling plan to test a crate of apples. The first sample of 150 apples has 52 nonconforming apples, so Joe takes another sample of 20 apples. This sample has three nonconforming apples. Joe rejects the crate. Did Joe make the correct decision based on the sample results? Why or why not?

8. Penny is using the new double acceptance sampling plan to test a crate of apples. The first sample of 150 apples has 49 nonconforming apples, so Penny takes another sample of 20 apples. This sample has six nonconforming apples. Penny accepts the crate. Did Penny make the correct decision based on the sample results? Why or why not?

9. Kim is using the new double acceptance sampling plan to test a crate of apples. The first sample of 150 apples has 53 nonconforming apples, so Kim takes another sample of 20 apples. This sample has zero nonconforming apples. Kim accepts the crate. Did Kim make the right decision based on the sample results? Why or why not?

10. If the company tightened their quality standards and wanted no more than 30 nonconforming apples in a sample of 150, what would happen to the shape of the OC curve?

11. If QA decided that 150 apples was too large a sample and decreased the sample size, but still allowed 50 nonconforming apples in a sample to be acceptable, what would happen to the shape of the OC curve?

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* See Hint 1 on next page.
† See Hint 2 on next page.

Hint 1: Note that you will probably not find tables to help you out with the sampling as stated. You can use the following function in Excel to help you:

=BINOMDIST([c],[n],[p_o],TRUE)

What this formula does is use the binomial distribution to calculate the probability of accepting a lot at a given fraction defective of a population. Here's how to put the formula into Excel:

- Replace [c] with the cell that contains the acceptance criteria number.
- Replace [n] with the cell that contains the sample size.
- Replace [p_o] with the cell that contains the fraction nonconforming in the population.
- Leave TRUE alone, as it means that the function will calculate the cumulative probability (which is what you want).

In terms of what this might look like, see Figure 1.

If we wanted to calculate the probability of acceptance for the first row (100*p_o=0.1), the formula would be:

=BINOMDIST(C17,C18,(A3/100),TRUE)

Notice that we need to divide the p_o value by 100 so that it's a fraction and not a percent.

If you haven't seen dollar sign notation in Excel, the dollar sign will lock the particular value right after it so you can do a fill down or across without the value changing

Note: $C1 will lock the column (C) but not the row, so the row will change if you fill down. C$1 will lock the row (1) but not the column, so the column will change if you fill across. C1 will lock the column and row.

	A	B	C
1			
2	100*po	npo	Pa
3	0.1	0.075	0.0
4	0.2	0.15	0.0
5	0.3	0.225	0.0
6	0.4	0.3	0.0
7	0.5	0.375	0.0
8	0.6	0.45	0.0
9	0.7	0.525	0.0
10	0.8	0.6	0.0
11	0.9	0.675	0.0
12	1	0.75	0.0
13			
14			
15			
16	Desired acceptability		
17		c	12
18		n	75

Figure 1. A sample Excel table.

Hint 2: This plan looks extremely complicated, but here are some tips to help you get started:

- Consider the accept/reject/retest criteria on the first sample. There are only five cases where you need to test again. Write these five cases down.
- Consider the final accept/reject criteria and remember that the allowable number of nonconforming apples in the second sample if the crate is to be accepted is dependent on the number of nonconformities in the first sample.
- Excel is very helpful with this problem. This formula will calculate individual probabilities:
  =BINOMDIST([c],[n],[p_o],FALSE)
  The FALSE designator will return the probability of that particular c and n combination only, not the cumulative probability.
- Let's run through the first point to get you started:
  - Say there are 51 nonconforming apples in the first sample. If we want to accept the crate, there can't be any more than 4 nonconforming apples in the second sample.
  - Say there are 52 nonconforming apples in the first sample. If we want to accept the crate, there can't be any more than 3 nonconforming apples in the second sample.
  - Say there are 53 nonconforming apples in the first sample. If we want to accept the crate, there can't be any more than 2 nonconforming apples in the second sample.
  - Say there are 54 nonconforming apples in the first sample. If we want to accept the crate, there can't be any more than 1 nonconforming apples in the second sample.
  - Say there are 55 nonconforming apples in the first sample. If we want to accept the crate, there can't be any nonconforming apples in the second sample.
- But what we want is the probability of acceptance of the lot at a certain fraction nonconforming of the population. So we have to do a little addition. We need to add all of the bullets below together. Asterisks means multiply.
  - probability of 50 or fewer nonconformities in the first sample
  - (probability of 51 nonconformities in the first sample)*(probability of 4 or fewer nonconformities in the second sample)
  - (probability of 52 nonconformities in the first sample)*(probability of 3 or fewer nonconformities in the second sample)
  - (probability of 53 nonconformities in the first sample)*(probability of 2 or fewer nonconformities in the second sample)
  - (probability of 54 nonconformities in the first sample)*(probability of 1 or fewer nonconformities in the second sample)
  - (probability of 55 nonconformities in the first sample)*(probability of no nonconformities in the second sample)

  These are ALL of the possibilities we can have and still accept a crate. And that's the first point of your double acceptance curve. Repeat as needed for the other points. Each point will have a different fraction nonconforming, but everything else will be the same.