NATIONAL CENTER FOR CASE STUDY TEACHING IN SCIENCE

Through the Grater:

Creating Acceptance Sampling Plans

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Part I – Let the New Kid Try

Milton frowned at the large bag of shredded cheddar cheese he was holding. Although he was excited about his new job as a summer intern at a cheese co-packing facility and was eager to impress his boss with his knowledge and dedication to the job, he had a feeling that something was not right about the sampling procedure he was following.

Well, I'll just have to ask Chuck about this, he'll know, thought Milton. He took a deep breath and walked over to Chuck, who was shouting directions at several of the operators on the line. Although it wasn't that noisy in the production area, Chuck seemed incapable of speaking in a normal tone of voice. With his loud voice, burly physique, and bushy beard, Chuck reminded Milton of a pirate captain. *At least he's a good sort of pirate*, Milton mused. *He's broken all kinds of production records while keeping product quality really high. He'll be able to help me out with this.*

Clutching the large bag of cheese to his chest like a shield, Milton asked, "Hey Chuck, do you know why management has us sampling a bag of cheese for nitrogen and carbon dioxide levels every 15 minutes? I mean, we don't even have to write down what we measure, we just let an operator know. And then, if we're outside spec, we have to open all of the bags and rework the cheese. No one told me why we have to do that, so I was hoping you'd know…" Milton trailed off and squeezed the bag a little more tightly at the look on Chuck's face. *Uh oh, stormy seas for the cap'n*, he thought inanely.

"It's a bunch of horse pucky!" Chuck exploded. "Man, I swear! I think management does whatever it takes to keep us so busy we don't even have time to think," he said as he tried to calm down. "We have all sorts of job requirements that are never fully justified or explained to us."

"Yeah, that sucks," Milton said, doing his best not to flinch. "I think that's why they hired me to work here this summer. I just took a quality control course in college. When I told them about it in my interviews, they told me they really needed my help to make sense out of their quality control procedures."

"Hmph!" was the initial response from Chuck. Milton waited, not willing to risk his ears with another bout of yelling.

After a pause, Chuck squinted down at him. "Ok, college boy," said Chuck in a decidedly lower volume. "I'll believe it when I see it. The last couple of summer interns they hired just made more work for the rest of us. You seem like you have your head screwed on right, though. Now, I like you, but you're going to have to prove yourself here. I work hard all day, every day to make the best cheese out there. And I've gotten used to my bonus at the end of every quarter from getting profit margins higher. So if you've got any smart ideas that will make us more money, let me know. I want to buy a new four-wheeler with my bonus this quarter!"

Milton resisted the urge to flinch at the increase in volume as Chuck mentioned his bonus and a new four-wheeler. *Wow, I didn't think he'd want to listen to me, being the "new kid." Probably just wants to make sure I don't do anything to mess up his bonus. But it's a start, anyway.*

"Okay, Chuck, I've got some ideas about sampling that I want to run by you," Milton said. Watching Chuck's bushy eyebrows start to climb up his forehead, he hastily added, "I want to make sure we're doing the right thing, sampling all these bags. We might be able to sample fewer bags. That'll cut costs for sure. And if we can figure out what affects the gas levels in the bags, we can get better control over the process, which means reworking fewer bags. More money for the company and more free time to focus on the important stuff, then, right?"

Chuck smiled. "Kid, any day I don't have to cut open a couple hundred bags of cheese is a good day. Let's hear it."

Questions

1. What sampling plan design, if any, does the processing plant currently utilize? Explain your choice.

2. What are the advantages of this sampling plan?

3. What are the disadvantages of this sampling plan?

4. Chuck told Milton that he wants the simplest sampling plan with the least amount of quality costs (e.g., training, inspection, record keeping, etc.). Which of the sampling plan designs listed below should Milton suggest? Why?
   a. Single
   b. Double
   c. Multiple
   d. Sequential

5. The company Milton is interning with is a co-packer that fulfills many orders for various buyers in the retail and foodservice sectors. Typically, it takes 1–6 hours to fulfill each order. How might the sampling plan for the cheese need to be changed based on the particular order the company is processing? Think about the needs and specifications of the different customers as you develop your answer.

Part II – Custom Plans for Customers

"So okay, we're going with the single plan. That's your basic point with all that talk and these pictures," said Chuck, thoughtfully stroking his beard while staring at Milton.

Milton, who had spent the last ten minutes explaining exactly why using a single acceptance plan was their best option, held back a sigh. "Yes, that's right. We should use a single acceptance sampling plan."

"Good, so you can work up the plan and we'll run it by the boss," said Chuck. "We can do that tomorrow. Right now, I need to get back to the line."

"Uh, Chuck?" Milton asked hesitantly. "I don't think it's that easy. Most of our customers have different standards for their cheese, so that means that we're probably going to need different sampling sizes and acceptance criteria."

Chuck stared at Milton.

"What I mean is that this isn't a one-size-fits-all kind of thing," Milton said hurriedly. "We're going to have to customize for different standards."

Chuck squinted at Milton for a bit. "Mmmmf. Like those one-size-fits-all bump caps we got last year that didn't fit hardly anyone."

"Sure," said Milton. "But if I can get the specs for the typical orders we get from everyone, I should be able to come up with several sampling plans that we can switch in and out when we need to."

"OK, you do that," said Chuck. "The specs are in the QA office. You can get them and make some copies. Put the originals back as soon as you're done. And don't take too long putting this together; you've got other stuff to do."

"I don't think it'll take me that long," said Milton eagerly. "I'll have something together within the next couple of days."

Questions

1. Construct a single acceptance OC curve for a sample size of 50 and acceptance number of 2. You can use Excel to generate the probabilities or look them up in a table.

2. Propose a change that could be made to the OC curve constructed in the previous question to decrease the probability of acceptance at a higher percent nonconforming in the lot. Sketch how the OC curve would change. Explain how this change would impact management considerations of whether to adopt this sampling plan.

3. Normally the operators take samples directly from the conveyor belt as they test them for nitrogen and carbon dioxide gas levels. However, sometimes they are so busy that they may not be able to sample as product is made and so someone will need to take samples from finished product that is boxed and palletized in the warehouse.

   In the current warehouse setup, each pallet has 6 layers of boxes. Each layer contains 12 boxes, and each box has 6 bags of product. There are 50 pallets in each lot. Explain how Milton could propose a method for randomizing sample selections from pallets of product such that each sample is representative of the entire lot/pallet. Provide an example to support your answer.

Part III – Double Down

Milton was pleased. Management had agreed to try out the single acceptance sampling plan and it had been working pretty well so far. The time they spent sampling was a good deal shorter than it had been when they previously started, which made Chuck happy. On the other hand, they were still taking a fair number of samples.

"Hey Milton!" Milton looked up from his computer at the sound of his name. It was Chuck, and his voice sounded even louder in Milton's tiny cubicle. "Got any more college wizardry for me? We're still opening a truckload of bags every day and reworking all of that product is costing us time and money!"

"Actually, I was thinking about trying a new type of sampling plan," Milton told him. "Come here and see. It's called a double acceptance sampling plan, and it might cut down on the number of bags we have to sample. Here's how it works…"

Questions

1. Chuck agreed to try the double acceptance sampling plan. Construct an OC curve for a double-sampling sampling plan for the following criteria:
   $n1 = 25$ $n2 = 50$
   $c1 = 1$ $c2 = 4$
   $r1 = 3$ $r1 = 5$

2. Explain how the pros and cons of the single- vs. double-acceptance sampling plan. Which one should Milton recommend management incorporate? Why?

Part IV – Everything Has Risk

Milton looked over the double acceptance sampling plan one more time. *Looks pretty good,* he thought. *And Chuck likes it too, so there's that hurdle cleared. I should get the okay from him to go to management with this plan. I probably should do a little more to prove it'll be a better way to sample than they've been doing.*

Milton stared at his computer screen for a minute. *Wait, maybe I can figure out the AQL and LQ for a couple of different plans. They just told me in that meeting a few days ago that some of their customers have really strict quality requirements, and some are more lax about quality. If I can show management that they can adjust their plans to meet all kinds of customer requirements, that'll really sell it!* Milton pushed back his sleeves and set to work.

Questions

1. For the single acceptance sampling plan you developed in Part II, Question 3, calculate the acceptance quality limit (AQL) for a producer's risk of 1% and compare it to a producer's risk of 5%. Draft a memo that Milton could write to his supervisor that explains what AQL is and the differences between these two AQL values.

2. For the plan you developed in Part II, calculate the limiting quality (LQ) for a customer who demands high-quality product and has a risk value of 2%, and a more lenient customer, who has a risk value of 10%. Draft a memo that Milton could write to his supervisor that explains what LQ is and the differences between these two LQ values.

Part V – Quality In, Quality Out

Milton looked over the suggested acceptance sampling plan he had created. *Am I missing anything? Oh, I should do a quick check of the average outgoing quality and outgoing quality level. I'll have to make sure to explain what those mean, but if I can do that, I'll have even more support for this new sampling plan. Now where did I put the formulas for those?*

Questions

1. For the single acceptance sampling plan you developed in Part II, Question 1, calculate the average outgoing quality (AOQ) and average outgoing quality level (AOQL).

2. Draft a memo that Milton could write to his supervisor that explains what AOQ and AOQL are, and how they are useful and important to management.

Part VI – Choices, Choices

Milton was delighted. Management had been very receptive of his proposed sampling plan and was seriously considering accepting it. There had even been mention of a permanent job offer for him! But first, management wanted to take a look at a few more options. What would happen, Milton had been asked, if the producer's risk changed? Or the acceptance quality limit changed? What if these parameters stayed the same but the curve shape changed? Could that happen? They asked Milton to provide them just a few more sampling plans to look at so they could get an idea of how quality and risk changed with different plan parameters.

Questions

1. Management wants to see what an acceptance sampling plan would look like for a producer's risk of 5% and an AQL of 1.4. Propose three different sampling plans that would meet these requirements, and explain the management implications of these different plans.

2. Management also wants to see three options for a sampling plan that would satisfy the requirements of having a consumer's risk of 10% and an LQ of 5.0%. Construct these different sampling plans and explain the management implications of these different plans.

3. Management has looked over all of the sampling plans Milton has provided and decided that they want an LQ of 9% at a consumer's risk of 10% and an AQL of no more than 2.0% at a producer's risk of 5%. They also don't want a sample size larger than 200. Construct three sampling plans that fit these criteria. Which one should Milton recommend to management and why?

Part VII – Training Time

One of the sampling plans that Milton recommended was approved by management! He was told to work with Chuck to implement the sampling plan on Chuck's line to start with, then add in other lines if the trial on Chuck's line was successful. Milton hoped it was, not the least because Chuck would be furious about losing his quarterly bonus from a botched sampling plan. He was also nervous about explaining the plan to the operators on the line. *I'll have to make sure to explain everything simply and clearly. If no one understands what they need to do and why they need to do it the way it's laid out in the sampling plan, the whole thing could fall apart and it'll be a huge mess. Maybe I should make a handout or something for them.*

Questions

1. Milton is working on his handout for the operators, but he's not sure what to include or how to word the handout so the explanation of the sampling plan is easy to follow. Prepare a handout that Milton could use during a training session on the new acceptance sampling plan. This handout should contain the following components:
   a. Why are changes being made to the current sampling plan?
   b. How does the new acceptance sampling plan work?
   c. How were the parameters chosen in the new acceptance sampling plan?

   Note that the operators may not have a strong statistical background, so be sure to clearly explain any statistical terms you use. Assume one of the sampling plans that you created in Part VI, Question 3 was the plan accepted by management.

Part VIII – Distribution Confusion

Milton was feeling pretty good. Chuck had come around to the whole idea of statistically-based sampling and acceptance sampling plans, and the sampling plan that he had suggested was already helping to catch quality problems before they became a major issue on the line or with customers. *I think we're actually taking fewer samples*, he thought. *I should check the numbers from the past week and compare them to a few months ago to be sure.*

On his was to his computer, full of thoughts of record-checking, Milton bumped into Sandra, a new hire in the QA department. He always felt a bit anxious around Sandra, who had a PhD in food science with a statistics minor. Sandra had scrutinized Milton's sampling plans, and Milton often wondered if she was trying to find a mistake to make him look bad.

"Milton, I've been thinking," said Sandra. "Your sampling plans are based on the Poisson distribution, right?" Milton nodded nervously. "Yes, that's the distribution that you have to use when you have a finite lot," he said tentatively.

"But we don't have finite lots!" exclaimed Sandra. "We're a continuous packing operation, which means we have infinite lots and we should be using the binomial distribution. It makes a difference, you know. I hope that those OC curves of yours aren't telling us to do the wrong thing because they're made using the wrong distribution!"

Milton's stomach felt like it was somewhere around his feet. "Ummm…that's how I learned to make them, with the Poisson distribution. And I think ... "

"That's the problem with college kids today: you just do what you're told; you don't think about what you're doing and if it makes sense! I'm going to let the QA manager know about this. We shouldn't be using a plan that's based on bad assumptions." Sandra walked past Milton and down the hallway.

Shaken, Milton did his best to corral his wildly circling thoughts. *I can't have messed this up that badly; I know I thought about this carefully and looked at tons of data. The Poisson distribution is the right one, I know it! I just have to remember why I made that choice…*

Questions

1. In what cases would you want to use the Poisson distribution to create acceptance sampling plans? What about the binomial distribution? Provide an example of a process in which you would use the Poisson distribution and a process in which you would use the binomial distribution in your answer.

2. Construct an OC curve with a sampling size of 30 and an acceptance criteria of 1 using the Poisson distribution to calculate the probabilities of acceptance. Create a second OC curve with the same sample size and acceptance criteria, but use the binomial distribution to calculate the probabilities of acceptance. How do the OC curves compare?

3. Repeat Question 2 but use a sample size of 300 and an acceptance criteria of 1. How do the OC curves compare?

4. Using the results of the OC curves in questions 2 and 3, what can you say about using the Poisson distribution versus the binomial distribution?

5. Who is right in this scenario, Milton or Sandra? Explain your answer.

OC Curve Generation for Single Acceptance Plans

There are two main ways to generate the OC curves for this case study. The first way is to use the Poisson distribution tables to determine the probability of acceptance at each given percent nonconforming in the lot, sample size, and acceptance criteria. The second way is to use Excel to perform those calculations for you. You can use the following function in Excel to create points for your OC curve:

=POISSON.DIST([c],[np$_o$],TRUE)

What this formula does is use the binomial distribution to calculate the probability of accepting a lot at a given population fraction nonconforming. Here's how to put the formula into Excel:

- Replace [*c*] with the cell that contains the acceptance criteria number.
- Replace [np_o] with the cell that contains the sample size multiplied by the fraction nonconforming in the population.
- Leave TRUE alone, as it means that the function will calculate the cumulative probability (which is what you want).

In terms of what this might look like, below is a sample Excel table. Here, p_o is the fraction of nonconforming units in the population, *n* is the number of units in each sample (meaning that npo is n multiplied by p_o), and *Pa* is the probability of acceptance. Green cells are cells that can be changed manually, every other cell is calculated. Note that the *Pa* column is set to 0.0 to prevent answers from being copied.

	A	B	C
1			
2	100*po	npo	Pa
3	0.1	0.075	0.0
4	0.2	0.15	0.0
5	0.3	0.225	0.0
6	0.4	0.3	0.0
7	0.5	0.375	0.0
8	0.6	0.45	0.0
9	0.7	0.525	0.0
10	0.8	0.6	0.0
11	0.9	0.675	0.0
12	1	0.75	0.0
13			
14			
15			
16	Desired acceptability		
17		c	12
18		n	75

If we wanted to calculate the probability of acceptance for the first row (100*p_o=0.1), the formula would be:

=POISSON.DIST(C17,C3,TRUE)

If you haven't seen dollar sign notation in Excel, the dollar sign will lock the particular value right after it so you can fill down or across without the value changing

Example: $C1 will lock the column (C) but not the row, so the row will change if you fill down. C$1 will lock the row (1) but not the column, so the column will change if you fill across. C1 will lock the column and row.

OC Curve Generation for Double Acceptance Plans

You can use the same two methods to calculate OC curves for double acceptance sampling plans. Calculation of double acceptance sampling plans is a little more complex, since you need to add probabilities together. Here are some tips to help you get started:

- Consider the accept/reject/retest criteria on the first sample. Under what conditions would you need to test again? Write each separate case down.
- Consider the final accept/reject criteria and remember that if the sample is to be accepted, the allowable number of nonconforming units in the second sample is dependent on the number of nonconformities in the first sample.
- Excel is very helpful with this problem. This formula will calculate individual probabilities:

  =POISSON.DIST([c],[np_o],FALSE)

The FALSE designator will return the probability of that particular *c* and np_o combination only, not the cumulative probability.

As an example, let's examine a sampling plan where c1 is 2, c2 is 4, and r1 is 5.

- We can accept the lot on the first sample if there are 2 or fewer nonconforming units in our sample.
- We can reject the lot on the first sample if there are 5 or more nonconforming units in our sample.
- If there are 3 or 4 nonconforming units in the first sample, we need to resample.
  - If we want to accept the lot, there can't be any more than 1 nonconforming unit in the second sample.
  - Say there are 3 nonconforming units in the first sample. If we want to accept the lot, there can't be any more than 1 nonconforming unit in the second sample.
  - Say there are 4 nonconforming units in the first sample. If we want to accept the lot, there can't be any nonconforming units in the second sample.

But what we want is the total probability of acceptance of the lot at a certain fraction nonconforming of the population. So we have to do a little addition. We need to add all of the bullets below together. Asterisks means multiply.

- probability of 2 or fewer nonconformities in the first sample
- (probability of 3 nonconformities in the first sample)*(probability of 1 or fewer nonconformities in the second sample)
- (probability of 4 nonconformities in the first sample)*(probability of no nonconformities in the second sample)

These are *all* of the possibilities we can have and still accept a lot. And that's the first point of your double acceptance curve. Repeat as needed for the other points. Each point will have a different fraction nonconforming, but everything else will be the same.